Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	**Engr Phir Meher Ali Shah**	**Total Marks:**	30

Student Details

Name: **Sajid Ahmad** **Student ID:** **12671**

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \left\{2, \underset{\uparrow}{1}, -2, 3, -4\right\} \quad , h[n] = \left\{\underset{\uparrow}{3}, 1, 2, 1, 4\right\}$$	Marks 5 CLO 2

	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $$x(n) = \begin{cases} (\frac{1}{4})^{n}, & n \ge 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$$ ii. $$x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$$	Marks 10 CLO 2

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Subject : DSP

Module : 10th Semester
Instructor: Engr. Pir Meher Ali shah
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Q1
(a) Consider the following analog signal

$$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$$

(i) Determine the minimum sampling rate required to avoid ailiasing.

Nyquist criteria:

$$F_s \geq 2f_{max}$$

$$f_1 = \frac{100\pi}{2\pi}$$

$$\boxed{f_1 = 50\,Hz}$$

$$f_2 = \frac{200\pi}{2\pi}$$

$$\boxed{f_2 = 100\,Hz}$$

These f_2 is maximum

So

$$f_s = 2 \times f_{max}$$

$$f_s = 2 \times 100$$

$$\boxed{f_s = 200\,Hz}$$

ii) Suppose that the Signal is sampled at the rate $F_s = 100\,Hz$. what is the discrete-time Signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal.

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(ii) $F_s = 100 Hz$

$= \frac{100}{2} =$

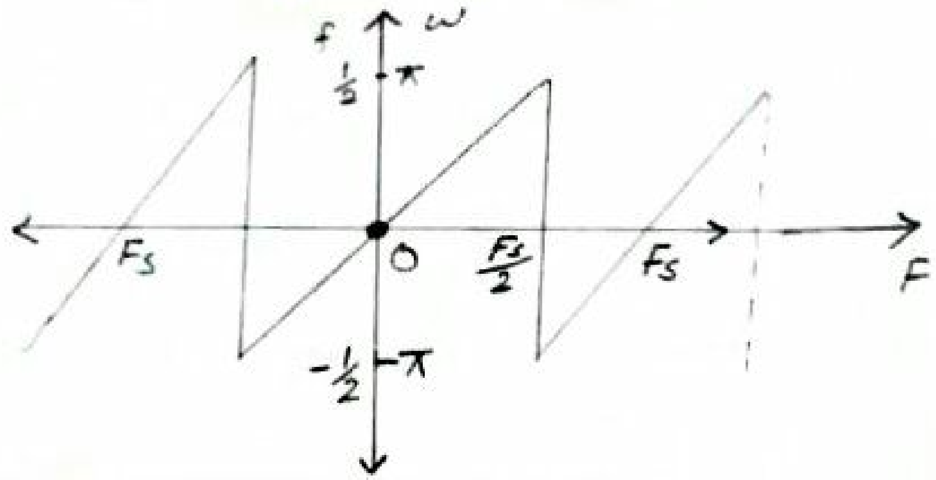
$\boxed{F_s = 50 Hz}$

This is the maximum frequence which is representing by the sampled Signal.

As

$$x_a(n) = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin 2\pi\left(\frac{100}{100}\right)n$$

$$= 3\cos 2\pi\left(\frac{5}{10}\right)n + 4\sin 2\pi n$$



This is the result of Discreate time Signal

(iii) What is the analog Signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation.

Sol:→
Folding frequence of sampled Signal

$$F_{fold} = \frac{F_s}{2} = \frac{100}{2}$$

②

$\boxed{f_{fold} = 50 Hz}$

And the frequency of original Signal is:

$\boxed{f_1 = 50 Hz}$

$\boxed{f_2 = 100 Hz}$

This frequency ineiather equal or greater than the folding frequency.

Hence for ideal interpulation we can construct the original Signal.

$$x_a(t) = 3\cos 100\pi t + 4\sin\pi t$$

We use a sampling frequency at Nyquist rate so the original Signal is constructed.

Q 1 (b) Consider a descreate time Signal which is given by

$$x(n) = \begin{cases} 0.5^n , & n \geq 0 \\ 0 , & n < 0 \end{cases}$$

Sol:

$$F_s = 2HZ$$

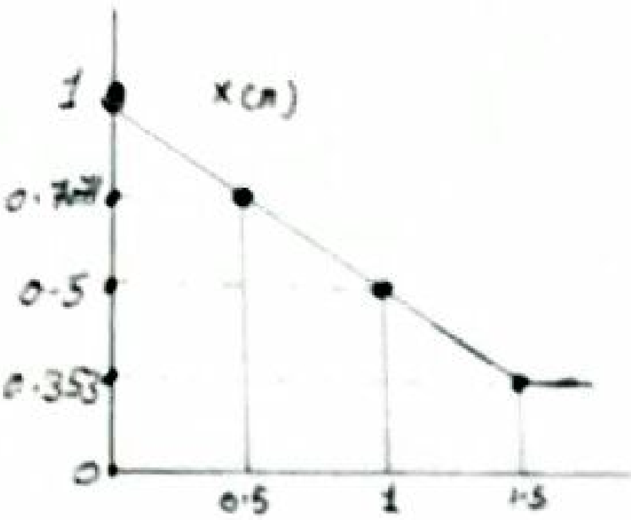
$$F_s = \frac{1}{T}$$

$$T = \frac{1}{F_s}$$

$$\boxed{T = \frac{1}{2} = 0.5 sec}$$

(i) Draw the sampled Signal.

$x \cdot n$	0.5^n
0	1
0.5	0.7071
1	0.5
1.5	0.353



$$\boxed{T = 0.5 sec}$$

(ii) The samples of the Signals are intended to carry 3 bits per simple.
Determine the quantization level & quantization resolution to quantized the sampled Signal achieved in part (i)

Sol →

$$l = 2^n$$

$$n = \text{bits} = 3$$

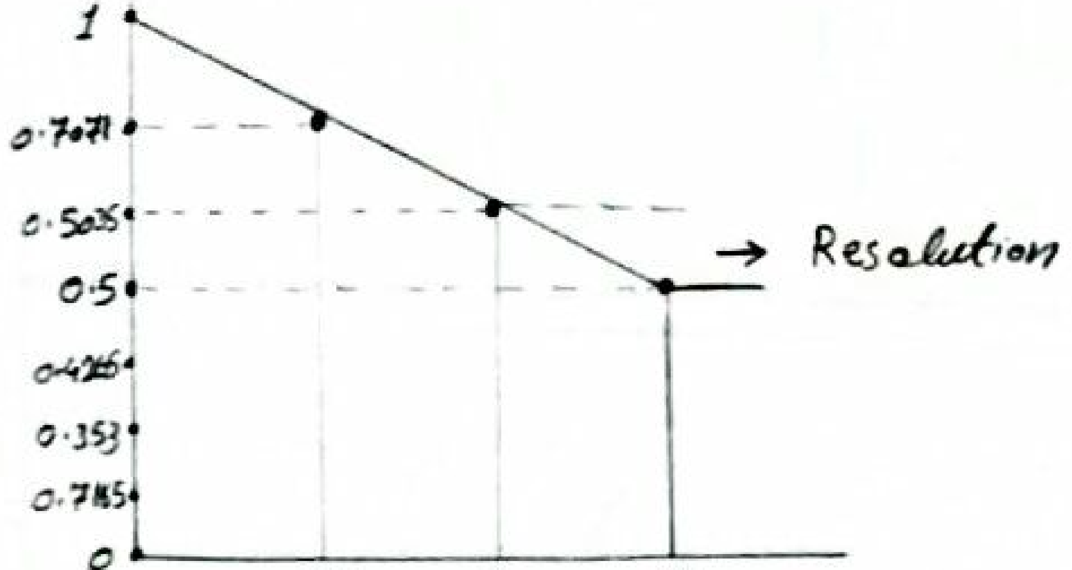
$$l = 2^3$$

$$\boxed{l = 8 \text{ levels}}$$

$$\text{Resolution} = \frac{x_{max} - x_{min}}{l}$$

$$= \frac{1-0}{8} = \frac{1}{8}$$

$$\boxed{\text{Resolution} = 0.125}$$



(iii) Performed the process of Truncation and rounding off on all the values of the sampled Signal and find the quantization error for each of the sampled data.

Express your answer in tabular form.

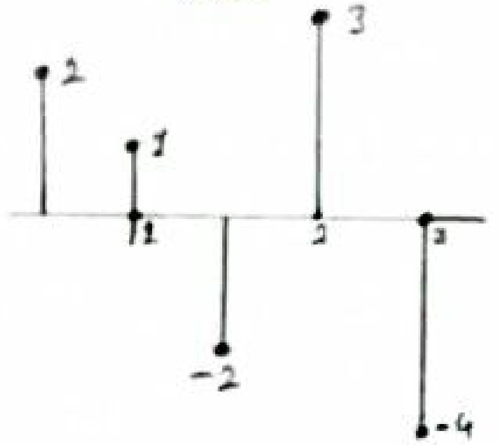
S.No	Discrete-Time Signal	Truncation	Rounding	error
0	1	1.0	1.0	0.0
1	0.8535	0.8	0.9	-0.1
2	0.7071	0.7	0.7	0.0
3	0.6835	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.4265	0.4	0.4	0.0
6	0.353	0.3	0.4	-0.1
7	0.1765	0.1	0.2	-0.1

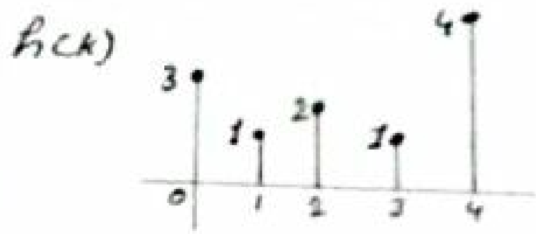
Q:2 (a) Determine the response of the system to the following input signal with given impulse response

$$x(n) = \{2, \underset{\uparrow}{1}, -2, 3, -4\} \qquad h(n) = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}$$

Sol:->

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$





h(-k) folded Signal

$$y(0) = \sum_{k=1}^{0} x(-1)\, h(-1) + x(0)\, h(0)$$

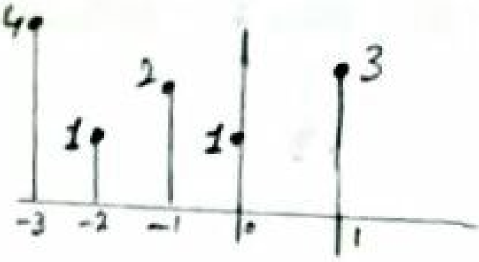
$$y(0) = (2)(1) + (1)(3)$$

$$y(0) = 2 + 3$$

$$\boxed{y(0) = 5}$$

for $n = 1$

$h(1-k)$



$$y(1) = \sum_{x=1}^{1} x(n)\, h(1-k)$$

$$y(1) = x(-1)\,h(-1) + x(0)\,h(0) + 0\,x(1)\,h(1) + (x)\,h(1) + x(2)\,x(2) + 0(x)(3)\,h(3)$$

For $n = 2$

$$y(2) = (2)(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$$

$$y(2) = 8 + 1 - 4 + 3 - 12$$

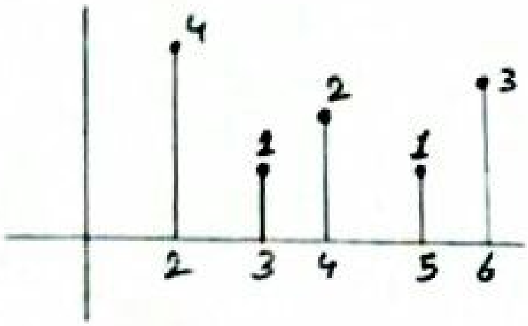
$$y(2) = 9 - 4 + 3 - 12$$

$$y(2) = 12 - 4 - 12$$

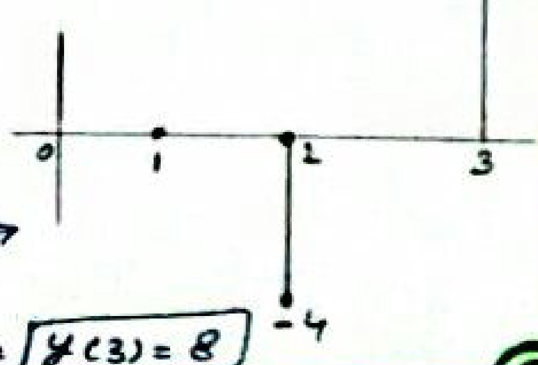
$$\boxed{y(2) = -4}$$

for $n = 3$

$h(3-k)$



$$y(3) = \sum_{k=2}^{3} x(x)\, h(n-k)$$



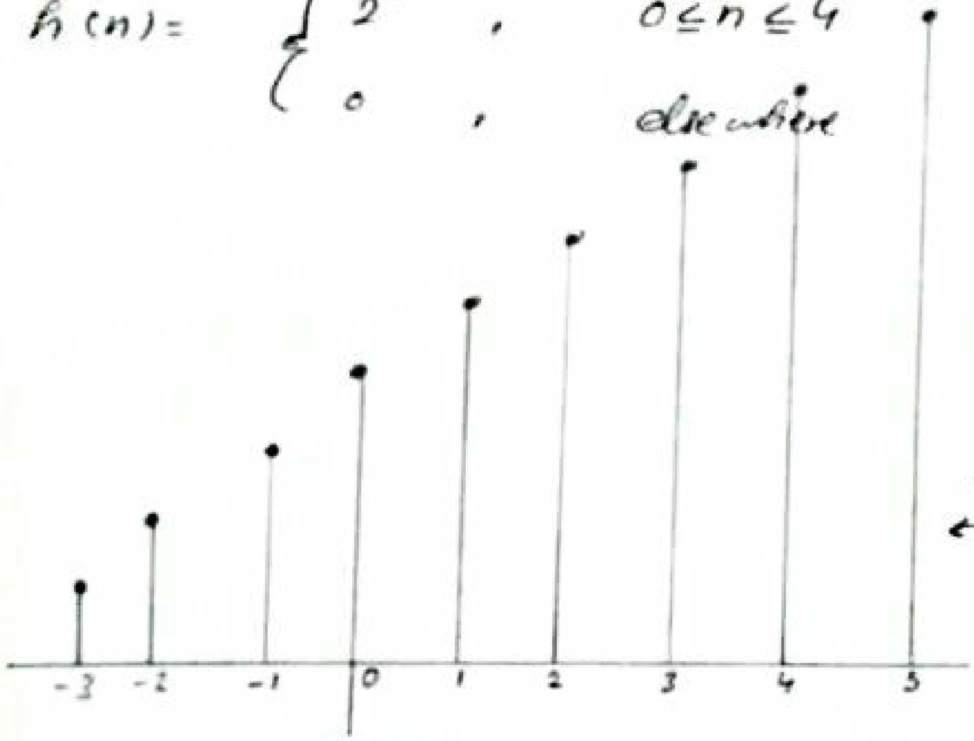
$$= x(2)\,h(2) + x(3)\,h(3)$$

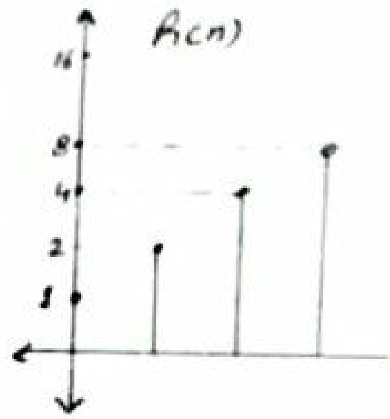
$$= (3)(4) + (-4)(1) = 12 - 4 = \boxed{y(3) = 8}$$

Q2 (b) Compute the convolution $y(n)$ of the following Signal.

$$y(n) = \begin{cases} a^{n+1} , & -3 \le n \le 5 \\ 0 , & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 2^{n} , & 0 \le n \le 4 \\ 0 , & \text{elsewhere} \end{cases}$$

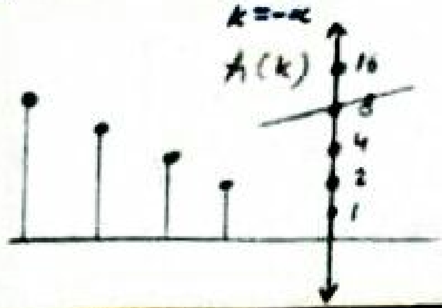




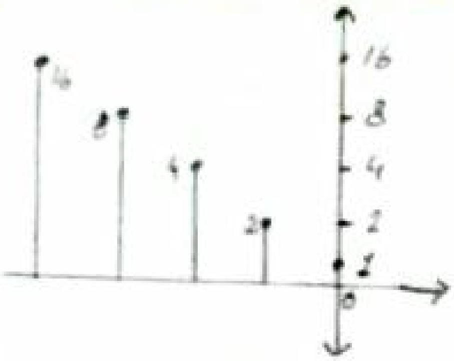
$$x(n) = \{ a^{-2}, a^{-1}, 1, a, a^{2}, a^{3}, a^{4}, a^{5}, a^{6} \}$$

$$h(n) = \{ 1, 2, 4, 8, 16 \}$$

$$y(n_0) = \sum_{k=-\infty}^{\infty} x[k]\, h(n_0 - k)$$

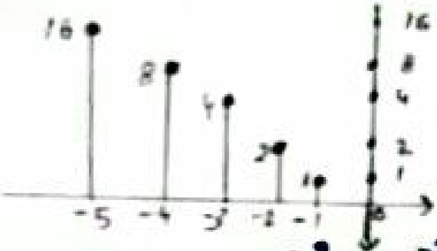


$y(0) = a - 2 - 4a^{-1} + 8a^{-2}$



$h(k)$

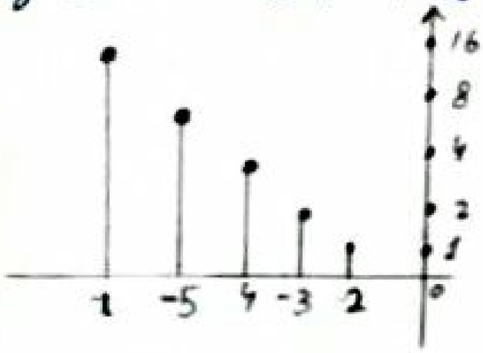
$y(-1) = 1 + 2a^{-1} + 4a^{-2}$



$h(-1-k)$

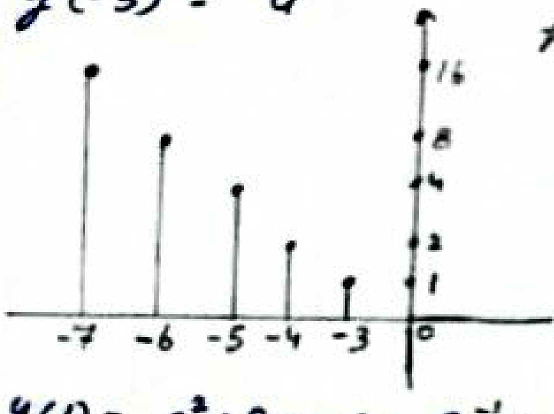
$y(-2) = 2a^{-2} + a^{-1}$

$h(-2-k)$

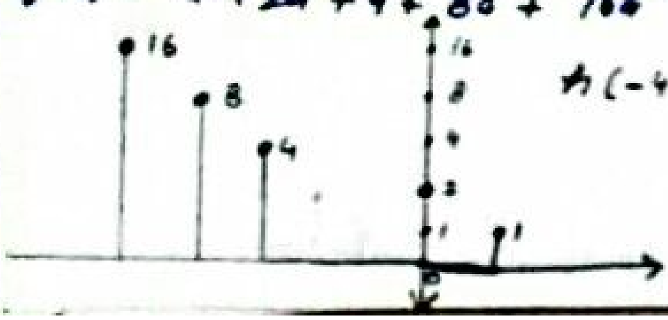


$y(-3) = a^{-2}$

$h(-3-k)$

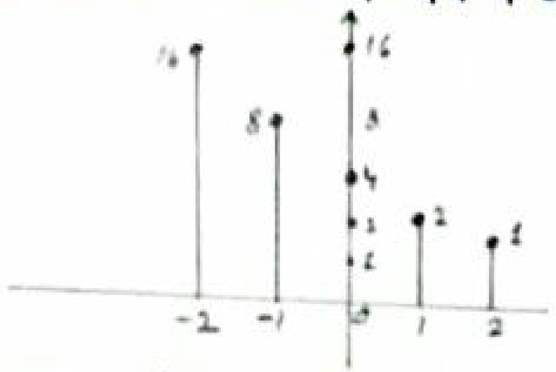


$y(1) = a^2 + 2a + 4 + 8a^{-1} + 16a^{-2}$



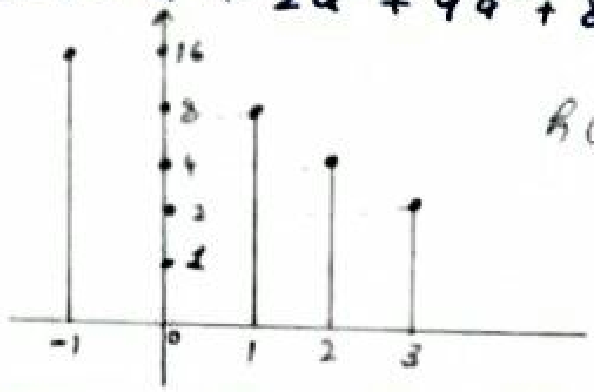
$h(-4-k)$

$y(2) = a^3 + 2a^2 + 4a + 8 + 16a^{-1}$



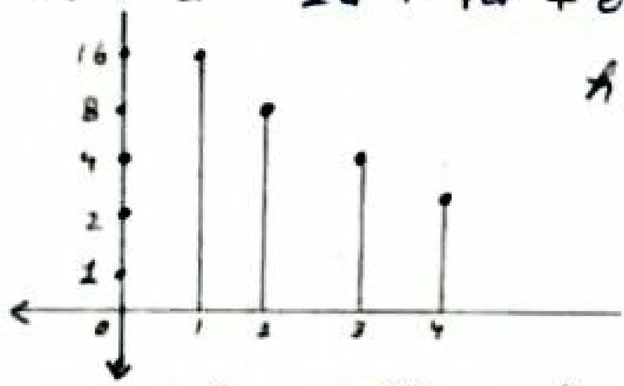
$h(-5-k)$

$y(3) = a^4 + 2a^3 + 4a^2 + 8a + 16$



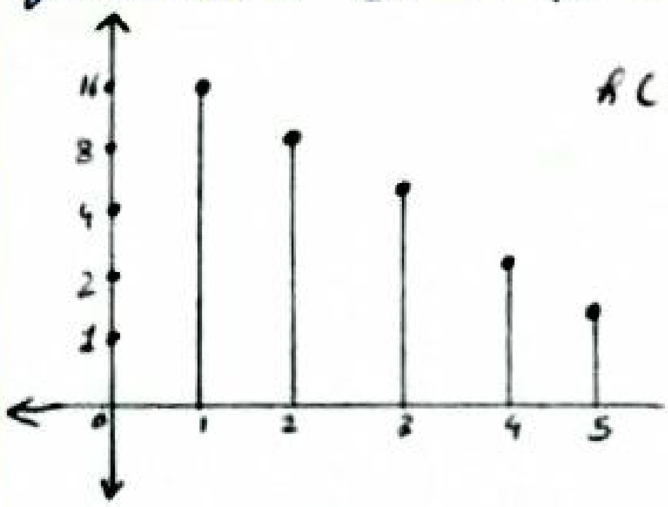
$h(3-k)$

$y(4) = a^5 + 2a^4 + 4a^3 + 8a^2 + 16a$



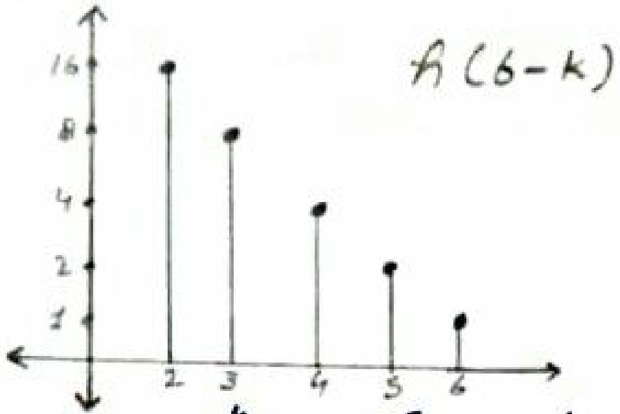
$h(4-k)$

$y(5) = 16a^2 + 8a^3 + 4a^4 + 2a^5 + a^6$

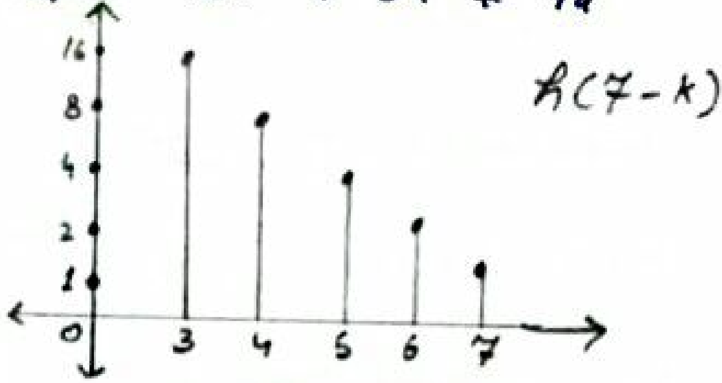


$h(5-k)$

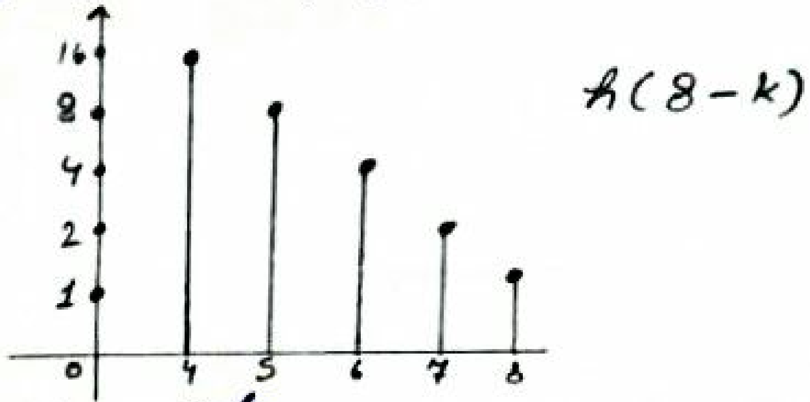
$y(6) = 16a^3 + 8a^4 + 4a^5 + 2a^6$



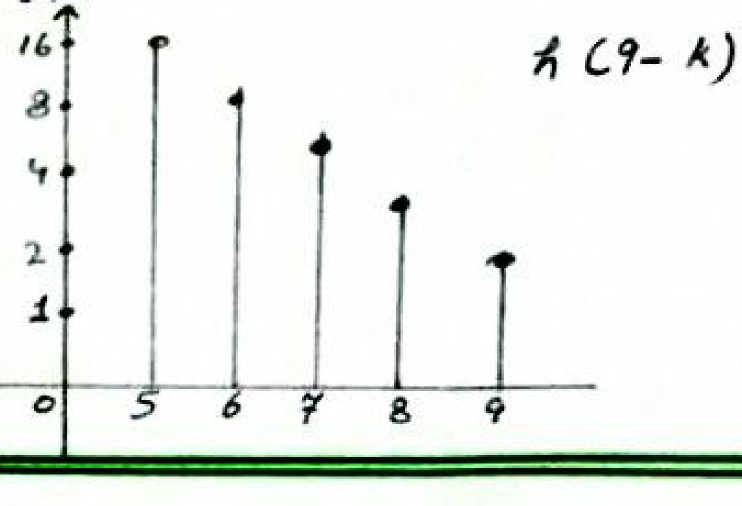
$y(7) = 16a^4 + 8a^5 + 4a^6$



$y(8) = 16a^5 + 8a^6$



$y(9) = 16a^6$



Q3. Determine the Z-Transform of the following Signals and also sketch its Region of Convergence. (ROC)

(i) $$x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$$

Sol:-

The Z-transform pair is

$$x(n) = a^n c(x) \Leftrightarrow x(z) \frac{1}{1-\alpha z^{-1}} \quad \text{Roc } |z|$$

put the values of the above Equation we get

$$x(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^4 z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} - 1$$

Using geometric series

$$x(z) = \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z} - 1$$

Taking L.C.M

$$x(z) = \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - (1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1 - \frac{1}{3} z + 1 - \frac{1}{4} z^{-1} - \left(1 - \frac{1}{3} z - \frac{1}{4} z^{-1} + \frac{1}{12} \cancel{z^{-1}} \cancel{z}\right)}{\left(1 - \frac{1}{4} z\right)\left(1 - \frac{1}{3} z\right)}$$

$$= \frac{\cancel{1} - \cancel{\frac{1}{3} z} + \cancel{1} - \cancel{\frac{1}{4} z^{-1}} - \cancel{1} + \cancel{\frac{1}{3} z} + \cancel{\frac{1}{4} z^{-1}} + \frac{1}{12}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

$$= \frac{1 + \frac{1}{12}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

Taking L.C.M

$1 + \frac{1}{12}$

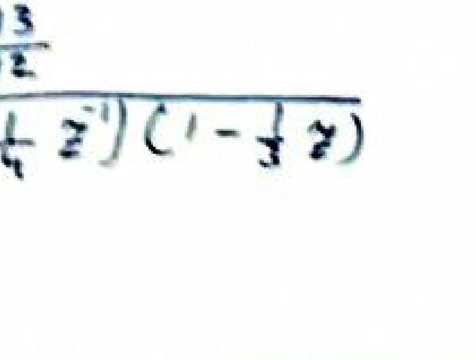
$\frac{12+1}{12} = \frac{13}{12}$

$$= \frac{\frac{13}{12}}{\left(1 - \frac{1}{4} z^{-1}\right)\left(1 - \frac{1}{3} z\right)}$$

So

$$\boxed{\text{ROC is } \frac{1}{4} < |z| < 3}$$

Sketch



Q 3.
(ii) $$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n, & n \geq 0 \\ 0, & \text{else where} \end{cases}$$

Sol:→

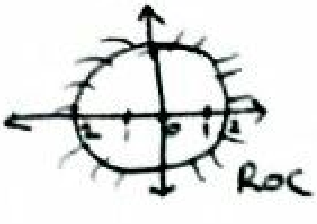
$$x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3, & n \geq 0 \\ 0, & \text{elsewhere} \end{cases}$$

So in Z-Transform form

$$x(z) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

Sketch

$$\frac{1}{1 - \frac{1}{2} z^{-1}} - \frac{1}{1 - 3z^{-1}}$$



Taking L.C.M

$$x(z) = \frac{\cancel{1} - 3z^{-1} - \cancel{1} - \frac{1}{2} z^{-1}}{(1 - \frac{1}{2} z^{-1})(1 - 3z^{-1})}$$

$$= \frac{-\frac{5}{2} z^{-1}}{(1 - \frac{1}{2} z^{-1})(1 - 3z^{-1})}$$

$\boxed{\text{Hence the Roc is } |z| > 3}$

The end