Q1 Solve the initial value problem

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$$\frac{dy}{dx} = e^{y-1} \sec(y)(1+t^2) \quad y(0) = ?$$

Solution:-

$$\frac{dy}{dx} = e^{y-1} \sec(y)(1+t^2) \quad (y)(0) = ?$$

$y(0) = 0$ so $x = 0$, $y = 0$

$$dy = e^{y} \cdot e^{-t} \sec(y)(1+t^2)\,dt$$

$$\frac{1}{e^{y} \cdot \sec(y)} dy = (1+t^2)\, e^{-t}\, dt$$

As $\cos(y) = \frac{1}{\sec(y)}$

$$\int e^{-y} \cos y \, dy = \int (1+t^2) e^{-t}\, dt$$

using integration by parts

$$e^{-y} \int \cos y\, dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y}\right) =$$
$$(1+t^2) \int e^{-1} - \int \left(\int e^{-t} \frac{d}{dt} (1+t^2)\right)$$

eq(1)

L.H.S

$$e^{-y} \int \cos y\, dx - \int \left(\int \cos y \cdot \frac{d}{dy} e^{-y}\right)$$

$= e^{-y}\sin y + \int(\sin y \cdot e^{-y})$

$= e^{-y}\sin y + \int(e^{-y} \sin y)$

= Again using integration by parts.

$= e^{-y}\sin y + e^{-y}(-\cos y) - \int(\int \sin y \frac{d}{dy} e^{-y})$

$= e^{-y}\sin y + e^{-y}(-\cos y) - \int(-\cos y \frac{e^{-y}}{-1})$

$= e^{-y}\sin y - e^{-y}\cos y - \int(\cos y\, e^{-y})$

= Since $\int(\cos y\, e^{-y} = L.H.S$

Since it is again some to the first one so L.H.S will become

$= L.H.S = e^{-y}(\sin y - \cos y) - L.HS$

$= 2L.H.S = e^{-y}(\sin y - \cos y)$

= $\div$ ing by 2

$= \frac{\cancel{2}.L.H.S}{\cancel{2}} = \frac{e^{-y}(\sin y - \cos y)}{2}$

$= L.H.S = \frac{e^{-y}(\sin y - \cos y)}{2}$

Now taking R.H.S

$= \int (1+t^2) e^{-t} dt$

$= (1+t^2) \int e^{-t} - \int (\int e^{-t} \frac{d}{dt}(1+t^2)$

$= (1+t^2) e^{-t} - \int (-e^{-t} (2t)$

$= (1+t^2 e^{-t} + \int (2t) e^{t})$

again using integeration by parts.

$= -(1+t^2) e^{-t} + (2t \int e^{-t} - \int (\int e^{-t} dt \frac{2t}{dt}$

$= -(1+t^2) e^{t} + (-2te^{-t} + \int (2e^{-t})$

$= -(1+t^2) e^{-t} + (-2te^{-t} - 2e^{-t}) + c$

$= -(1+t^2) e^{-t} - 2te^{-t} - 2e^{-t} + c$

$= -e^{-t} - e^{-t} t^2 - 2te^{-t} + c =$ R.H.S

Now take L.H.S = R.H.S

$$\frac{e^{-y}(\sin y - \cos y)}{2} = -t^2 + 2t(3)e^{-t} + c$$

weknow that.

$t = 0 \ , y = 0$

put it above

$\frac{1}{2}(0-1) = -3 + c$

$c = 5/2$

put value of C

$$\frac{e^{-y}}{2}(\sin y - \cos y) = -(x^2 + 2t + 3e^{-t} + 5/2.$$

$$\boxed{\frac{e^{-y}}{2}(\sin y - \cos y) = -(x^2 + 2t + 3e^{-t} + 5/2.\ \text{Ans.}}$$

Q #2

Question#2

$$(\sqrt{x+y}+\sqrt{x-y})dx-\sqrt{x+y}-\sqrt{x-y}\,dy=0$$

$$\frac{dy}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{xy}-\sqrt{x-y}} \quad \text{——— (1)}$$

This is homogenous differential equation in x and y to solve this put

$$y=vx$$

$$\Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$$

This equation (1) become.

$$=vx\,x\frac{dv}{dx}=\frac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}$$

$$=v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$=v+x\frac{dv}{dx}=\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}\times\frac{\sqrt{1+v}+\sqrt{1-v}}{\sqrt{1+v}-\sqrt{1-v}}$$

$$=v+x\frac{dv}{dx}=\frac{1+v+1-v+2\sqrt{1-v2}}{2v}$$

$\Rightarrow v + x\frac{dv}{dx} = \frac{\cancel{x}\,1+\sqrt{1-v^2}}{\cancel{x}v}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1+\sqrt{1-v^2}}{v}$

$\Rightarrow \frac{x\,dv}{dx} = \frac{1+\sqrt{1-v^2}-v^2}{v}$

$\Rightarrow x\frac{dv}{dx} = \frac{\sqrt{1-v^2}(1+\sqrt{1-v^2})}{x.}$

$\Rightarrow x\frac{dv}{dx} \Rightarrow \frac{1+\sqrt{1-v^2}-v}{v}$

$x\frac{dv}{dx} = \frac{\sqrt{1-v^2}(1+\sqrt{1-v^2})}{v}$

$\Rightarrow \frac{v\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2})} = \frac{dx}{x}$

= taking integeration on both side.

$= \int \frac{v\,dv}{\sqrt{1-v^2}(1+\sqrt{1-v^2}} = \int \frac{dx}{x}$

put $1+\sqrt{1-v^2} = t$

$\Rightarrow \Rightarrow \frac{1}{2}(1-v^2)^{-1/2}(-2v)dv = dt$

$$\frac{v\,dv}{\sqrt{1-v^2}} = -dt$$

$$\int \frac{-dt}{t} = \int \frac{dx}{x}$$

$$= -\ln t = \ln x + \ln c.$$

$$= -\ln(1+\sqrt{1-v^2} = \ln cx$$

$$= \ln(1+\sqrt{1-v^2}) = -\ln cx$$

$$= \cancel{\ln}(1+\sqrt{1-v^2} = \cancel{\ln}(cx)^{-1}$$

$$= 1+\sqrt{1-v^2} = \frac{1}{cx}$$

$$= 1+\sqrt{\frac{x^2-y^2}{x^2}} = \frac{1}{cx}$$

$$= x+\sqrt{x^2-y^2} = \frac{1}{c}.$$

$$= x+\sqrt{x^2-y^2} = c_1 \quad \because \frac{1}{c} = c_1$$

= which is required solution

Q #3

x

x

Q#3

$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$

Solution:.

$(D^4 + D^2)y = 3x^2 + 4\sin x - 2\cos x$

$\rightarrow f(D)y = f(x)$

As it is a non-homogenous linear equation
so
Solution will be

$y = y_c + y_p$ — ①

Complementary Solution y_c

$D^4 - D^2 = 0 \implies D^2(D^2+1) = 0$

either $D^2 = 0 \implies D = 0$

$D^2 + 1 = 0 \implies D^2 = -1$

$D = \sqrt{-1} \implies D = i$ or $D = 0 + i$

Root are real & complex.

$y_c = c_1 e^{0x} + c_2 e^{0x} (c_2 \cos x + c_3 \sin x)$

$y_c = c_1 + c_2 \cos x + c_3 \sin x)$

$y_p = \frac{1}{f(D)} f(x).$

$$y_P = \frac{1}{D^4+D^2}(3x^2 + 4\sin x - 2\cos x)$$

$$= \frac{3x^2}{D^4+D^2} + \frac{4\sin x}{D^4+D^2} - \frac{2\cos x}{D^4+D^2}$$

$fD = D^4+D^2$

at $D=0 \Rightarrow f(D)=0$

So $f'(D) = 4D^3 + 2D$

again differentiating.

$f''(D) = 12D+2$

So fo $D=0$

$f''(D) = 12(0)+2 = 2$

So replacing $\frac{1}{f(D)}$ with $\frac{x^2}{f''(D)}$

$$\Rightarrow y_P = \frac{x^2 3x^2}{12D+2} + \frac{x^2}{12D+2} 4\sin x - \frac{x^2}{12D+2} 2\cos x$$

putting $D=0$ in all

$$y_P = \frac{x^2 3x^2}{12(0)+2} + \frac{x^2}{12(0)+2} 4\sin x - \frac{x^2}{12(0)+2} 2\cos x$$

$$y_p = \frac{3x^4}{2} + \frac{\cancel{4}^{2}x^2 \sin x}{\cancel{2}} - \frac{\cancel{2}x^2 \cos x}{\cancel{2}}$$

$$= \frac{3}{2}x^4 + 2x^3 \sin x - x^2 \cos x$$

So putting in equation (1)

$$y = c_1 + c_2 \cos x + c_3 \cos x + 3/2 + 2x^2 \sin x - x^2 \cos x$$

$$\boxed{y = c_1 + (c_2 - x^2)\cos x + (c_3 + 2x^2)\sin x + \tfrac{3}{2}x^4}$$

I.D 7701

Section B

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