Iqra National University, Peshawar
Department of Electrical Engineering

IQRA NATIONAL UNIVERSITY

Assignment
Date:20/4/2020

Course Code:	MTH 102			**Course Title:**	Calculus and analytic geometry
Prerequisite:				**Instructor:**	HIMAYATULLAH
Module:	3	**Program:**	BEE	**Total Marks:**	30

Q1.	(a)	. Identify $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$	Marks 5 CLO1 C1
	(b)	Find the first order derivatives of the function $y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right)$	Marks 5 CLO1 C1
Q2	(a)	. A dynamite blast blows up a heavy rock with launch velocity of 160m/sec reaches a hight of $s = 160t - 16t^2$ ft after t sec, (i) How high does the rock go (ii) Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down (iii) find the acceleration of the rock at time 5sec	Marks 10 CLO2 C2
Q3	(a)	Does the curve $y=x^4-2x^2+2$ have nay horizontal tangent if so where ?	Marks 10

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Subject :- Calculas.

Teacher :- Mr. Himytullah.

Exam :- Mid-terms

Question Number 1

- (a) -

Identity :- $\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$

$\lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h}$ $\left(\frac{0}{0}\right)$.

Multiplay and devide with $\sqrt{2+h} + \sqrt{2}$.

$= \lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \times \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}}$

$= \lim_{h \to 0} \frac{(\sqrt{2+h})^2 - (\sqrt{2})^2}{h\{(\sqrt{2+h} + \sqrt{2})\}}$

$= \lim_{h \to 0} \frac{2+h-2}{h\{\sqrt{2+h} + \sqrt{2}\}}$

$= \lim_{h \to 0} \frac{h}{h\{\sqrt{2+h} + \sqrt{2}\}}$

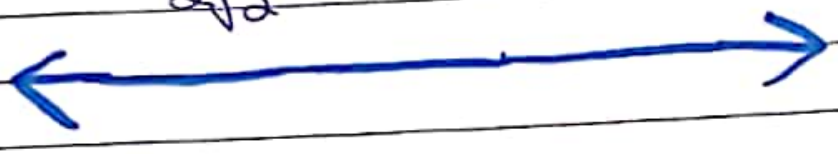
$= \lim_{h \to 0} \frac{1}{\sqrt{2+h} + \sqrt{2}}$

Now by Putting $\lim$

$$= \frac{1}{\sqrt{2+0}+\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}+\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} \quad \text{Ans/}$$



-(b)-

Find the first order derivate of the function.

$$y = \left(x + \frac{1}{x}\right)\left(x - \frac{1}{x} + 1\right).$$

$$y = \left(x^2 - \frac{x}{x} + x + \frac{x}{x} - \frac{1}{x^2} + \frac{1}{x}\right).$$

$$y = \left(x^2 - 1 + x + 1 - \frac{1}{x^2} + \frac{1}{x}\right).$$

$$y = \left(x^2 + x - \frac{1}{x^2} + \frac{1}{x}\right)$$

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Taking derivative on both sides.

$$\frac{dy}{dx} = \frac{d}{dx}\left(x^2 + x - \frac{1}{x^2} + \frac{1}{x}\right)$$

$$= \frac{d}{dx}(x)^2 + \frac{dx}{dx} - \frac{d}{dx}(x)^{-2} + \frac{d}{dx}(x)^{-1}$$

$$= 2x + 1 - (-2)x^{-3} - 1(x)^{-2}$$

$$= 2x + 1 + 2x^{-3} - x^{-2}$$

Question Number 2

-(a)-

A dynamite blast blows up a heavy rock with launch velocity of 160 m/sec reaches a height of $S = 160t - 16t^2$ ft after 1 sec.

-(i)- How high does the rock go.

Given

$$S = 160t - 16t^2$$

At any time "t" the velocity

$$v = \frac{ds}{dt} = \frac{d}{dt}(160t - 16t^2)$$

$$v = 160 - 32t$$

At maximum height $v = 0$

So,

$$160 - 32t = 0 \Rightarrow t = \frac{160}{32} = 5$$

$$t = 5 \text{ sec}$$

Then

$$S_{max} = S(5) = 160(5) - 16(5)^2$$

$$\boxed{S_{max} = 400 \text{ ft}}$$

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(11)- Find the velocity and speed of the rock when it is 256 ft above the ground on the way up and down.

Given that:

$$S = 256 + 96t - 16t^2$$

then:

$$160t - 16t^2 = 256$$

$$16t^2 - 160t + 256 = 0$$

$$16(t^2 - 10t + 16) = 0$$

$$t^2 - 10t + 16 = 0$$

$$t^2 - 8t - 2t + 16 = 0$$

$$t(t-8) - 2(t-8) = 0$$

$$(t-8)(t-2) = 0$$

$t - 8 = 0$; $t - 2 = 0$

$\boxed{t = 8}$ $\boxed{t = 2}$

Since:

$$v = 160 - 32t$$

$t_1 = 2$ sec

$$v(2) = 160 - 32(2)$$

$$= \boxed{96 \text{ m/sec}}$$

At $t_2 = 8$ sec

$$v(8) = 160 - 32(8)$$

$$= \boxed{-96 \text{ m/sec}}$$

- (iii) Find the acceleration of the rock at time 5 sec.

Since:

$$v = 160 - 32t$$

So, acceleration:..

$$a = \frac{dv}{dt} = \frac{d}{dt}(160 - 32t)$$

$$a = 0 - 32 = -32 \text{ m/sec}^2$$

$$\boxed{a = -32 \text{ m/sec}^2}$$

Question Number 3

-(a)-

Does the curve $y = x^4 - 2x^2 + 2$ have any horizontal tangent if so where?

Solution:-

$$y = x^4 - 2x^2 + 2$$

Taking derivate on both sides

$$\frac{dy}{dx} = \frac{d}{dx}\left\{x^4 - 2x^2 + 2\right\}$$

$$\frac{dy}{dx} = \frac{d}{dx}(x)^4 - 2\frac{d}{dx}(x)^2 + \frac{d}{dx}2$$

$$\frac{dy}{dx} = 4x^3 - 4x + 0$$

$$\frac{dy}{dx} = 4x^3 - 4x$$

If the tangent is horizontal then

$$\frac{dy}{dx} = 0$$

So,

$$4x^3 - 4x = 0$$

$$4x(x^2-1)=0$$

$4x=0$; $x^2-1=0$

$x=0$; $\sqrt{x^2}=\sqrt{1}$

So, $x=\pm 1$

$x=1$ $x=-1$ $x=0$

Corresponding point "x"

if $x=0$

$$y=x^4-2x^2+2$$

$$y=(0)^4-2(0)^2+2$$

$$\boxed{y=2}$$

if $x=1$

$$y=(1)^4-2(1)^2+2$$

$$y=1-2+2$$

$$\boxed{y=1}$$

if $x=-1$

$$y=(-1)^4-2(-1)^2+2$$

$$y=1-2+2$$

$$\boxed{y=1}$$

When :-

$(0,2)$ $(1,1)$ and $(-1,1)$