Name :- Mian Zeeshanullah.

ID :- 7906

Section :- A.

Subject :- Diffrential.

Teacher = Mam Shomaila.

Assignment :- 02.

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Cauchy Euler method

QUESTION #01

The Cauchy Euler equation

(1) $x^3y''' + 2x^2y' + 2y - 10x + \frac{10}{x}$.

Solution:-

$$x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3 D^3 y + 2x^2 D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3 D^3 + 2x^2 D + 2)y = 10x + 10x^{-1} \quad \text{--- (1)}$$

let $x = e^t \Rightarrow t = \ln x$.

$$xD = D$$

$$x^2D^2 = D(\Delta - 1) = \Delta^2 - \Delta$$

$$x^3D^3 = \Delta(\Delta - 1)(\Delta - 2).$$

Subtituting into eq(1).

$$(\Delta - 3\Delta^2 + 2\Delta + 2(\Delta^2 - \Delta) + 2)y = 10^x + 10x^{-1}$$

$(D^3 - D^2 + 2)y = 10x + 10x^{-1}$

$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}$

using Synthetic division

$$\begin{array}{c|cccc} & 1 & -1 & 0 & 2 \\ -1 & & -1 & 2 & -2 \\ \hline & 1 & -2 & 2 & \llcorner 0 \end{array}$$

$D^2 - 2D + 2 = 0$

Now using Quadratic Formula.

$a=1,\ b=-2,\ c=2.$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = \frac{-(-2) \pm \sqrt{-2^2 - 4(1)(2)}}{2a}$$

(5)

$$D = \frac{2 \pm \sqrt{4-8}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm \sqrt{-4}}{2}$$

$$D = \frac{2 \pm \sqrt{-1} \times \sqrt{4}}{2}$$

$$D = \frac{2 + 2i}{2}$$

$$D = \frac{\cancel{2}(1 \pm i)}{\cancel{2}}$$

$$D = 1 \pm i$$

Since roots are Complex.

$$y_c = e^{-x}(c_1 \cos t + c_2 \sin t).$$

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Now particular integration

$$y_p = \frac{1}{D^3-D^2+2} \cdot 10e^t + \frac{1}{D^3-D^2+2} \cdot 10/e^t$$

$$= \frac{10e^t}{(1)^3-(1)^2+2} + \frac{10e^{-t}}{(1)^3-(1)^2+2}$$

$$= \frac{\cancel{10}^{5}e^t}{\cancel{2}} + \frac{\cancel{10}^{5}e^{-t}}{\cancel{2}}$$

$$= 5e^t + 5e^{-t}$$

$$y_p = 5e^t + 5e^{-t}$$

General Solution

$$y = y_c + y_p$$

$$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^t + 5e^{-t}$$

put $e^t = x$ and $t = \ln x$.

$$y = e^{-x}(c_1 \ln x + c_2 \sin \ln x) + 5e^x + 5e^{-x}$$

Ans.

QUESTION# 02

2) $$x^3\frac{d^3y}{dx} + 4x^2\frac{d^2y}{dx} - 5x\frac{dy}{dx} - 15y = x^4$$

Solution:-

Let $\frac{d}{dx} = D$

$$x^3D^3y + 4x^2D^2y - 5xDy - 15y = x^4$$

$$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$$

Let $x = e^t \Rightarrow t = \ln x$.

$$xD = \Delta.$$

$$x^2D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta.$$

$$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta.$$

Now subtituting.

$$(x^3D^3+4x^2D^2-5xD-15)y = x^4.$$

$$(\Delta^3 - 3\Delta^2 + 2\Delta + 4(\Delta^2-\Delta) - 5(\Delta) - 15)y = e^{4t}$$

(b)

$$(\Delta^3 + \Delta^2 - 7\Delta - 15)y = e^{4x}$$

Synthetic division

5	1	+1	−7	−15
		3	12	15
	1	4	5	0

$$\Delta^2 + 4\Delta + 5 = 0$$

Quadratic formula.

$$\Delta = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2 - 4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{16 - 20}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$\Delta = \frac{\cancel{2}(-2 \pm i)}{\cancel{2}}$$

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$$y_c = e^{3x}(c_1 \cos t + c_2 \sin t).$$

For $y_p = ?$

$$y_p = \frac{1}{D^3+D^2-7D-15} \cdot e^{3x}$$

$$= \frac{1}{(4)^3+(4^2)-7(4)-15} e^{4t}$$

$$= \frac{1}{64+16-28-15} e^{4t}$$

$$= \frac{1}{80-43} e^{4t}$$

$$y_p = \frac{1}{37} e^{4t}.$$

Hence

$$y = y_c + y_p.$$

$$y = (C_1 \cos t + C_2 \sin t) + \frac{1}{37} e^{4t}$$

again put $t = \ln x$ and $x = \ln x$.

$$y = e^{3x}(c_1 \cos \ln x + c_2 \sin \log x) + \frac{1}{37} e^{4x}$$ Ans

Question #03

$$x^2 y'' + 2xy' - 6y = 10x^2$$

Solution:

$y(1) = 1$ and $y'(1) = -6$

$$x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} - 6y = 10x^2$$

$$\Rightarrow \left(x^2 d^2/dx^2 + 2x\, d/dx - 6\right) y = 10x^2$$

Put $xD = \Delta \Rightarrow x^2 D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta$

$x = e^t$ and $\log x = t$

$$(\Delta^2 - \Delta + 2\Delta - 6)y = 10e^{2t}$$

$$(\Delta^2 + \Delta - 6)y = 10e^{2t}$$

The characteristic equation

$$\Delta^2 + \Delta - 6 = 0$$

$$\Delta^2 + 3\Delta - 2\Delta - 6 = 0$$

$\Rightarrow D(D+3)-2(D+3)=0$

$\Rightarrow (D+3)(D-2)=0$

$D+3=0, \quad D-2=0$

$D=2, \quad D=-3$

Since roots are real and distinct

For $y_c = ?$

$y_c = C_1 e^{-3t} + C_2 e^{2t}$

For $y_p = ?$

$$y_p = \frac{1}{D^2-D-6} \cdot 10^{2t}.$$

$$= \frac{10}{D^2-D-6} e^{2t}$$

$$= 10 \frac{1}{0} e^{2t} \text{ fails}$$

Now

$$10 \frac{1}{d/dD(D^2+D-6)} e^{2t}$$

$$\Rightarrow 10 \frac{t}{2D+1} e^{2t}$$

$$= 10 \frac{1 \cdot t}{4+1} e^{2t}$$

$$y_p = 2t e^{2t}$$

General solution

$$y = y_c + y_p$$

$$= c_1 e^{-3t} + c_2 e^{2t} + 2te^{2t}$$

$$y = c_1 x^{-3} + c_2 x^2 + 2(\log x) x^2 \text{ — (B)}$$

Put $y(1) = 1$ i.e $x = 1, y = 1$ in (B)

$$1 = c_1(1)^{-3} + c_2(1)^2 + 2\log(1)$$

$$1 = c_1 + c_2 \longrightarrow (C)$$

Now differentiate eq (B) w.r.t x.

$y' = -3c_1 x^{-4} + 2c_2 x + \frac{2}{x}(x^2) + 4x\log x$

Now put $y'(1) = -6$ i-e $y' = -6$ and $x = -6$.

$-6 = -3c_1 + 2c_2 + 2 + 0$

$\Rightarrow -6 = -3c_1 + 2c_2 + 2.$

$\Rightarrow -6 - 2 = -3c_1 + 2c_2 + 2$

$-8 = -3c_1 + 2c_2 \quad -(1)$

x ing eq (1) with (2) and − ing from (1)

$$\begin{array}{r} 2c_1 + \cancel{2c_2} = 2 \\ \mp 3c_1 \pm \cancel{2c_2} = \mp 8 \\ \hline 5c_1 \qquad = 10 \end{array}$$

$c_1 = \frac{10}{5} = 2 \quad \boxed{c_1 = 2}$

$-8 = -3(2) + 2c_2$

$-8 = -6 + 2c_2$

$2c_2 = -8 + 6$

$2c_2 = -2.$

(12)

$$C_2 = \frac{-\cancel{2}\ 1}{\cancel{2}}$$

$$\boxed{C_2 = -1}$$

now put the value of C_1 and C_2.
in eq (B).

$$y = 2x^{-3} - x^2 + 2\ln x\, x(x^2)$$

$$y = \frac{2}{x^3} - x^2 + 2x^2 \log x \text{ Ans}$$

QUESTION # 04

$x^2y'' + 7xy' + 5y = x^5$

$y(0) = 2$ and $y'(1) = 2$

SOLUTION :-

$$x^2\frac{dy^2}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$$

$$\Rightarrow \left(x^2\frac{d^2}{dx^2} + 7x\frac{d}{dx} + 5\right)y = x^5 \quad \text{--- (A)}$$

Put $xD = \Delta \Rightarrow x^2D^2 = \Delta(\Delta - 1) = \Delta^2 - \Delta.$

$x = e^t \Rightarrow \log x = t$ in eq (A)

$$\Rightarrow (\Delta^2 - \Delta + 7\Delta + 5)y = e^{5t}$$

$$\Rightarrow (\Delta^2 + 6\Delta + 5)y = e^{5t}$$

By Quadratic Formula,

$$\Delta = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

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$$D = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36 - 20}}{2}$$

$$= \frac{-6 \pm \sqrt{16}}{2}$$

$$= \frac{-6 \pm \sqrt{4^2}}{2}$$

$$= \frac{\cancel{2}(-3 \pm 2)}{\cancel{2}}$$

$D = -3 \pm 2$ Since roots are real and distinct

$$y_c = C_1 e^{-5t} + C_2 e^{-t}$$

For $y_p = ?$

$$y_p = \frac{1}{D^2 + 6D + 5} e^{5t}$$

$$= \frac{1}{(5)^2 + 6(5) + 5} e^{5t}$$

$$= \frac{1}{60} e^{5t}$$

Now General solution is

$$y = y_c + y_p.$$

$$y = C_1 e^{-5t} + C_2 e^{-t} + \frac{1}{60} e^{5t}$$

$$y = C_1 x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5 \longrightarrow (B)$$

$x=0$ put in this equation

No in eq (B) $\quad e^0 = 1$

put $y(0) = 2$ i-e $y = 2$ and $x = 2$.

$$2 = C_1(2)^{-5} + C_2(2)^{-1} + \frac{1}{60}(2)^5$$

$$2 = -32c_1 - 2c_2 + \frac{1}{\cancel{60}_{15}}(\cancel{32}^{8})$$

$$2 = -32c_1 - 2c_2 + \frac{8}{15}$$

$$2 - \frac{8}{15} = -32c_1 - 2c_2.$$

$$\frac{22}{15} = -32c_1 - 2c_2 \longrightarrow (C)$$

Now differentiate eq(B) w.r.t (x)

$$y' = -5c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12}x^4 \longrightarrow$$

put $y'(1)=2$ i-e $y'=2$ and $x=2$ in above equation

$$2 = -5c_1(2)^{-6} - c_2(2)^{-2} + \frac{1}{12}(2)^4$$

$$2 = -5c_1(-64) - c_2(4) + \frac{1}{12}(16)$$

$$2 = 320c_1 + 4c_2 + 4/3$$

$\Rightarrow 2 - \frac{4}{3} = 320c_1 + 4c_2.$

$\Rightarrow 2/3 = 320c_1 + 4c_2 \longrightarrow$ (D)

$\times$ing eq (C) with 2 and then $-$ing eq (C) from (D)

$$\frac{-44}{15} = 64c_1 + 4c_2.$$

$$\frac{-44}{15} = 64c_1 + 4c_1$$

$$\frac{+2/3 = +320c_1 \pm 4c_2}{\frac{34}{15} = -256c_1}$$

$$c_1 = \frac{34}{15} \times 256.$$

$$\boxed{c_1 = 580}$$

Put the value of c_1 in eq (c).

$$\frac{22}{15} = -32(580) - 2c_2.$$

$$\Rightarrow \frac{22}{15} = -13560 - 2C_2.$$

$$\Rightarrow \frac{22}{15} + 13560 = -2C_2$$

$$\Rightarrow \frac{13561}{-2} = C_2.$$

$$\boxed{-9280 = C_2}$$

Now put the value of C_1 and C_2.
in eq (B).

$$y = 530x^{-5} - 9280x^{-1} + \frac{1}{60}x^5$$

$$y = \frac{530}{x^5} - \frac{9280}{x} + \frac{1}{60}x^5$$

Ans

QUESTION #05

(41)

$$(x+1)^2 y'' - 3(x+1)' y' + 4y = x^2$$

SOLUTION:

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$$

$$\Rightarrow \left((x+1)^2 \frac{d^2}{dx^2} - 3(x+1)\frac{d}{dx} + 4\right) y = x^2$$

$$\Rightarrow \left[(x+1)^2 D^2 - 3(x+1)D + 4\right] y = x^2 \longrightarrow (A)$$

Put $(x+1)D = \Delta \Rightarrow (x+1)^2 D^2 = \Delta(\Delta-1) = \Delta^2 - \Delta$

$x = e^t$ in eq (A)

$$\Rightarrow \left[\Delta^2 - \Delta - 3\Delta + 4\right] y = e^{2t}$$

$$\Rightarrow \left(\Delta^2 - 4\Delta + 4\right) y = e^{2t}$$

$$\Rightarrow (\Delta^2 - 4\Delta + 4) 2 = e^{2t}$$

for y_c we find the roots

$$\Delta^2 - 4\Delta + 4 = 0$$

$$\Delta^2 - 2\Delta - 2\Delta + 4 = 0$$

$$\Delta(\Delta - 2) - 2(\Delta - 2) = 0$$

$D - 2 = 0$, $D = 2$.

$D - 2 = 0$, $D = 2$

so the roots are real and reapet.

The General solution are.

$y = (c_1 + c_2 x)^{mx}$.

$y = (c_1 + c_3 x)^{2x}$.

For $yp = ?$

$yp = \frac{1}{D^2 - 4D + 4}$.

$(2)^2 - 4(2) + 4$
$\Rightarrow 0$

$yp = \frac{2}{2D - 4} e^{2t}$

if we put "2"

$2D - 4 \Rightarrow 2(2) - 4 = 0$

we take again derivatea.

$yp = \frac{2}{2} \cdot e^{2t}$

$y = (c_1 + c_2 x)^{2t} + e^{2t}$ → General solution

Ans