(2) Solution of $\frac{dy}{dx} + 2xy = y$?

$$\frac{dy}{dx} + 2xy = y$$

$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1 - 2x)$$

$$\int \frac{1}{y} dy = \int (1 - 2x) dx$$

$$\ln y = x - \frac{2x^2}{2} + c$$

$$\boxed{y = e^{x - x^2} + C}$$

①

QUESTION # 01

i. The order of matrix A is $m \times p$ and the order of B is $p \times n$. Then the order of matrix AB is

Sol: $[A]_{m \times p}[B]_{p \times n} = [AB]_{m \times n}$

ii. The number of non-zero rows in an Echelon form is one

Ans: one.

iii. If B is $= \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$ is a singular matrix then $a =$ 8.

iv. If $A = \begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ then $|A| = ?$

Ans: $|A| = 3$

v. The matrix $A = \begin{bmatrix} 9 & 0 \\ 0 & 9 \end{bmatrix}$ is scalar matrix.

vi ② Solution of $\frac{dy}{dx} + 2xy = y$?

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$$\frac{dy}{dx} = y - 2xy$$

$$\frac{dy}{dx} = y(1-2x)$$

$$\int \frac{1}{y}\,dy = \int (1-2x)\,dx$$

$$\ln y = x - \frac{2x^2}{2} + c$$

$$\boxed{y = e^{x-x^2} + C}$$

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vii:- The Order and Degree of differential Equation

$$\left(\frac{dy}{dx}\right)^3 = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \text{ is}$$

Order = 1

Degree = ~~6~~ 3

viii. The Order and Degree of

$$\frac{d^2y}{dx} - 4xy = \sin\left(\frac{d^2y}{dx^2}\right) \text{ is ?}$$

Order = 2

Degree = Undefined.

(ix) (x)

The differential equation

$$\frac{2\,dy}{dx} + x^2 y = 2x + 3 \quad y(0) = 5 \text{ is ?}$$

$$2\frac{dy}{dx} + x^2 y = 2x + 3$$

$$\int 2\,dy = \int (2x + 3 - x^2 y)\,dx$$

$$2y = \frac{2x^2}{2} + 3x - y\frac{x^3}{3} + c$$

$$y = \frac{2x^2}{2 \times 2} + \frac{3x}{2} - \frac{y \times x^3}{3 \times 2} + c$$

$\Rightarrow$ put $x = 0$, $y = 5$

$$5 = 0 + 0 - 0 + c$$

$$5 = c$$

then $\boxed{y = \frac{x^2}{2} + \frac{3x}{2} - \frac{x^3 y}{2} + 5}$

X.

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \text{ is ?}$$

Sol :-

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Expand by R_1

$$|A| = +1 \begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - a \begin{vmatrix} 1 & b^2 \\ 1 & c^2 \end{vmatrix} + a^2 \begin{vmatrix} 1 & b \\ 1 & c \end{vmatrix}$$

$$= 1(bc^2 - b^2c) - a(c^2 - b^2) + a^2(c-b)$$

$$|A| = bc^2 - b^2c - ac^2 + ab^2 + a^2c - a^2b$$

$$|A| = bc(c-b) - ac(c-a) - ab(b-a).$$

QUESTION # 02

i) Express Determinant.

$$\begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{bmatrix};$$

Sol:. Expand by Row 1

$$= a\begin{vmatrix} b^2 & c^2 \\ b^3 & c^3 \end{vmatrix} - b\begin{vmatrix} a^2 & c^2 \\ a^3 & c^3 \end{vmatrix} + c\begin{vmatrix} a^2 & b^2 \\ a^3 & b^3 \end{vmatrix}.$$

$$= a(b^2c^3 - b^3c^2) - b(a^2c^3 - a^3c^2) + c(a^2b^3 - a^3b^2)$$

$$= ab^2c^3 - ab^3c^2 - a^2bc^3 + a^3bc^2 + a^2b^3c - a^3b^2c$$

Take (abc) common

$$= abc(bc^2 - b^2c - ac^2 + a^2c + ab^2 - a^2b)$$

Ans.

(2)
ii Find the Eigen Value

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

Sol:-

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}$$

:--> we know that
characteristic equ $|A - \lambda I| = 0$

$$\begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Determinent:

$|A - \lambda I| = 0$

$$\begin{bmatrix} 2-\lambda & -1 & -1 & 0 \\ -1 & 3-\lambda & -1 & -1 \\ -1 & -1 & 3-\lambda & -1 \\ 0 & -1 & -1 & 2-\lambda \end{bmatrix} = 0$$

(3)

Expand by R_1.

$$2-\lambda \begin{vmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$

$$-1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix} = 0 \quad — (4)$$

Expand by R_1

$$3-\lambda \begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1) \begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - 1 \begin{vmatrix} -1 & 3-\lambda \\ -1 & -1 \end{vmatrix}$$

$$= (3-\lambda)\left[(3-\lambda)(2-\lambda) - (-1)(-1)\right] + 1\left[(-1)(2-\lambda) - (-1)(-1)\right] - 1$$

$$\left[(-1)(-1) - (-1)(3-\lambda)\right]$$

$$= (3-\lambda)(6-3\lambda-2\lambda+\lambda^2-1) + 1(-2+\lambda-1) - 1(1+3-\lambda)$$

$$= (3-\lambda)(\lambda^2-5\lambda+5)+(-3+\lambda)-(4-\lambda)$$

$$= 3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda-4+\lambda$$

$$= -\lambda^3+8\lambda^2-18\lambda+8 \longrightarrow (i)$$

$$\Rightarrow +1\begin{vmatrix} -1 & -1 & -1 \\ -1 & 3-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$ Expand by C_1

$$\Rightarrow -1\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} -(-1)\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0$$

$$\Rightarrow -1(6-3\lambda-2\lambda+\lambda^2-1)+(-2+\lambda-1)$$

$$\Rightarrow -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda^2+6\lambda-8 \text{ — } (ii)$$

5

$$\Rightarrow -1 \begin{vmatrix} -1 & 3-\lambda & -1 \\ -1 & -1 & -1 \\ 0 & -1 & 2-\lambda \end{vmatrix}$$ Expand by C_1

$$\Rightarrow -1\left[-1\begin{vmatrix} -1 & -1 \\ -1 & 2-\lambda \end{vmatrix} - (-1)\begin{vmatrix} 3-\lambda & -1 \\ -1 & 2-\lambda \end{vmatrix} + 0\right]$$

$$= -\left[-(-2+\lambda-1) + 1(6-3\lambda-2\lambda+\lambda^2-1)\right]$$

$$= -(3-\lambda+\lambda^2-5\lambda+5)$$

$$= -\lambda^2+5\lambda-5-3+\lambda$$

$$= -\lambda+6\lambda-8 \text{ --- (iii)}$$

put i, ii and iii in eq (*)

$$= (2-\lambda)(-\lambda^3+8\lambda^2-18\lambda+8)-\lambda^2+6\lambda-8-\lambda^2+6\lambda$$
$$-8=0$$

$$= -2\lambda^3+16\lambda^2-36\lambda+16+\lambda^4-8\lambda^3+18\lambda^2-8\lambda-\lambda^2-8-\lambda^2+6\lambda+6\lambda$$
$$-8=0$$

$$\Rightarrow \lambda^4-10\lambda^2-32\lambda^2-32\lambda=0$$

(6) By synthetic equation

$$\begin{array}{c|cccc} & 1 & -10 & 32 & -32 \\ 2 & & & -16 & 32 \\ \hline & 1 & -8 & 16 & 0 \end{array}$$

here we get

$(\lambda-2)(\lambda^3-8\lambda+16\lambda)=0$

$\lambda(\lambda-2)(\lambda^2-8\lambda+16)=0$

$\lambda=0$ | $\lambda-2=0$, $\lambda=2$ | $\lambda^2-8\lambda+16=0$

$\lambda^2-4\lambda-4\lambda+16=0$

$\lambda(\lambda-4)-4(\lambda-4)=0$

$(\lambda-4)=0$ | $(\lambda-4)=0$

$\lambda=4$ | $\lambda=4$

So,

$$\boxed{\begin{aligned}\lambda_1&=0\\ \lambda_2&=2\\ \lambda_3&=4\\ \lambda_4&=4\end{aligned}}$$

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QUESTION # 3:- (1)

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$x = 2 \quad , y = 6$$

Sol:-

$$(x^2+3y^2)dx - 2xy\,dy = 0$$

$$(x^2+3y^2)dx = 2xy\,dy$$

Dividing both sides by $2xy\,dx$

$$\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}$$

$$\frac{dy}{dx} = \frac{x}{2y} + \frac{3y}{2x}$$

$$\frac{dy}{dx} = \frac{1}{2}\left[x/y + 3y/x\right] \text{ ---(i)}$$

Let $y = gx$

Diff $dy = g\,dx + x\,dg$

(Both side ÷ (2) by dx

$$\frac{dy}{dx} = g + x\frac{dg}{dx} \quad — (a)$$

put eq (a) in (1)

$$g + x\frac{dg}{dx} = \frac{1}{2}\left[\frac{x}{xg} + 3g\frac{\cancel{x}}{\cancel{x}}\right]$$

$$g + x\frac{dg}{dx} = \frac{1}{2}\left[\frac{1}{g} + 3g\right]$$

Multiplying by "2" bothsides

$$2g + 2x\frac{dg}{dx} = \frac{1}{g} + 3g$$

$$2x\frac{dg}{dx} = \frac{1}{g} + 3g - 2g$$

$$2x\frac{dg}{dx} = \frac{1}{g} + g$$

$$2x\frac{dg}{dx} = \frac{1+g^2}{g}$$

(3) Multiply both sides by $\frac{dx}{dy}$

$$2x\,dy = \frac{1+y^2}{y}\,dx$$

Multiply by $\frac{y}{x(1+y^2)}$

we get

$$\frac{y}{1+y^2}\,dy = \frac{1}{x}\,dx$$

Take "$\int$" both sides

$$\int \frac{2y}{1+y^2}\,dy = \int \frac{1}{x}\,dx + c$$

$$\ln|1+y^2| = \ln x + \ln c$$

Now Take "e" both sides

$$e^{\ln|1+y^2|} = e^{\ln|xc|}$$

$$1+y^2 = xc$$

1)

$1 + g^2 = xc$

put $g = y/x$

$1 + (y/x)^2 = xc$

$$\frac{x^2 + y^2}{x^2} = xc$$

$x^2 + y^2 = x^3 c \longrightarrow$ ii

put $x = 2$, $y = 6$

$(2)^2 + (6)^2 = (2)^3 c$

$4 + 36 = 8c$

$$c = \frac{40}{8}$$

$c = 5$ put in ii

So $x^2 + y^2 = 5x^3$

$y^2 = 5x^3 - x^2$

$y^2 = x^2(5x - 1)$

(5) Taking "$\sqrt{\quad}$" both sides

$$y = x\sqrt{5x-1} \quad , \quad y = -x\sqrt{5x-1}$$