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Section: A

Paper: Differential Equation

Examination: Mid-Term

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Q1 Solve the following Objective type questions.

i The order of matrix A is $m \times p$ and the order of matrix B is $p \times n$. Then the order of matrix AB is ?

Solution:

The order of matrix AB = $m \times n$

ii The number of non-zero rows in an Echelon form?

Solution:
The number of non-zero rows in an echelon form is called rank of a matrix.

iii If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular

matrix then $a = ?$

Solution (iii)

For Singular Matrix

$|B| = 0$

$$\begin{vmatrix} 1 & 4 \\ 2 & a \end{vmatrix} = 0$$

$$(1 \times a) - (4 \times 2) = 0$$

$$\Rightarrow a - 8 = 0$$

$$\boxed{\Rightarrow a = 8}$$

iv. If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Solution:

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$|A| = 2i(-i) - i(i)$

$= -2i^2 - i^2$ $\quad \because i^2 = -1$

$= -2(-1) - (-1)$

$= 2 + 1$

$= 3$

v. The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is ?

Solution:

Scaler Matrix

vi. Solution of $\frac{dy}{dx} + 2xy = y = ?$

Solution:

$$\frac{dy}{dx} + 2xy = y$$

$$\Rightarrow \frac{dy}{dx} = y - 2xy$$

$$\Rightarrow \frac{dy}{dx} = y(1 - 2x)$$

$$\Rightarrow \frac{dy}{dx} = y(1-2x)$$

$$\Rightarrow \frac{dy}{y} = (1-2x)\,dx$$

$$\Rightarrow \int \frac{1}{y}\,dy = \int 1\,dx - \int 2x\,dx$$

$$\Rightarrow \ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\Rightarrow \boxed{\ln y = x - x^2 + c}$$

vii. The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1+\left(\frac{dy}{dx}\right)^2} \text{ is ?}$$

Solution:

Order = 1

Degree = 3

viii. The order and degree of differential equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right)$$ is ?

Solution:

Order = 2

Degree = 1

(ix) The differential equation

$$2\frac{dy}{dx} + x^2y = 2x+3 \ , \ y(0) = 5$$ is ?

Solution:

$$2\frac{dy}{dx} + x^2y = 2x+3$$

$$\Rightarrow 2\frac{dy}{dx} = 2x+3-x^2y$$

$$\Rightarrow 2\,dy = (2x+3-x^2y)\,dx$$

Integrating both sides

$$\int 2dy = \int (2x + 3 - x^2y)\,dx$$

$$2\int 1\,dy = \int 2x\,dx + \int 3\,dx - \int x^2y\,dx$$

$$\Rightarrow 2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + c$$

$$\Rightarrow y = \frac{\cancel{2}x^2}{\cancel{2}\times 2} + \frac{3x}{2} - \frac{x^3y}{3\times 2} + c$$

$$\Rightarrow y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + c \quad \text{—— (A)}$$

put $x = 0$, $y = 5$

$$5 = 0 + 0 - 0 + c$$

$c = 5$ then eq (A) becomes

$$\Rightarrow y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3y}{6} + 5$$

(x) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$ is?

Solution:

$$|A| = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by R_1

$= \cancel{bc^2}$

$$1\begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a\begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2\begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$= bc^2 - cb^2 - a(c^2 - b^2) + a^2(c-b)$

$= bc(c-b) - a(c-b) + a^2(c-b)$

$= (c-b)(bc - a + a^2)$

Q2 (i) Express the determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the product of factors which are linear in a, b, c.

Solution:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

⑨

$\Rightarrow ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c$

Taking common abc

$\Rightarrow abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$

$\Rightarrow abc[bc(c-b) - ac(c-a) + ab(b-a)]$

Ans

Q2: (ii) Find the Eigen value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

The characteristic equation for solution is

$$|A - \lambda I| = 0 \quad \text{— (A)}$$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determinant which is

$$|A - \lambda I| = 0$$

(11)

$$|A-\lambda I| = \begin{vmatrix} 2-\lambda & -1-0 & -1-0 & 0-0 \\ -1-0 & 3-\lambda & -1-0 & -1-0 \\ -1-0 & -1-0 & 3-1 & -1-0 \\ 0-0 & -1-0 & -1-0 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$\Rightarrow (2-\lambda)\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 - ⓐ$$

Again expand the matrix

$$\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$$

Expand by R_1

$$\Rightarrow (3-\lambda)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix}$$

$$-1\begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$\Rightarrow (3-\lambda)\Big[\big((3-\lambda)(2-\lambda)-(-1)(-1)\big)+1\big((-1)(2-\lambda)-(-1)(-1)\big)-1\big((-1)(-1)-(-1)(3-\lambda)\big)\Big]$$

$$\Rightarrow (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)-(+1+3-\lambda)$$

$$\Rightarrow (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$\Rightarrow 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$\Rightarrow \boxed{-\lambda^3+8\lambda^2-18\lambda+8} \longrightarrow *$$

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+1(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$\Rightarrow \boxed{-\lambda^2+6\lambda-8} \longrightarrow (B)$$

$$\Rightarrow -1\begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

Expand by C_1

$$-\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$\Rightarrow -[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)$$

14

$= -(3-\lambda+\lambda^2-5\lambda+5)$

$= -\lambda^2+5\lambda-5-3+\lambda$

$= \boxed{-\lambda^2+6\lambda-8} \longrightarrow$ C

Put ~~eq~~, e.q ✱, B and C in
(a)

$(2-\lambda)[-\lambda^3+8\lambda^2-18\lambda+8]-\lambda^2+6\lambda-8-\lambda^2+6\lambda-8$

Q3.

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$x = 2, \quad y = 6$$

Solution:

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$\Rightarrow (x^2+3y^2)dx = 2xy\,dy$$

Dividing both sides by $2xy\,dx$
we get

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^2}{2xy} + \frac{3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \quad \text{——— (A)}$$

Let $y = vx$

Differentiste w.r.t to x

$$\frac{dy}{dx} = \frac{v\cancel{dx}}{\cancel{dx}} + \frac{x\,dv}{dx}$$

$$\frac{dy}{dx} = v + \frac{x\,dv}{dx} \longrightarrow (B)$$

Put eq (B) in (A)

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{\cancel{x}v} + \frac{3(v\cancel{x})}{\cancel{x}}\right]$$

$$v + \frac{x\,dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying both sides by 2

$$2v + \frac{2x\,dv}{dx} = \frac{1}{v} + 3v$$

$$\frac{2x\,dv}{dx} = \frac{1}{v} + 3v - 2v$$

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$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying both sides by dx we get

$$2x\,dv = \frac{1+v^2}{v}dx$$

Multiplying both sides by $\frac{v}{x(1+v^2)}$

we get

$$\frac{v}{\cancel{x}(1+v^2)} \times 2\cancel{x}\,dv = \frac{\cancel{1+v^2}\,dx}{\cancel{v}} \times \frac{\cancel{v}}{x(\cancel{1+v^2})}$$

$$\Rightarrow \frac{2v\,dv}{1+v^2} = \frac{1}{x}dx$$

Integrating both sides

$$\int \frac{2v}{1+v^2}dv = \int \frac{1}{x}dx + C$$

$$\ln(1+v^2) = \ln x + \ln c$$

(18)

$\cancel{\ln}(1+v^2) = \cancel{\ln}(xc) \quad \because \ln(mn) = \ln m + \ln n$

$\Rightarrow 1+v^2 = xc$ — (B)

put $v = \frac{y}{x}$ in (B)

$\Rightarrow 1+\left(\frac{y}{x}\right)^2 = xc$

$\Rightarrow 1+\frac{y^2}{x^2} = xc$

$\Rightarrow \frac{x^2+y^2}{x^2} = xc$

$\Rightarrow x^2+y^2 = x^3c$ — (C)

put $x=2, y=6$ in (C)

$\Rightarrow (2)^2+(6)^2 = (2)^3c$

$\Rightarrow 4+36 = 8c$

$\Rightarrow 8c = 40$

$\boxed{\Rightarrow c = 5}$

So by putting the value of C in e.q (c), it becomes

$$x^2+y^2=5x^3$$
$$y^2=5x^3-x^2$$

$$y^2=5x^3-x^2$$

Taking "$\sqrt{\ }$" on b. sides

$$\boxed{y=\pm\sqrt{5x^3-x^2}}$$

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