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Assignment :- Digital signal processing

Q1 a)-

Sol:- Consider the difference equation

$$y(n)-3y(n-1)-4y(n-2)=x(n)+2x(n-1) \rightarrow (1)$$

The homogenous eq. of the system is

$$y(n)-3y(n-1)-4y(n-2)=0.$$

The chara- eq of the system is

$$\lambda-3\lambda^{-1}-4\lambda^{2}=0$$

$$\lambda^{2}-3\lambda-4=0.$$

Determine the root of the chara. eq.

$$\lambda^{2}-4\lambda+\lambda-4.$$

$$\lambda(\lambda-4)+1(\lambda-4).$$

$$(\lambda-4)(\lambda+1).$$

$$\lambda=4,\ \lambda=-1 \Rightarrow \lambda=4 \ \& \ \lambda=-1$$

The homogenous sol is :-

$$y_h(n)=C_1(-1)^n u(n)+C_2(4)^n u(n).$$

Since 4 is a characteristic root of the excitation is :-

$$x(n)=4^n u(n).$$

We assume a particular solution of the form

$$y_p(n)=Kn4^n u(n).$$

Then

$$Kn\,4^n u(n) - 3k(n-1)4^{n-1}u(n-1) - 4k(n-2)\,4^{n-2}u(n-2)$$
$$= 4^n u(n) + 2(4)^{n-1}\,u(n-1).$$

For $n=2$

$$K(32-12) = 4^2+8 = 24 \rightarrow \quad K = 6/5$$

The total solution is

$$y(n) = yp(n) + yh(n).$$
$$= \left[6/5\, n4^n + c_1 4^n + c_2(-1)^n\right] u(n).$$

To solve for c_1 & c_2 we assume that $y(-1) = y(-2) = 0$ Then

$$y(0) = 1 \quad \& \quad y(1) = 3y(0) + 4 + 2 = 9$$

Hence

$$c_1 + c_2 = 1 \quad \&$$
$$24/5 + 4c_1 - c_2 = 9$$
$$4c_1 - c_2 = 21/5$$

therefore

$$(c_1 = 26/25) \quad \& \quad c_2 = (-1/25)$$

The total solution is:

$$y(n) = \left[6/5\, n4^n + 26/25\, 4n - 1/25(-1)^n\right] u(n).$$

x ~~★~~ x

Q1:-

b).

Sol:- Determine the difference equation.

$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$.

$y(n) = 0.6y(n-1) + 0.08y(n-2) = x(n)$.

To obtain homogenous equation set input.

$x(n) = 0$.

$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0$.

Determine the solution to the homogenous equation.

$\lambda^n - 0.6\lambda^{n-1} + 0.08\lambda^{n-2} = 0$.

$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$.

$\lambda^2 - 0.6\lambda + 0.08 = 0$.

$(\lambda - 0.2)(\lambda - 0.4) = 0$.

Therefore, roots are

$\lambda_1 = 0.2, \quad \lambda_2 = 0.4$

Thus, the general form of the solution to the homogenous equation is:

$y_h(n) = c_1(\lambda_1)^n + c_2(\lambda_2)^n$.

$y(n) = c_1(0.2)^n + c_2(0.4^n) \longrightarrow (1)$

$\lambda = 0.2, \quad \lambda = 0.4$ Hence

$y_h(n) = c_1 \frac{1^n}{5} + c_2 \frac{2^n}{5}$.

with $x(n) = \delta(n)$ the initial condition are

$y(0) = 1,$

$y(1) - 0.6\, y(0) = 0.$

$y(1) = 0.6$

Hence $c_1 + c_2 = 1$ & $\frac{1}{5} c_1 + \frac{2}{5} = 0.6$

$\Rightarrow c_1 = -1, \quad c_2 = 3$

therefore $h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n).$

the step response is:

$$S(n) = \sum_{k=0}^{n} h(n-k), \quad n > 0.$$

$$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right].$$

$$= \left\{ \frac{1}{0.12}\left[\frac{2^{n+1}}{5} - 1\right] - \frac{1}{0.16}\left[\left(\left(\frac{1}{5}\right)^{n+1} - 1\right)\right] ub \right\}$$

Q2:-
a).-

Sol:- The Z-transform is

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

The expression is written as:-

$$Y(z) = \frac{1}{(1-\frac{2}{z})(1-\frac{1}{z})^2}$$

$$= \frac{1}{\left(\frac{z-2}{z}\right)\left(\frac{z-1}{z}\right)^2}$$

$$= \frac{1}{\frac{(z-2)(z-1)^2}{z^3}}$$

$$= \frac{z^3}{(z-2)(z-1)^2}$$

$X(z)$ has a simple pole at $p_1 = 2$ & a double $p_2 = p_3 = 1$ In such case the approcipate partial expansion

$$Y(z) = \frac{z^3}{(z-2)(z-1)^2} = \frac{A_1}{(z-2)} + \frac{A_2}{(z-1)} + \frac{A_3}{(z-1)^2}.$$

The problem is to determine the co-efficient A_1, A_2, A_3

we proced as in the case of the distinct pole to determine A_1, we multiply both side of $3y(z-2)$ & evaluate the result $z=-2$.

$$(z-2)\,x(z) = \frac{A_1 + z - 2}{z-1} A_2 + \frac{z-2}{(z-1)^2} A_3.$$

which we evaluate at $z=2$

$$A_1 = \frac{(z-2)\,x(2)}{2}\Big|_{z=2}$$

$(A_1 = 4)$, $A_2 = A_1 + \frac{z-2}{z-1}$

$(A_2 = -3)$

$$A_3 = \frac{A_1 + z - 2}{z-1} A_2$$

$\therefore\ x(n) = [4[2]^n - 3 - n]\,u(n).$

$(= -1)$

x ~~~~ * ~~~~ x

Q2
b)
Ans: First we eliminate the negative power by "xing" both num. & den. by z^2 Thus,-

$$x(z) = \frac{z^2}{z^2 - 1.5z + 0.5}.$$

The poles $X(z)$ are $P_1 = 1$ & $P_2 = 0.5$ consequently the expansion of the form

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

A very simple method to determine A_1 & A_2 to multiply the equation by the deterministic term $(z-1)(z-0.5)$ thus we obtain

$$z = (z-0.5)A_1 + (z-1)A_2 \rightarrow (1)$$

Now if we set $z = p_1 = 1$ in eq (1) we eliminate the term involving A_2. Hence $1 = (1-0.5)A_1$.

Thus we obtain the result $A_1 = 2$

Next we return eq (1) & $z = p_2 = 0.5$

Thus eliminating the term involving A_1, so we have.

$$0.5 = (0.5-1)A_2$$

And hence $A_2 = -1$, therefore the result of the partial fraction expansion is $\frac{X(z)}{z} = \frac{z}{z-1} - \frac{1}{z-0.5}$ Ans

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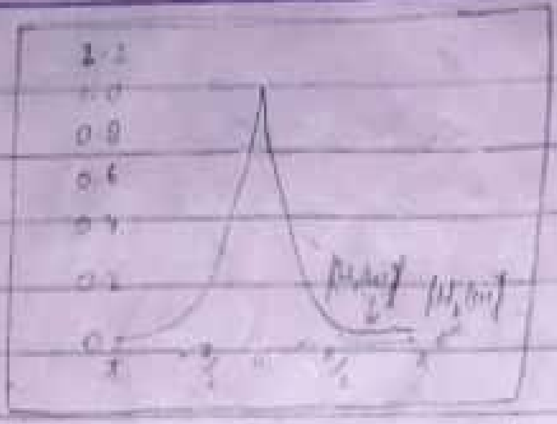
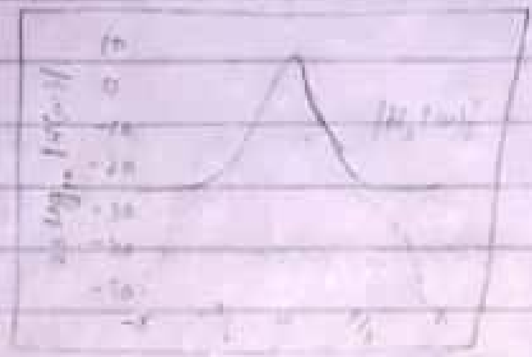
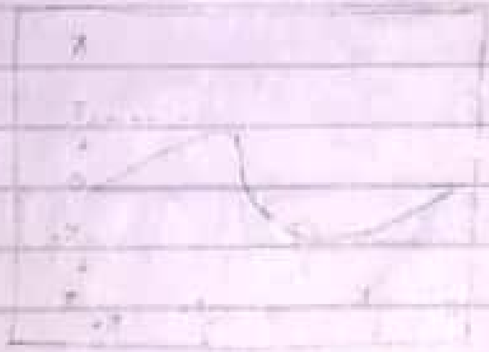
Q3:-

a) Ans:-







$H(0) = 1$. statisfying the conditions.

$$\left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2}$$

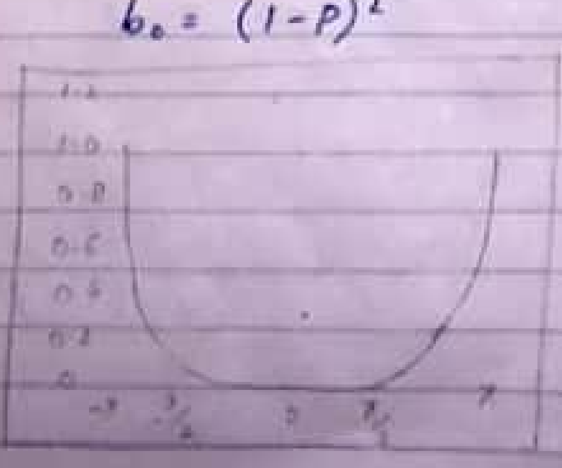
Solution:-

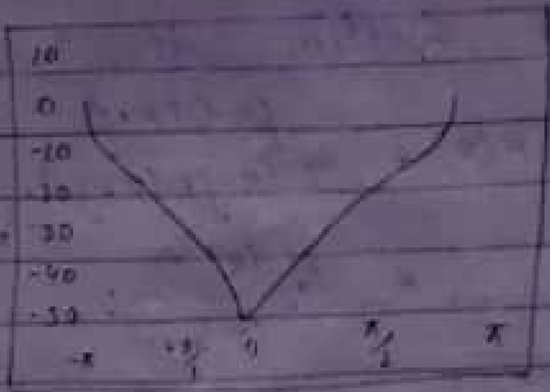
At $\omega = 0$ we have

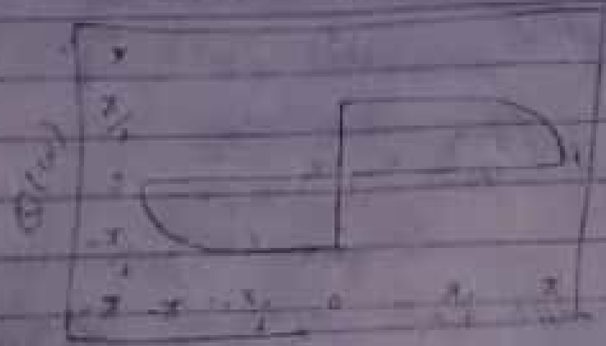
$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

So:-

$$b_0 = (1-p)^2$$







At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2} + jp\sqrt{2})^2}$$

Hence :-

$$= \frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + (p^2\sqrt{2})^2}$$

$$= \left(\frac{1}{2}\right)$$ Ans.

x ——— x ——— x

Q3:-
b)
Ans:- Clearly the filter must poles at:

$$P_{1,2} = re^{j\pi/2}$$

and zeros at $z=1$ & $z=-1$,
Then:-

System function is

$$H(z) = G\,\frac{(z-1)(z+1)}{(z-jr)(z+jr)}.$$

$$= G\,\frac{(z^2-1)}{(z^2+r^2)}.$$

The gain factor becomes:- by $H(\omega)$ at $\omega = \pi/2$. Thus

$$H(\pi/2) = G\,\frac{2}{1-r^2} = 1$$

$$\Rightarrow G = 1-r^2/2$$

The value of "r" is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$

Thus:-

$$\left|H\left(4\pi/9\right)\right|^2 = \frac{(1-r^2)^2}{4}\,\frac{2-2\cos(8\pi/9)}{1+r^2+2r^2\cos(8\pi/9)}.$$

$$= 1/2$$

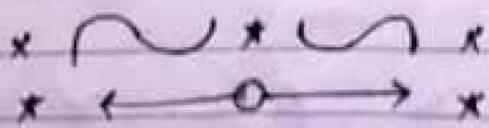
or equivalently:-

$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

So:-

The value of $r^2 = 0.7$ satisfies the equation. Therefore the system function for the desired filter is

$$\left\{H(z) = 0.15\,\frac{1-z^{-2}}{1+0.7z^{-2}}\right\}$$



Q4:-

a)-

Ans:-

P.T.O →

(7)

The fourier transform of this sequence is

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{L-1} e^{-j\omega n} = \frac{1-e^{-j\omega L}}{1-e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{(\sin \omega/2)} e^{-j\omega(L-1)/2}$$

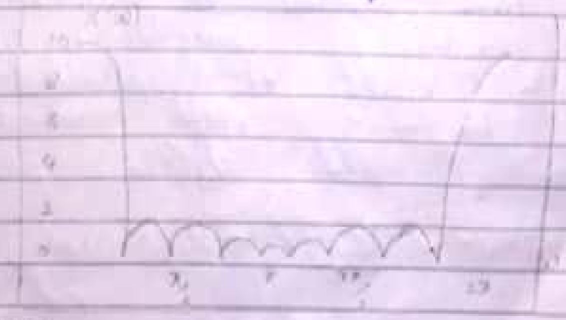
The magnitude & phase of $X(\omega)$ are $L = 10$. The L-point DFT of $x(n)$ is simply $X(\omega)$ at set of N equally spaced frequencies $\omega_k = 2\pi k/N$, $k = 0, 1, \cdots N-1$

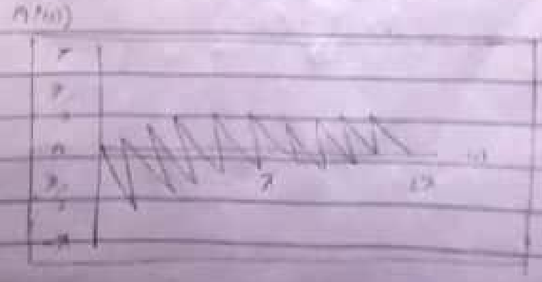
Hence

$$X(k) = \frac{1-e^{-j2\pi kL/N}}{1-e^{-j2\pi k/N}}$$

$; k = 0, 1, \cdots \cdots N-1.$

$$= \frac{\sin(\pi kL/N)}{(\sin \pi k/N)} e^{-j\pi k(L-1)/N}.$$





(8)

If "N" is selected such that $N=L$, then DFT becomes

(8)

If "N" is selected such that $N = L$, then DFT becomes

$$X(k) = \begin{cases} L & k = 0 \\ 0 & k = 1, 2, \cdots L-1 \end{cases}$$

If we wish to have a better picture, we must evaluate $X(\omega)$ at more closely spaced frequencies, say $\omega_k = 2\pi k/N$ where $N > L$.

Q4-

b)- The first is to determine the matrix W_4, By exploiting the periodicity property of W_4 the symmetry property.

$$W_N^{k+N/2} = -W_N^k.$$

The matrix W_4 may be exposed as:-

$$W_4 = \begin{bmatrix} W_{40}^{0} & W_{41}^{0} & W_{42}^{0} & W_{43}^{0} \\ W_{40}^{0} & W_{41}^{1} & W_4 & W_4 \\ W_4^{0} & W_{4,1}^{2} & W_4^{4} & W_4^{4} \\ W_4^{0} & W_4^{3} & W_4^{6} & W_4^{5} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^{1} & W_4^{2} & W_4^{3} \\ 1 & W_4^{2} & W_4^{0} & W_4^{2} \\ 1 & W_4^{3} & W_4^{2} & W_4^{1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then:-

$$Y_4 = W_4 Y_4 = \begin{bmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{bmatrix}$$

The DFT of Y_4 may be determined by conjugating the element in W_4 to obtain W_4' then applying the formula.