Q # 1 :-

(i) The order of Matrix AB is

$\underline{m \times n}$.

(ii) The number of non-Zero rows in on Echelon forms is $\underline{\text{one}}$.

(iii) If $B = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a = \underline{\quad 8 \quad}$

(iv) If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$

$$|A| = \begin{vmatrix} 2i & i \\ i & -i \end{vmatrix}$$

$$= -2i^2 - i^2 \qquad \therefore i^2 = -1$$

$$= -2(-1) - (-1)$$

$$= 2 + 1$$

$$= \underline{3} .$$

(V) The Matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is Scalar Matrix

The given Matrix A is a scalar matrix because the diagonal elements are same and non-diagonal are zero.

(Vi) Solution of $\frac{dy}{dx} + 2xy = y$?

Sol:-

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$ (y taking common)

$$\frac{dy}{dx} = y(1-2x)$$

$$\frac{dy}{y} = (1-2x)dx$$

take Integration

$$\int \frac{1}{y} dy = \int (1-2x)\,dx$$

$$\ln y = \int 1\,dx - \int 2x\,dx$$

$$\Rightarrow \ln y = x - \frac{\cancel{2}x^2}{\cancel{2}} + c$$

$$\ln y = x - x^2 + c$$

$$\ln y = x - x^2 + c$$

$$e^{\ln y} = e^{x - x^2 + c}$$

$$\boxed{y = e^{x(1-x)+c}}$$

(vii) The order and degree of differential equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

sol: order : $\underline{1}$.

Degree: $\underline{3}$.

(viii) The order and degree of differential equation

$$\frac{d^2y}{dx^2} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is}$$

sol:- order = Two.

Degree = one.

(ix) The differential equation $2\,{}^{dy}/_{dx} + x^2y = 2x+3$, $y = y'(0) = 5$ is ______.

sol:-

$$2y' + x^2y = x^2+3, \quad y(0) = 5$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{x^2+3}{2}$$

$$y' + \left(\frac{x^2}{2}\right)y = \frac{1}{2}(x^2+3)$$

$$\mu = \frac{x^2}{2}$$

$$e^{\int \frac{x^2}{2} dx} = e^{x^3/6}$$

$$e^{x^3/6} y' + e^{x^3/6}\left(\frac{x^2}{2}\right) y = \frac{1}{2} e^{x^3/6}(x^2+3)$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6} + c}{2e^{x^3/6}}$$

$$y(0) = \frac{0+3}{2} = 3/2$$

$$y(x) = \frac{e^{x^3/6} x^2 + 3e^{x^3/6}}{2e^{x^3/6}} + \frac{3}{2} \quad \underline{\underline{\text{Ans:-}}}$$

(X)

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by C_1

$$1 \begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1 \begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1 \begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix}$$

$$\Rightarrow 1(bc^2 - cb^2) - 1(ac^2 + a^2c) + 1(ab^2 - a^2b)$$

$$= bc^2 - cb^2 - ac^2 - a^2c + ab^2 - a^2b$$

$$= ab^2 - cb^2 + a^2c - a^2b - ac^2 + bc^2$$

$$= a^2c - a^2b + ab^2 - cb^2 + bc^2 - ac^2$$

$$= a^2(c-b) + b^2(a-c) + c^2(b-a) \quad \underline{\text{Ans.}}$$

Q #2 :- A Part

Express the Determinant

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

as the Product of factors which are linear in a, b, c -

Sol:-

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$$

Expand by R_1

$$a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2).$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2cb^3 - a^3b^2c$$

common abc

$$\Rightarrow abc\left(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b\right)$$

$$\Rightarrow abc\left[bc(c-b) - ac(c+a) + ab(b-a)\right] \quad \underline{\text{Ans:}}$$

Q # 2:- $\underline{\text{B Part}}$:- $\begin{vmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix}$

$\underline{\text{Sol}}$:-

$$\begin{vmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{vmatrix}$$

characteristic equ $\rightarrow |A - \lambda I| = 0 \longrightarrow$ (A)

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now take determiment

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

Expand by R_1

$$\Rightarrow 2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \quad \text{— (A)}$$

Again $\begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix}$ Expand by R_1

$$\Rightarrow 3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\Big[\big((3-\lambda)(2-\lambda)\big) - (-1)(-1)\big) + 1\big((-1)(2-\cancel{\lambda}) - (-1)(-1)\big)$$
$$-1\big((-1)(-1) - (-1)(3-\lambda)\big)\Big]$$

$$= (3-\lambda)(6-3\cancel{\lambda}-2\lambda+\lambda^2+1) + (-2+\lambda-1) - (+1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5) + (-3+\lambda) - (4-\lambda)$$

$$= 3\lambda^2 - 15\lambda + 15 - \lambda^3 + 5\lambda^2 - 5\lambda - 3 + \lambda - 4 + \lambda$$

$$= \boxed{-\lambda^3 + 8\lambda - 18\lambda + 8} \rightarrow (a)$$

$$\Rightarrow +1 \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} \quad \text{Expand by } C_1$$

$$\Rightarrow -1 \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6 - 3\lambda - 2\lambda + \lambda^2 - 1) + 1(-2 + \lambda - 1)$$

$$\Rightarrow -\lambda^2 + 5\lambda - 5 - 3 + \lambda$$

$$= \boxed{-\lambda^2 + 6\lambda - 8} — (b)$$

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} \quad \text{Expand by } C_1$$

$$- \left[-1 \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0 \right]$$

$$\Rightarrow -\left[-(-2+\lambda-1)+1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= \boxed{-\lambda^2+6\lambda-8} \text{ --- } ⓒ$$

Put ⓐ, ⓑ and ⓒ in Ⓑ

$$(2-\lambda)\left[-\lambda^3+8\lambda^2-18\lambda+8\right]-\lambda^2+6\lambda-8-\lambda^2$$
$$+6\lambda-8.]$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-$$
$$-\lambda^2+6\lambda-8-\lambda^2+16\lambda-8$$

$$\Rightarrow \lambda^4-2\lambda^3-8\lambda^3+16\lambda^2+16\lambda^2+16\lambda^2-\lambda^2-\lambda^2$$
$$-36\lambda-8\lambda+6\lambda+6\lambda+16-16$$

$$\Rightarrow \lambda^4-10\lambda^3+32\lambda^2-32\lambda=0$$

By synthatic division

we get,

$$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$$

$$(\lambda=0)$$

$$\lambda-2=0 \Rightarrow \boxed{\lambda=2}$$

$$\lambda^2-8\lambda+16=0$$

By factorization Method

$$\lambda^2-4\lambda-4\lambda+16=0$$

$$\lambda(\lambda-4)-4(\lambda-4)=0$$

$$\lambda=4, \quad \lambda=4$$

$$\boxed{\lambda_1=0, \quad \lambda_2=2, \quad \lambda_3=4, \quad \lambda_4=4}$$ Ans.

Q #3 $(x^2+3y^2)dx - 2xy\,dy = 0$

$X=2$, $y=6$

Sol:-

$(x^2+3y^2)dx - 2xy\,dy = 0$

$\Rightarrow (x^2+3y^2)dx = 2xy\,dy$

Dividing both sides by $2xydx$ we get.

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x^{\cancel{2}}}{2\cancel{x}y} + \frac{3y^{\cancel{2}}}{2x\cancel{y}}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{x}{y} + \frac{3y}{x}\right] \longrightarrow (*)$$

Let $y = vx$

Diff: $dy = v\,dx + x\,dv$

Diving by dx

$$\frac{dy}{dx} = v + x\frac{dv}{dx} \longrightarrow (a)$$

Put ⓐ in ⊛

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{\cancel{x}}{xv} + 3\frac{v\cancel{x}}{x}\right]$$

$$v + x\frac{dv}{dx} = \frac{1}{2}\left[\frac{1}{v} + 3v\right]$$

Multiplying Both sides by "2"

$$2v + 2x\frac{dv}{dx} = \frac{1}{v} + 3v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + 3v - 2v$$

$$2x\frac{dv}{dx} = \frac{1}{v} + v$$

$$2x\frac{dv}{dx} = \frac{1+v^2}{v}$$

Multiplying both sides by $\frac{dx}{dv}$

we get:

$$2x\,dv = \frac{1+v^2}{v}dx$$

Multiplying both sides by $\frac{v}{x(1+v^2)}$

we get,

$$\frac{v}{\quad}dv = \frac{1}{\quad}dx$$

Take "$\int$" on both sides

$$\int \frac{2v}{1+v^2} dv = \int \frac{1}{2x} dx + c$$

$$\ln|1+v^2| = \ln x + \ln c$$

Take "e" on both sides

$$e^{\ln|1+v^2|} = e^{\ln|xc|}$$

$$1+v^2 = xc$$

$$1+v^2 = xc$$

Put $v = y/x$

$$1 + (y/x)^2 = xc$$

$$\frac{x^2+y^2}{x^2} = xc$$

$$x^2+y^2 = x^3c \quad \text{———} \quad (*)(*)$$

Put $x = 2$, $y = 6$ in equ $(**)$

$(4) + (36) = 8c$

$c = \frac{\cancel{40}^{5}}{\cancel{8}}$

$\boxed{c = 5}$ → put in $(**)$

So

$x^2 + y^2 = 5x^3$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x - 1)$

Taking "$\sqrt{\ }$" on both sides

$\boxed{y = +x\sqrt{5x-1} \quad , \quad y = -x\sqrt{5x-1}}$

$|y = \pm x\sqrt{5x-1}|$ Ans:-

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