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Section:- B

Subject:- Applied Calculus

Department of Civil Engineering

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Final Exam

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Question#1

$P(4,1,3) = 4\hat{i} + \hat{j} + 3\hat{k}$

$Q(1,2,4) = \hat{i} + 2\hat{j} + 4\hat{k}$

Sol:- Distance b/w $PQ = ?$

$$= |PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$= \sqrt{(4-1)^2 + (1-2)^2 + (3-4)^2}$$

$$= \sqrt{3^2 + (-1)^2 + (-1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11}$$

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Now find the position vector of the point dividing PQ in the ratio of $\underline{1}:3$

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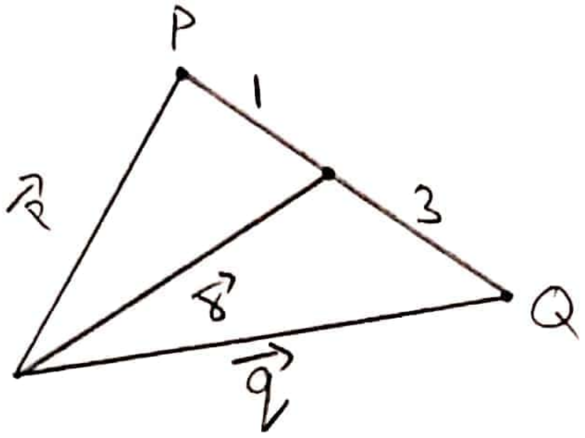

$a:b = 1:3$

$$\vec{r} = \frac{b\vec{p} + a\vec{q}}{b+a}$$

$$= \frac{3(4i+j+3k)+1(\hat{i}+2\hat{j}+4\hat{k})}{3+1}$$

$$= \frac{12\hat{i}+3\hat{j}+9\hat{k}+i+2\hat{j}+4\hat{k}}{4}$$

$$\boxed{\vec{r} = \frac{13}{4}\hat{i} + \frac{5}{4}\hat{j} + \frac{6}{4}\hat{k} \quad \text{Ans}}$$

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Question # 2

Evaluate $\displaystyle\int \frac{4x^3+10x+4}{2x^2+x}$

By long division

$$
\begin{array}{r|l}
 & 2x-1 \\
\hline
2x^2+x & \cancel{4x^3}+10x+4 \\
 & \cancel{\pm 4x^3} \quad \pm 2x^2 \\
\hline
 & \cancel{-2x^2}+10x+4 \\
 & \cancel{\mp 2x^2} \mp x \\
\hline
 & 11x+4
\end{array}
$$

So

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx = \int \left(2x-1+\frac{11x+4}{2x^2+x}\right)dx$$

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$$\int 2x\,dx - \int 1\,dx + \int \frac{11x+4}{2x^2+x}\,dx$$

$$x^2 - x + \int \frac{11x+4}{2x^2+x}\,dx$$

By using partial fraction

$$\frac{11x+4}{x(2x+1)} = \frac{A}{x} + \frac{B}{2x+1}$$

$$11x+4 = A(2x+1) + Bx$$

$$11x+4 = 2Ax + A + Bx$$

By comparing

$A=4$ and $B=3$ so

$$x^2 - x + \int \left(\frac{4}{x} + \frac{3}{2x+1}\right) dx$$

$$x^2 - x + \int \frac{4}{x}\,dx + \int \frac{3}{2x+1}\,dx$$

$$\boxed{x^2 - x + 4\ln|x| + \frac{3}{2}\ln|2x+1| + C}$$

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Question # 3

Part (a)

$$\int_0^2 x^2 e^x \, dx$$

Sol:- $\int_0^2 x^2 e^x \, dx$

Integrating by parts

$$x^2 \int e^2 - \int \left(e^x dx \cdot \frac{d}{dx} x^2 \right) dx$$

$$x^2 e^x - \int e^x (2x) \, dx$$

$$x^2 e^x - \int 2x e^x \, dx$$

$$x^2 e^x - 2 \int x e^x \, dx$$

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By using integration by parts

$$x^2e^x - 2\left[x\int e^x\,dx - \int\left(\int e^x\,dx \cdot \frac{dx}{dx}\right)dx\right.$$

$$x^2e^x - 2\left[xe^x - \int e^x\,dx\right]$$

$$xe^x - 2xe^x + 2e^x$$

$$(x^2 - 2x + 2)\,e^x$$

Applying limits

$$(2^2 - 2(2) + 2)\,e^2 - (0^2 - 2(0) + 2)\,e^0$$

$$\boxed{2e^2 - 2 \text{ Ans}}$$

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Question # 3

Part (b)

$$\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

Sol:- $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx \longrightarrow (1)$

let $u = \sqrt{x} \longrightarrow (ii)$

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$dx = 2\sqrt{x}\, du \rightarrow$ Put in eq (1)

$\int 2 \sin u \, du$

$-2 \cos u$

$-2 \cos \sqrt{x} \longrightarrow$ from eq (ii)

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Applying limits

$$= 2\cos\sqrt{2} - 2\cos(1)$$

$$= \boxed{-0.769}$$

As area can not be negative so

$$= \boxed{0.769}$$

Question #4

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

Laplace for $u(x,y,z)$ is given by

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z} = 0$$

$$u(x,y,z) = \frac{1}{x^2+y^2+z^2}$$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial u}\left(x^2+y^2+z^2\right)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}(\cancel{2}x)$$

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$$\frac{\partial u}{\partial x} = -x\left(x^2+y^2+z^2\right)^{-3/2}$$

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$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left[-x\left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(-3/\cancel{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}\right.$$

$$\left.\cdot \cancel{2}x + \left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial u}{\partial y} = \frac{2}{2y}\left(x^2+y^2+z^2\right)^{-1/2}$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(x^2+y^2+z^2\right)^{-1/2}$$

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$$\frac{\partial u}{\partial y} = -\frac{1}{2}\left(x^2+y^2+z^2\right)^{-3/2}(2y)$$

$$\frac{\partial u}{\partial y} = -y\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left[-y\left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(-\frac{3}{\cancel{2}}\right)\left(x^2+y^2+z^2\right)^{-5/2}(\cancel{2}y) + \left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial y^2} = \left[3y^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}\right]$$

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$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(x^2+y^2+z^2\right)^{-1/2}$$

$$\frac{\partial u}{\partial z} = \frac{-1}{\cancel{2}}\left(x^2+y^2+z^2\right)^{-3/2}\left(\cancel{2}z\right)$$

$$\frac{\partial u}{\partial z} = -z\left(x^2+y^2+z^2\right)^{-3/2}$$

$$\frac{\partial u}{\partial z} = \frac{\partial}{\partial z}\left(-z\left(x^2+y^2+z^2\right)\right)$$

$$\frac{\partial^2 u}{\partial z^2} = 3z^2\left(x^2+y^2+z^2\right)^{-5/2} - \left(x^2+y^2+z^2\right)^{-3/2}$$

Now $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial^2 z^2} = 0$

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$$3x^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2} +$$

$$3y^2(x^2+y^2+z^2)^{-3/2} - (x^2+y^2+z^2)^{-1/2}$$

$$+ 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

$$(x^2+y^2+z^2)^{-5/2}\Big[\cancel{3x^2} - \cancel{x^2} - \cancel{y^2} - \cancel{z^2} + \cancel{3y^2}$$

$$- \cancel{x^2} - \cancel{y^2} - \cancel{z^2} + \cancel{3x^2} - \cancel{x^2} - \cancel{y^2} - \cancel{z^2}\Big]$$

$$(x^2+y^2+z^2)(0) = 0$$

Hence $u(x,y,z)$ satifies solution of laplace solution