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Paper linear Algebra.

: Question No 1 :-

Solution

$$v_1 = \begin{bmatrix} 1 \\ 6 \\ 7 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 6 \\ 7 \\ 2 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 7 \\ 2 \\ 9 \end{bmatrix}$$

need to solve

$$x_1\vec{v_1} + x_2\vec{v_2} + x_3\vec{v_3} = \vec{0}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 6 & 7 & 2 & 0 \\ 7 & 2 & 9 & 0 \end{array}\right], R_2 + 1$$

$$\sim \left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 6 & 7 & 2 & 0 \\ 7 & 2 & 9 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 7 & 8 & 3 & 0 \\ 7 & 2 & 9 & 0 \end{array}\right], R_2 - R_3$$

$$\left[\begin{array}{ccc|c} 1 & 9 & 7 & 0 \\ 0 & 6 & -6 & 0 \\ 7 & 2 & 9 & 0 \end{array}\right] R_3 - 7 \rightarrow \left[\begin{array}{ccc|c} 1 & 9 & 7 & 0 \\ 0 & 6 & -6 & 0 \\ 0 & -5 & 2 & 0 \end{array}\right], R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 0 & 6 & -6 & 0 \\ 0 & -5 & 2 & 0 \end{array}\right] R_2 + R_3$$

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$$\left[\begin{array}{ccc|c} 1 & 6 & 7 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & -5 & 2 & 0 \end{array}\right], R_3 + 2$$

$$\begin{bmatrix} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & -3 & 4 \end{bmatrix}, R_2 + R_3$$

$$\begin{bmatrix} 1 & 6 & 7 \\ 0 & -2 & 0 \\ 0 & -3 & 4 \end{bmatrix}, R_2 + 3$$

$$\begin{bmatrix} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & -3 & 4 \end{bmatrix}, R_3 + R_2 \rightarrow \begin{bmatrix} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & -2 & 0 \end{bmatrix} \rightarrow R_3 \times 0$$

$$= \begin{bmatrix} 1 & 6 & 7 \\ 0 & 1 & -4 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 6 & 7 & 0 \\ 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

this show that
that matrix is
infinite
this is not linea[r]
independent

Q2 part 2

we want to prove $T(cU) = cT(U)$

$$T(cU) = T(c[x_1, y_1, z_1])$$
$$= T([cx_1, cy_1, cz_1])$$
$$= [cx_1 + cy_1, cx_1 - cy_1, cz_1]$$

and

$$cT(U) = cT([x_1, y_1, z_1])$$
$$= c[x_1 + y_1, x_1 - y_1, z_1]$$
$$= [c(x_1 + y_1), c(x_1 - y_1), cz_1]$$
$$= [cx_1 + cy_1, cx_1 - cy_1, cz_1]$$

So $T(cU) = cT(U)$.

Q3 part a

Sol: polynomials of degree n does not from a vector space because don't form a set closed under addition.

For instance:

$$x^n - x^n = 0$$
which is not of degree n.

So, don't get confused with the set of polynomials of degree less or equal then n, which form a vector space of dimension n + 1 we often work with this space.

Question no 3

part. A

Solution:

This implies that B,

$$k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} ka & b \\ kc & d \end{bmatrix} \text{ for } k \in R$$

$$k \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ka_1 & kb_1 \\ kc_1 & kd_1 \end{bmatrix}$$

$$= \begin{bmatrix} a' & b' \\ c' & d' \end{bmatrix} \in R \left(\text{taking } a' = ka_1 \in R,\ b' = kb_1 \text{ \& } c' = kc_1 \text{ and } d' = kd_1 \in R \right)$$

Now

$$a' - 2b' = ka_1 - 2kb_1$$
$$= k(a_1 - 2b_1)$$
$$= k \cdot 0$$
$$= 0$$

$$c' - 2d' = kc_1 - 2kd_1$$
$$= k(c_1 - 2d_1)$$
$$= k \cdot 0$$
$$= 0$$

So $kA_1 \in R$

This implies that $V \subset M_{22}$ Satisfied $A_1 + A_2 \in V$ and $kA_1 \in V$, where $k[A] \in R$ and $k \in R$ is any Constant

Q5

Part A $\quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$= \frac{1}{\det(A)} \times \operatorname{adj}(A)$$

$$= \frac{1}{a \cdot d - (b \cdot c)} \times \begin{bmatrix} d & -b \\ c & a \end{bmatrix}$$

$$= \begin{bmatrix} \frac{d}{a \cdot d - (b \cdot c)} & \frac{-b}{a \cdot d - (b \cdot c)} \\ \frac{c}{a \cdot d - (b \cdot c)} & \frac{a}{a \cdot d - (b \cdot c)} \end{bmatrix} \text{ Ans}$$

Part B) $\quad A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\det(A) = |A| = a \cdot d - b \cdot c$$

part(C)

Question No 4
part a)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 6 & 7 & 6 \\ 2 & 1 & 9 \end{bmatrix}$$

$$= 1\det\begin{bmatrix} 7 & 6 \\ 1 & 9 \end{bmatrix} - 1\det\begin{bmatrix} 6 & 6 \\ 2 & 9 \end{bmatrix} + 1\det\begin{bmatrix} 6 & 7 \\ 2 & 1 \end{bmatrix}$$

$$= 1[63-6] - 1[54-12] + 1[14-6]$$

$$= 1[57] - 1[42] + 1[10]$$

$$= 25 \text{ Ans}$$