**Department of Electrical Engineering**

**Assignment**

**Date: 13/04/2020**

**Course Details**

| | | | |
|---|---|---|---|
| **Course Title:** | Digital Signal Processing | **Module:** | 6th |
| **Instructor:** | Pir meher ali sha | **Total Marks:** | 30 |

**Student Details**

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|---|---|---|---|
| | (a) | Consider the following analog signal<br>$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$<br>i. Determine the minimum sampling rate required to avoid aliasing.<br>ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal.<br>iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation? | Marks 5<br>CLO 1 |
| | (b) | Consider a discrete time signal which is given by | Marks 5 |

| | | | |
|---|---|---|---|
| | (b) | Compute the convolution y(n) of the following signal<br>$$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$<br>$$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$ | Marks 5<br>CLO 2 |
| Q3. | | Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC).<br>i. $$x(n) = \begin{cases} (\frac{1}{4})^{n}, & n \ge 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$$<br>ii. $$x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$$ | Marks 10<br>CLO 2 |

Q1(a)

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) Minimum Sapling rate?
(ii) Suppose $F_s = 100\,Hz$. Obtained signal after Sampling?
(iii) What is $y_a(t)$ we can reconstruct from samples if we use ideal interpolation?

Solution: → (i) The Minimum Sampling rate is. ~~we also~~ find by the Nyquist Criteria.

As. $$f_s \geq 2 f_{max}$$

we know

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi}$$

$\omega_1 = 100\pi$ $\qquad\qquad$ $\omega_2 = 200\pi$

$f_1 = \frac{100\pi}{2\pi}$ $\qquad\qquad$ $f_2 = \frac{200\pi}{2\pi}$

$f_1 = 50\,Hz$ $\qquad\qquad$ $f_2 = 100\,Hz$

As $f_2 > f_1$ so $f_2$ is max

$$f_s \geq 2 \times 100\,Hz$$

$$f_s \geq 200\,Hz$$

Hence proved the Sampling frequency avoid aliasing effect.

Q'9 (ii) if $F_s = 100Hz$ obtained signal after Sampling?

Sol

\# As we know

$$F_s = 100Hz$$

As the sampling frequency is now 100 Hz then $f_1$ and $f_2$ will become

$$f_1' = \frac{f_1}{F_s} \qquad f_2' = \frac{f_2}{F_s}$$

$$f_1' = \frac{50}{100} = 0.5Hz \qquad f_2' = \frac{100}{100} = 1Hz$$

Thus By changing $f_1$ and $f_2$ the angular frequency will also changed and becomes

$$\omega_1' = 2\pi f_1' \qquad \omega_2' = 2\pi f_2'$$

$$\omega_1' = 2\pi(0.5Hz) \qquad \omega_2' = 2\pi(1Hz)$$

$$\omega_1' = \pi \qquad \omega_2' = 2\pi$$

So the obtained signal after sampling is

$$x[n] = 3\cos\pi n + 4\sin 2\pi n$$

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Q1(a)(iii) $Y_a(t)$ we can reconstruct from samples if we use ideal interpolation?

Sol

$$x[n] = 3\cos 100\pi n + 4\sin 200\pi n$$

As we know

$$\text{Folding frequency} = \frac{F_s}{2}$$

$$F_f = \frac{100}{2} = 50Hz$$

The orignal signal frequencies are

$$f_1 = 50Hz \quad \& \quad f_2 = 100Hz$$

So Both frequencies $f_1$ and $f_2$ are either equal or greater than the Folding frequency

Hence for ideal interpolation we can construct the orignal signal

$$y_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

The orignal signal is constructed Because we use sampling frequency at Nyquist rate. We can also reconstruct the signal for sampling again at Nyquist rate above.

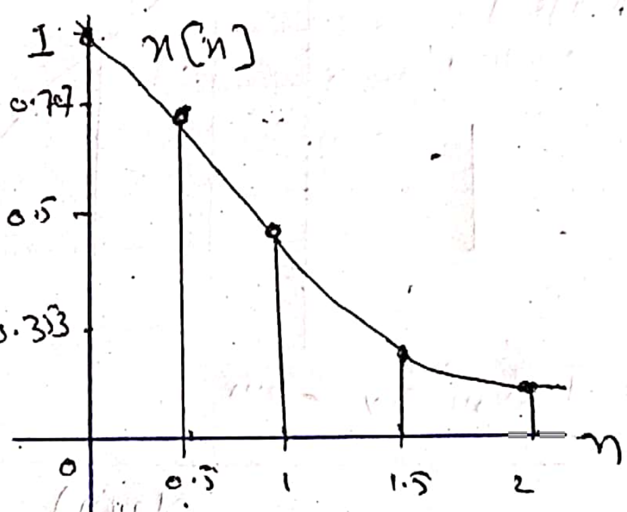
Q1(b):-

Sol
\# (i) $F_s = 2Hz$

$$F_s = \frac{1}{T}$$

$$T = \frac{1}{F_s} = \frac{1}{2} = 0.5 sec$$

| $n$ | $x[n]$ |
|---|---|
| 0 | $0.5^{0} = 1$ |
| 0.5 | $0.5^{0.5} = 0.707$ |
| 1 | $0.5^{1} = 0.5$ |
| 1.5 | $0.5^{1.5} = 0.353$ |




(ii) Quantization level
we need to do it for 3bit
as we know $L = 2^n$
$n$ = bit which is 3
$L = 2^3 = 8$
we have 8 levels

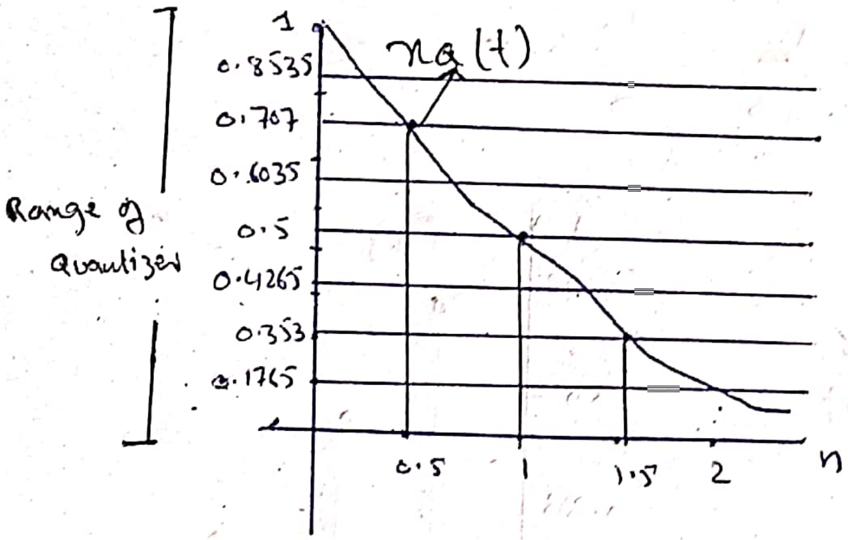
Ali Raza Khan 12647 5

Now for Resolution

$$= \frac{X_{max} - X_{min}}{L}$$

Putting values

$$= \frac{1 - 0}{8} = 0.125$$




(iii) Tabular form

| n | | x(n) | xq(n) Truncation | xq(n) Rounding | xq(n) − x(n) |
|---|---|---|---|---|---|
| 0 | 0 | 1 | 1.0 | 1.0 | 0.0 |
| 0.5 | 1 | 0.8535 | 0.8 | 0.9 | 0.0465 |
| | 2 | 0.707 | 0.7 | 0.7 | −0.07 |
| 1.5 | 3 | 0.6035 | 0.6 | 0.6 | −0.035 |
| 2 | 4 | 0.5 | 0.5 | 0.5 | 0.0 |
| | 5 | 0.4265 | 0.4 | 0.4 | −0.0265 |
| | 6 | 0.353 | 0.3 | 0.4 | 0.047 |
| | 7 | 0.1765 | 0.1 | 0.2 | 0.0235 |

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(6)

Q2 (a):- Determine the response of system with given impulsive response.

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$$

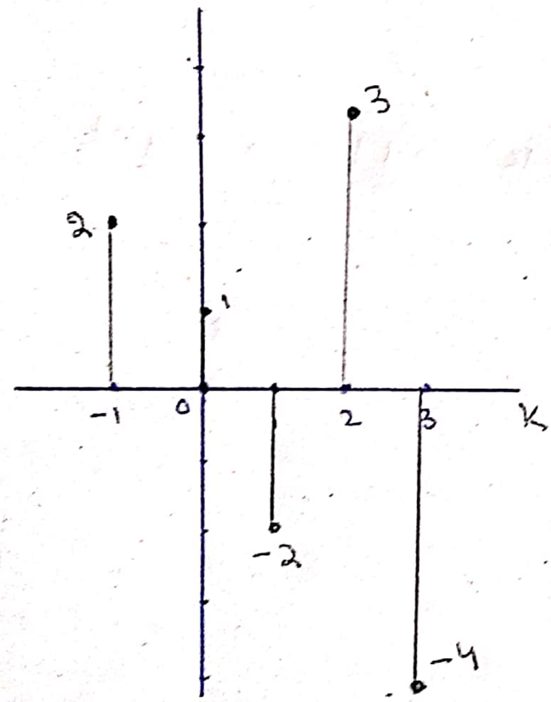
$$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

Sol

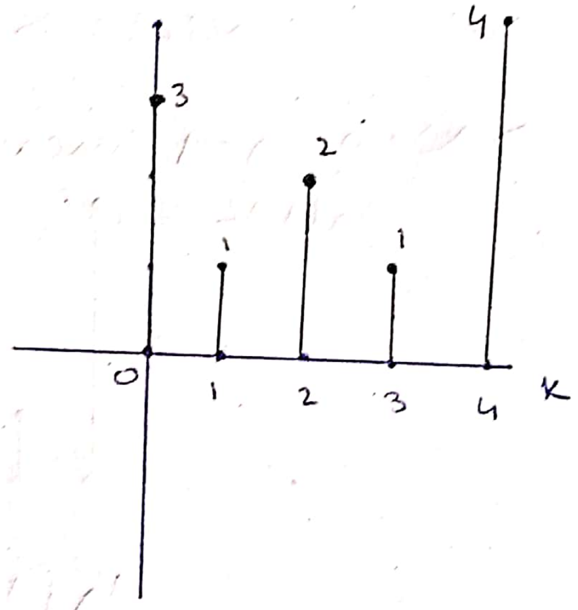
\# As we know

$$Y[n] = \sum_{K=\infty}^{\infty} x[K]\,h[n-K]$$

(i) change variable n by K




$x[K]$




$h[K]$

(ii) Now fold $h[K]$ to get $h[-K]$

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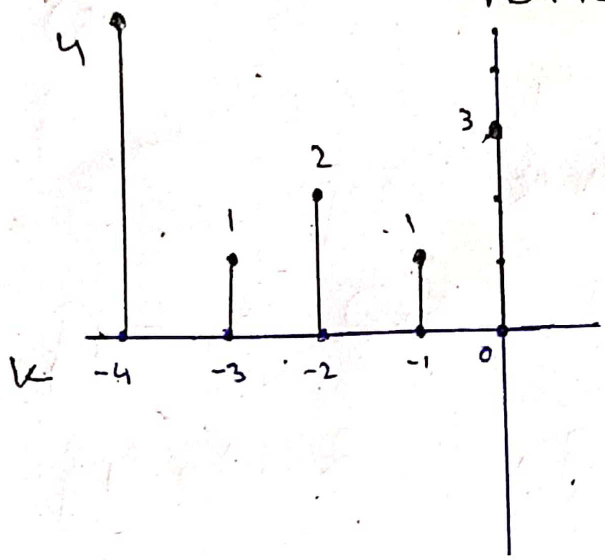
7




$h[-k]$

for $n=0$

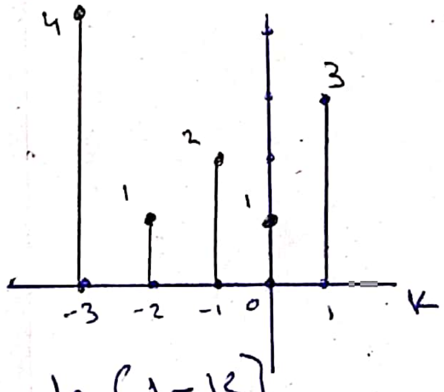
$$Y[0] = \sum_{k=-1}^{0} x(n)\,h(n-k]$$

$$Y[0] = x(-1)\,h(-1) + x(0]\,h[0]$$

$$Y[0] = 2(1) + (1)(3)$$

$$Y[0] = 5$$

→ Now $n=1$ mean shif $h(-k)$ by 1 to get $h(1-k$




$h[1-k]$

$$Y[1] = \sum_{k=-1}^{1} x[n]\,h[n-k]$$

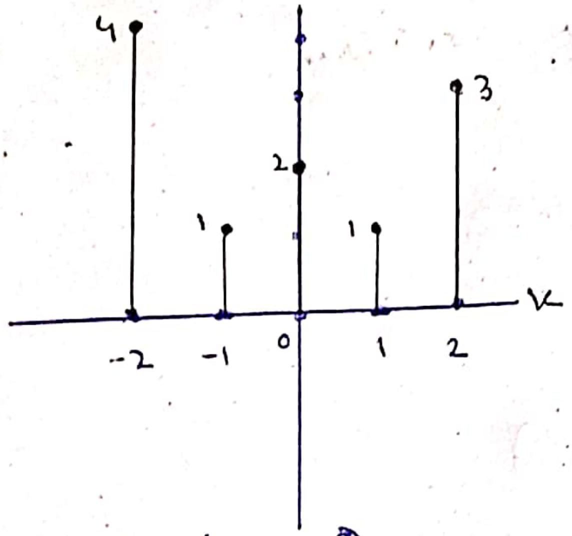
$$Y[1] = x[-1]h[-1] + x[0]h[0] + x(1)\,h(1)$$

$$Y(1) = (2)(2) + (1).(1) + (-2)(3)$$

$$Y[1] = 4 + 1 - 6$$

$$Y[1] = -1$$

Now for $n=2$ Ali Rajarkham 12647 (8)




$h(2-k)$

$$Y[2] = \sum_{k=-1}^{2} x[n]\,h[n-k]$$

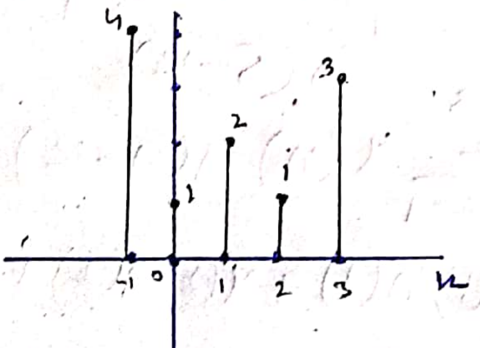
$$Y[2] = x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$$

$$= (2)(1) + (1)(2) + (-2)(1) + (3)(3)$$

$$= 2+2-2+9$$

$$Y[2] = 11$$

Now for $n=3$




$h[3-k]$

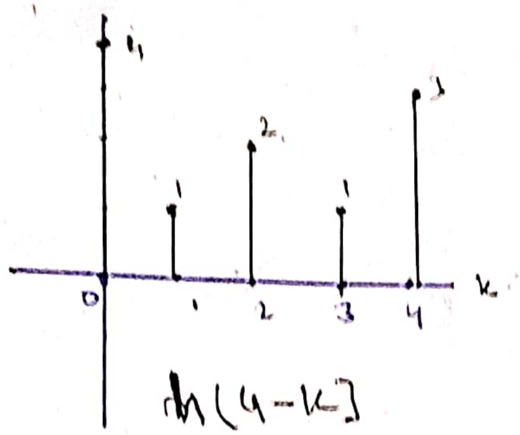
$$Y[3] = \sum_{k=-1}^{3} x[n]\,h[n-k]$$

$$Y[3] = x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$Y(3) = (1)(4) + (1)(1) + (-2)(2) + (3)(1) + (3)(-4$$

$$Y(3) = 4+1-4+3-12 = -8$$

for $n=4$



$h(4-k)$

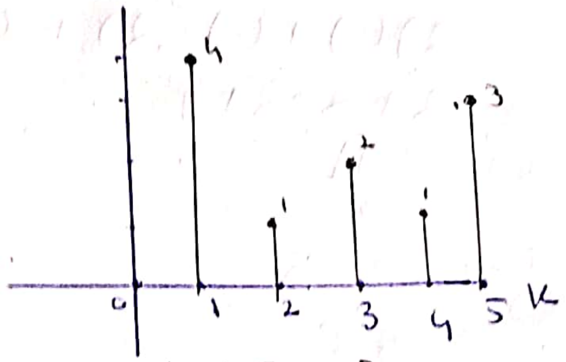
$$Y[4] = \sum_{k=0}^{3} x(n)h(n-k)$$

$$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= (1)(4) + (-2)(1) + (3)(2) + (-4)(1)$$

$$Y[4] = 4$$

for $n=5$



$h(5-k)$

$$y[5] = \sum_{k=1}^{3} x(n)h(n-k)$$

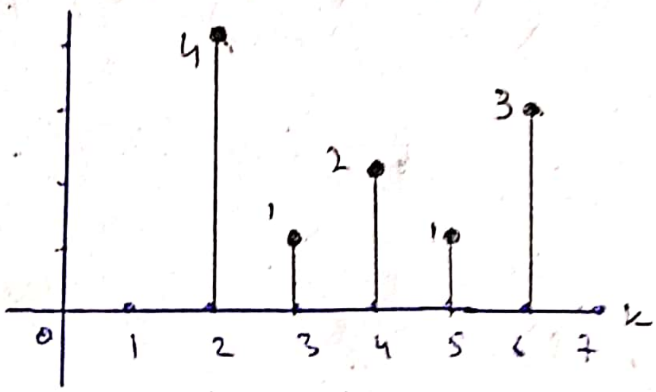
$$y[5] = x(1)h(1) + x(2)h(2) + x(3)h(3)$$

$$= (-2)(4) + (3)(1) + (-4)(2)$$

$$y[5] = -8+3-8$$

$$y[5] = -13$$

for $n=6$ 6 shift to right




$h(6-k)$

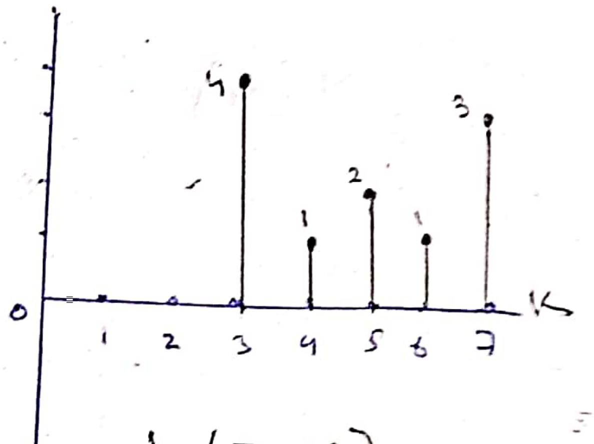
$$y[6] = \sum_{k=2}^{3} x(n]\,h(n-k]$$

$$y(6) = x(2)h(2) + x(3)h(7)$$

$$= (3)(4) + (-4)(1)$$

$$= 12-4 = 8$$

for $n=7$




$h(7-k)$

$$y[7] = \cancel{\sum_{k=3}}\, x(3)\,h(3)$$

$$= (-4)(4)$$

$$y(7) = -16$$

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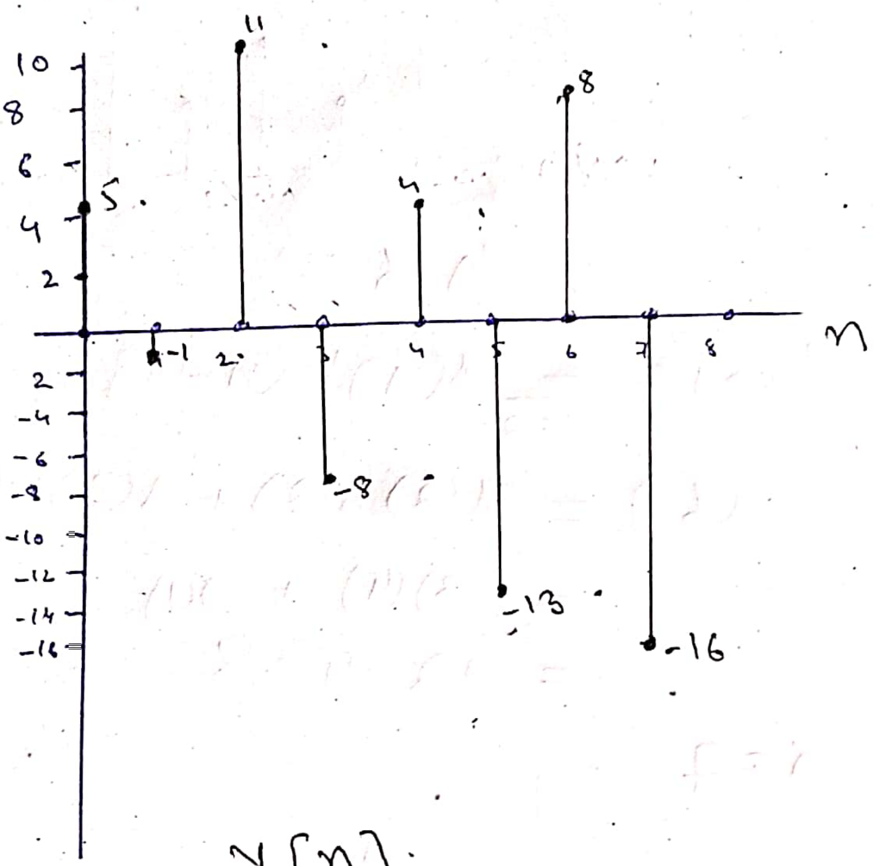
$$y[n] = \{\underset{\uparrow}{5}, -1, 11, -8, 4, -13, 8, -16\}$$




Y[n]

x x

(Ali Raza Khan 12647) (12)

Q2(b) :- Compute the convolution $y[n]$ of the following signal

$$x[n] = \begin{cases} \alpha^{n+1} & -3 \le n \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

$$h[n] = \begin{cases} 2^n & 0 \le n \le 4 \\ 0 & \text{elsewhere} \end{cases}$$

Sol
#

$$x(n) = [\alpha^{-3+1}, \alpha^{-2+1}, \alpha^{-1+1}, \alpha^{0+1}, \alpha^{1+1}, \alpha^{2+1}, \alpha^{3+1}, \alpha^{4+1}, \alpha^{5+1}]$$

$$x(n) = [\alpha^{-2}, \alpha^{-1}, \alpha^{0}, \alpha^{1}, \alpha^{2}, \alpha^{3}, \alpha^{4}, \alpha^{5}, \alpha^{6}]$$

$$h[n] = [1, 2, 4, 8, 16]$$

Convolution formula

$$y[n] = \sum_{k=0}^{n} x(k)\, h(n-k)$$

Therefore

$$y[-2] = \alpha^{-2}$$

$$y[-1] = \alpha^{-2} + \alpha^{-1}$$

$$y[0] = \alpha^{-2} + \alpha^{-1} + 1$$

$$y[1] = \alpha^{-2} + \alpha^{-1} + 1 + \alpha$$

$$y[2] = \alpha^{-2} + \alpha^{-1} + 1 + \alpha^{1} + \alpha^{2}$$

$$y[3] = \alpha^{-1} + 1 + \alpha + \alpha^{2} + \alpha^{3}$$

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13

$y[4] = \alpha^4 + \alpha^3 + \alpha^2 + \alpha + 1$

$y[5] = \alpha + \alpha^2 + \alpha^3 + \alpha^4 + \alpha^5$

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Q3:(a)

Determine the z-transform of the following Signals and also Sketch the Region of Convergence (ROC).

(i) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n} & n < 0 \end{cases}$

Sol #

$$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=-\infty}^{0} \left(\frac{1}{3}\right)^{-n} z^{-n} - 1$$

Using the Geometric Series

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{n} - 1$$

$$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z} - 1$$

$$= \frac{1-\frac{1}{3}z + 1-\frac{1}{4}z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)} - 1$$

$$= \frac{1-\frac{1}{3}z + 1-\frac{1}{4}z^{-1} - (1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1-\frac{1}{3}z + 1-\frac{1}{4}z^{-1} - 1+\frac{1}{3}z + \frac{1}{4}z^{-1} + \frac{1}{2}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

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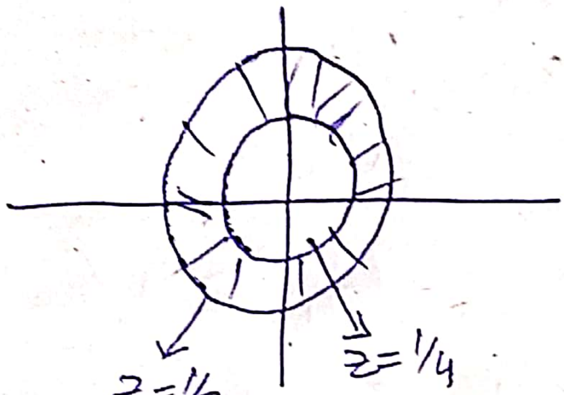
Hence the Region of Convergence is

$\frac{1}{4} < |z| < \frac{1}{3}$




(ii)

Q3(b) $x(n) \begin{cases} (\frac{1}{2})^n - 3^n & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$

Sol

$$x(n) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

Using geometric series to Simplify

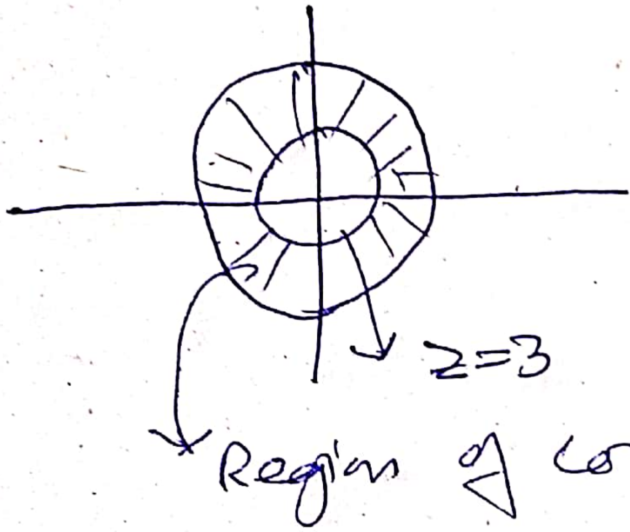
$$= \frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

$$= \frac{\cancel{1} - 3z^{-1} - \cancel{1} + \frac{1}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

$$= \frac{-\frac{5}{2}z^{-1}}{(1-\frac{1}{2}z^{-1})(1-3z^{-1})}$$

Ali Raza Khan. 12647 (18)

Hence the Region of convergence is $|z| > 3$

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