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FINAL PAPER :

PROBABILITY AND STATISTIC.

Question # 01

Construct a grouped distribution table for the following data and calculate Mean, Mode Median and Quartiles.

423,, 390

Solution

363, 369, 371, 372, 377, 381, 382, 386,
 387, ~~389~~, ³⁸⁹390, 391, 392, 393, 394, 396, 390,
 400, 401, 405, 408, 409, 410, 411, 415
 419, 422, 423, 428, 431

$$\text{max value} = 431$$

$$\text{min value} = 363$$

$$\begin{aligned} \text{Range} &= 431 - 363 \\ &= 68 \end{aligned}$$

Since range is 68 we will choose an interval length of 10

Class Interval	Frequency	midpoint	F.m	cf
360-369	2	364.5	729	2
370-379	3	374.5	1123.5	5
380-389	5	389.5	1922.5	10
390-399	7	394.5	2761.5	17
400-409	5	404.5	2022.5	22
410-419	4	414.5	1658	26
420-429	3	424.5	1273.5	29
430-439	1	434.5	434.5	30
	30		11925	

$$\text{mean} = \frac{\sum f \cdot m}{\sum f}$$

$$= \frac{11925}{30}$$

$$\text{mean} = 397.5$$

$$\text{medium} = l_m + \left(\frac{\frac{\sum f}{2} - c_{bcb}}{f_m} \right) c$$

$$= 389.5 + \left(\frac{\frac{30}{2} - 10}{7} \right) (399.5 - 389.5)$$

$$= 389.5 + \left(\frac{15 - 10}{7} \right) 10$$

$$= 389.5 + \left(\frac{5}{7} \right) 10$$

$$= 389.5 + \frac{50}{70}$$

$$= 389.5 + 7.1428$$

$$\text{median} = 396.6$$

$$\text{mode} = l_m + \left(\frac{A_1}{A_1 + A_2} \right) C$$

$$= 389.5 + \left(\frac{2}{2+2} \right) 10$$

$$= 389.5 + \left(\frac{2}{4} \right) 10$$

$$= 389.5 + \frac{10 \cdot 2}{2}$$

$$= 389.5 + 5$$

$$\text{mode} = 394.5$$

$$l_m = 389.5$$

$$A_1 = 7 - 5 = 2$$

$$A_2 = 7 - 5 = 2$$

$$C = 399.5 - 389.5 = 10$$

Quartile

class interval	frequency	C.F
360-369	2	2
370-379	3	5
380-389	5	10
<u>390-399</u>	<u>7</u>	17
400-409	5	22
410-419	4	26
420-429	3	29
430-439	1	30
	<u>30</u>	

$$Q = D = \frac{Q_3 - Q_1}{2}$$

$$Q_1 = l + \frac{h}{\delta} (\Sigma_1 - c) \quad \Sigma_1 = \frac{n}{4} = \frac{30}{4} = 7.5$$

$$l = 390 \quad \delta = 7 \quad h = 4 \quad c = 10$$

$$= 390 + \frac{4}{7} (7.5 - 10)$$

$$= 390 + 0.5(-2.5)$$

$$= 390 - 2$$

$$\text{Quartile} = 388$$

Question # 02

By multiplying each of the number 3, 6, 2, 1, 7, 5 by 2
 two sets.

X	$X_i - U$	$(X_i - U)^2$
		36
7	-6	16
9	-4	4
11	-2	16
17	4	4
15	2	36
19	6	
$\Sigma X = 78$	$\Sigma (X_i - U) = 0$	$\Sigma (X_i - U)^2 = 112$

$$\text{mean} = \frac{\Sigma X}{N}$$

$$= \frac{78}{6}$$

$$= 13$$

$$S.D = \sigma = \sqrt{\frac{\Sigma (X_i - U)^2}{N}} = \sigma = \sqrt{\frac{112}{6}}$$

The mean is a measure of central tendency. The standard deviation is a measure of dispersion. Both are appropriate descriptive statistics for normally distributed data sets using ratio or interval

Scaling. Both mean and standard deviation are used in calculating some correlation coefficients, effect sizes, t scores, F scores (Analysis of Variance)

Question #03

Class	Frequency	Class-boundaries	X	X ²
64-84	15	63.5 - 84.5	74	5476
85-104	18	84.5 - 104.5	94.5	8930.25
105-124	27	104.5 - 124.5	114.5	13110.25
125-144	10	124.5 - 144.5	134.5	18090.25
145-164	6	144.5 - 164.5	154.5	23870.25
165-184	5	164.5 - 184.5	174.5	30450.25
185-204	13	184.5 - 204.5	194.5	37830.25
	= 94			

fX	fX ²
1110	82140
1701	160744.5
3091.5	353,976.75
1345	180902.5
927	143221.5
872.5	152251.25
2528.5	491,793.25
= 11575.5	1565029.75

Standard deviation:-

$$S = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{1565029.75}{94} - \left(\frac{11575.5}{94}\right)^2}$$

$$= \sqrt{16649.25 - (123.14)^2}$$

$$= \sqrt{16649.25 - 15163.46}$$

$$= \sqrt{1485.79}$$

variance:-

$$s^2 = (\overline{dS})^2$$

$$s^2 = 1485.79$$

Question # 04

If two fair dice are thrown, what is the probability of getting

1. A double six
2. A sum of 8 or more dots.

Solution

The sample space S is represented by the following 36 outcomes

$$S = \{ (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\ (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \\ (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \}$$

1. Let A be the event that double six occurs

$$A = \{ (6, 6) \} \text{ and thus}$$

$$P(A) = 1/36$$

2. Let B denotes that a sum of 8 or more dots occurs

$$B = \{ (2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6) \\ (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \}$$

Hence

$$P(B) = 15/36 = 5/12$$

Q5: Let C_1, C_2, \dots, C_M be a partition of the sample space S , and A and B be two events. Suppose we know that.

- A and B are conditionally independent given C_i , for all $i \in \{1, 2, \dots, M\}$;
- B is independent of all C_i 's.

Prove that A and B are independent.

Solution:

Since the C_i 's form a partition of the sample space, we can apply the law of total probability for $A \cap B$:

$$\begin{aligned}
 P(A \cap B) &= \sum_{i=1}^M P(A \cap B | C_i) P(C_i) \\
 &= \sum_{i=1}^M P(A | C_i) P(B | C_i) P(C_i) \\
 &= \sum_{i=1}^M P(A | C_i) P(B) P(C_i) \\
 &= \sum_{i=1}^M P(B) \sum_{i=1}^M P(A | C_i) P(C_i) \\
 &= P(B) P(A) \quad (\text{law of total probability})
 \end{aligned}$$

$\left\{ \begin{array}{l} A \ \& \ B \ \text{are} \\ \text{conditionally} \\ \text{independent} \end{array} \right\}$

$\left\{ \begin{array}{l} B \ \text{is independent} \\ \text{of all } C_i \text{'s} \end{array} \right\}$