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Paper Linear Algebra

∴ Question No 1:-

Solution

$$v_1 = \begin{bmatrix} 1 \\ 6 \\ 7 \end{bmatrix}, v_2 = \begin{bmatrix} 6 \\ 7 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$$

need to solve

$$x_1 \vec{v}_1 + x_2 \vec{v}_2 + x_3 \vec{v}_3 = \vec{0}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 6 & 7 & 2 & 0 \\ 7 & 2 & 9 & 0 \end{array} \right], R_2 + 1 \downarrow$$

$$\sim \left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 6 & 7 & 2 & 0 \\ 7 & 2 & 9 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 7 & 2 & 9 & 0 \\ 7 & 2 & 9 & 0 \end{array} \right] \Rightarrow R_2 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & 9 & 7 & 0 \\ 0 & 6 & -6 & 0 \\ 7 & 2 & 9 & 0 \end{array} \right] R_3 - 7 \rightarrow \left[\begin{array}{ccc|c} 1 & 9 & 7 & 0 \\ 0 & 6 & -6 & 0 \\ 0 & -5 & 2 & 0 \end{array} \right] \Rightarrow R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 0 & 6 & -6 & 0 \\ 0 & -5 & 2 & 0 \end{array} \right] R_2 + R_3$$

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$$\left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & -5 & 2 & 0 \end{array} \right], R_3 + 9$$

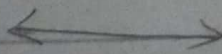
$$\left[\begin{array}{ccc} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & -3 & 4 \end{array} \right], R_2 + R_3$$

$$\left[\begin{array}{ccc} 1 & 6 & 7 \\ 0 & -2 & 0 \\ 0 & -3 & 4 \end{array} \right], R_2 + 3$$

$$\left[\begin{array}{ccc} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & -3 & 4 \end{array} \right], R_3 + R_2 \rightarrow \left[\begin{array}{ccc} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & -2 & 0 \end{array} \right], R_3 \times 0$$

$$= \left[\begin{array}{ccc} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

this show that
that matrix is
infinite
this is not linear
independent



Q2 part 2

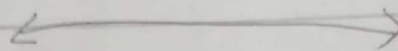
we want to prove $T(cu) = cT(u)$

$$\begin{aligned} T(cu) &= T(c[x_1, y_1, z_1]) \\ &= T([cx_1, cy_1, cz_1]) \\ &= [cx_1 + cy_1, cx_1 - cy_1, cz_1] \end{aligned}$$

and

$$\begin{aligned} cT(u) &= cT([x_1, y_1, z_1]) \\ &= c[x_1 + y_1, x_1 - y_1, z_1] \\ &= [c(x_1 + y_1), c(x_1 - y_1), cz_1] \\ &= [cx_1 + cy_1, cx_1 - cy_1, cz_1] \end{aligned}$$

So $T(cu) = cT(u)$.



Q3 part a

Sol: polynomials of degree n does not form a vector space because they don't form a set closed under addition.

For instance:

$$x^n - x^n = 0$$

which is not of degree n .

So, don't get confused with the set of polynomials of degree less or equal than n , which form a vector space of dimension $n+1$. We often work with this space.

Question no 3

Solution, part: A

This implies that B_1

$$k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka & kb \\ kc & kd \end{bmatrix} \text{ for } k \in \mathbb{R}$$

$$k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka_1 & kb_1 \\ kc_1 & kd_1 \end{bmatrix}$$

$$= \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \in \mathbb{R} \text{ (taking } a' = ka_1 \in \mathbb{R}, b' = kb_1 \in \mathbb{R} \\ \text{ } \searrow = kc_1 \text{ and } d' = kd_1 \in \mathbb{R})$$

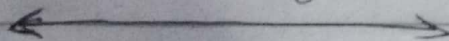
Now

$$\begin{aligned} a' - 2b' &= ka_1 - 2kb_1 \\ &= k(a_1 - 2b_1) \\ &= k \cdot 0 \\ &= 0 \end{aligned}$$

$$\begin{aligned} c' - 2d' &= kc_1 - 2kd_1 \\ &= k(c_1 - 2d_1) \\ &= k \cdot 0 \\ &= 0 \end{aligned}$$

So $kA_1 \in \mathbb{R}$

This implies that $\forall C \in M_{2,2}$ satisfied $A_1 + A_2 \in V$
and $kA_1 \in V$, where $k[A] \in \mathbb{R}$ and
 $k \in \mathbb{R}$ is any constant



Q5

Part A $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$= \frac{1}{\det(A)} \times \text{adj}(A)$$

$$= \frac{1}{a \cdot d - (b \cdot c)} \times \begin{bmatrix} d & -b \\ c & a \end{bmatrix}$$

$$= \begin{bmatrix} \frac{d}{a \cdot d - (b \cdot c)} & \frac{-b}{a \cdot d - (b \cdot c)} \\ \frac{c}{a \cdot d - (b \cdot c)} & \frac{a}{a \cdot d - (b \cdot c)} \end{bmatrix} \text{ Ans.}$$

Part B) $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\det(A) = |A| = a \cdot d - b \cdot c$$

Part C

Question No 4
part a)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 6 \\ 2 & 1 & 9 \end{bmatrix}$$

$$= 1 \det \begin{bmatrix} 7 & 6 \\ 1 & 9 \end{bmatrix} - 1 \det \begin{bmatrix} 6 & 6 \\ 2 & 9 \end{bmatrix} + 1 \det \begin{bmatrix} 6 & 7 \\ 2 & 1 \end{bmatrix}$$

$$= 1 [63 - 6] - 1 [54 - 12] + 1 [14 - 6]$$

$$= 1 [57] - 1 [42] + 1 [10]$$

$$= 25 \text{ Ans}$$

