



**ID : 14103**

**SUBJECT : STATISTICS**

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**FINAL TERM PAPER**

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**Question No: 01**

**Find averages (A.M, G.M, H.M) of the following table (s) also justify their logical relationships.**

**a.**

Number of children per family	Number of families
1	4
2	13
3	9
4	4
5	1

**b.**

marks	frequency
0 – 9	2
10 – 19	31
20 – 29	73
30 – 39	85
40 – 49	28

**Solution:**

Q1 (a)

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Number of children per family $x$	Number of families $f$	$F \cdot x$	$F \cdot \log x$	$F/x$
1	4	4	$4 \times \log 1 = 0$	$4/1 = 4$
2	13	26	$13 \times \log 2 = 3.913$	$13/2 = 6.5$
3	9	27	$9 \times \log 3 = 4.294$	$9/3 = 3$
4	4	16	$4 \times \log 4 = 2.408$	$4/4 = 1$
5	1	5	$1 \times \log 5 = 0.699$	$1/5 = 0.2$
$\Sigma f = 31$		$\Sigma Fx = 74$	$\Sigma F \log x = 11.314$	$\Sigma F/x = 14.7$

Arithmetic Mean

$$A.M = \frac{\Sigma Fx}{\Sigma f} = \frac{74}{31}$$

$$A.M = 2.3871$$

Geometric Mean

$$G.M = \text{Antilog} \left( \frac{\Sigma (F \log x)}{\Sigma f} \right)$$

$$G.M = \text{Antilog} \left( \frac{11.314}{31} \right)$$

$$G.M = \text{Antilog} (0.365)$$

$$G.M = 2.317$$

## HARMONIC MEAN

$$H.M = \frac{\sum F}{\sum \left(\frac{F}{x}\right)}$$

$$H.M = \frac{31}{14.7}$$

$$H.M = ~~2.1088~~ 2.1088$$

Logic relationship of A.M, G.M, H.M

$$A.M \geq G.M \geq H.M$$

$$2.516 > 2.317 > 2.1088$$

logical Relationship of A.M,  
G.M & H.M Justified.

(b)

Class Interval	Frequency (F)	mid Point	F · m	F log m	F/m
0-9	2	4.5	$2 \times 4.5 = 9$	$2 \log 4.5 = 1.306$	$\frac{2}{4.5} = 0.444$
10-19	31	14.5	$31 \times 14.5 = 449$	$31 \log 14.5 = 36.002$	$\frac{31}{14} = 2.158$
20-29	73	24.5	$73 \times 24.5 = 1789.5$	$73 \log 24.5 = 130.715$	$\frac{73}{24.5} = 2.99$
30-39	85	34.5	$85 \times 34.5 = 2932.5$	$85 \log 34.5 = 461.54$	$\frac{85}{34.5} = 2.464$
40-49	28	44.5	$28 \times 44.5 = 1246$	$28 \log 44.5 = 46.154$	$\frac{28}{44.5} = 0.629$
	$\Sigma F = 219$		$\Sigma F \cdot m = 6425$	$\Sigma (F \cdot \log m) = 315.586$	

Arithmetic Mean:

$$A.M. = \frac{\Sigma(F \cdot m)}{\Sigma F}$$

From the above data  $\Sigma(F \cdot m) = \frac{6425}{219}$

$$A.M. = 29.34$$

$$A.M. = \frac{6425}{219}$$

## Geometric Mean

$$G.M = \text{Antilog} \left( \frac{\sum (F \times \log m)}{\sum F} \right)$$

$$G.M = \text{Anti log} \left( \frac{315.586}{219} \right)$$

$$G.M = \text{Anti log} (1.441)$$

$$G.M = 27.607$$

## Harmonic Mean

$$H.M = \frac{\sum F}{\sum (F/m)}$$

$$H.M = \frac{219}{8.655}$$

$$H.M = \frac{219}{8.655} = 25.303$$

logical relationship of A.M, G.M, H.M

$$AM \geq GM \geq HM$$

**Question No: 02**

**Find Median & Mode of the following tables**

**a.**

Number of children per family	Number of families
1	4
2	13
3	9
4	4
5	1

**b.**

marks	frequency
0 – 9	2
10 – 19	31
20 – 29	73
30 – 39	85
40 – 49	28

Solution:

(5)

Q.2

(a)

x	f	C. f
1	4	4
2	13	17
3	9	26
4	4	30
5	1	31

$$\sum f = 31$$

$$n = \sum f = 31$$

n is odd.

Median is the central value

$$\text{Position of central value} = \frac{n+1}{2} = \frac{31+1}{2}$$

$$= 16^{\text{th}} \text{ term.}$$

From the table above it is clear that 16<sup>th</sup> term is 2. Therefore, [median = 2]



Mode:

Mode is the most frequent from the table. Therefore, it is clear that the highest frequency of 13 is 2.

Therefore;

$$\text{Mode} = 2$$

Q.2 (b)

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Class Interval	F	Class Boundaries	c.f	Mid-Point
0-9	2	-0.5-9.5	2	4.5
10-19	31	9.5-19.5	33	14.5
20-29	73 $F_0$	19.5-29.5	106	24.5
30-39	85 $F_1$	29.5-39.5	191	34.5
40-49	28 $F_2$	39.5-49.5	219	44.5

→ Median & mode lies there.

Median	OR	
$\left(\frac{n+1}{2}\right)^{\text{th}}$ term		$\frac{n}{2}$ th term
$\left(\frac{219+1}{2}\right)^{\text{th}}$ term		$\frac{219}{2}$ th term
$\left(\frac{220}{2}\right)^{\text{th}}$ term		109.5 <sup>th</sup> term
110 <sup>th</sup> term		

$L = 29.5, h = 10$   
 $F = 85, \Sigma F = 219$   
 $C = 106$

$$\text{Median} = L + \frac{h}{F} \left( \frac{\Sigma F}{2} - C \right)$$

$$\text{Median} = 29.5 + \frac{10}{85} \left( \frac{219}{2} - 106 \right)$$

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$$\text{Median} = 29.5 + 0.1176 (35)$$
$$\text{Median} = 29.5 + 0.4118$$
$$\text{Median} \approx 30$$

$$L_1 = 29.5 \quad F_1 = 85$$
$$i = 10 \quad F_2 = 28$$
$$f_0 = 73$$

$$\text{Mode} = L_1 + \left( \frac{F_1 - f_0}{2F_1 - f_0 - F_2} \right) i$$

$$\text{Mode} = 29.5 + \left( \frac{85 - 73}{2(85 - 73 - 28)} \right) \times 10$$

$$\text{Mode} = 29 + \left( \frac{12}{69} \times 10 \right)$$

$$\text{Mode} = 29 + 1.739$$

$$\text{Mode} = 30.74$$

**Question No: 03**

- a. Find Semi Quartile Range & Semi Inter Quartile Range of Q2(a)
- b. Find Variance and Co-efficient of variance of Q2(a)

**Solution :**

Q-3 (a)

Finding Semi Quartile Range of Q.2  
 & Semi Inter Quartile Range of Q.2

n	f	cf	
1	4	4	Semi Q. Formula
2	13	17	$Q.D = \frac{Q_3 - Q_1}{2}$
3	9	26	Formula for Quartile Deviation
4	4	30	
5	1	31	

$$\Sigma f = 31$$

Formula for Inter Quartile deviation Range =  $Q_3 - Q_1$

First finding the quartile values

Finding  $Q_1$  value

Formula,

$$Q_1 = \left( \frac{N+1}{4} \right)$$

$$= \frac{31+1}{4} = \frac{32}{4} = 8$$

$Q_1 = 8$ , the value of 8 exists in the second class i.e. 2.

So, the Quartile 1 = 12

Finding  $Q_3$  Value,

$$Q_3 = 3 \left( \frac{N+1}{4} \right) = 3 \left( \frac{32}{4} \right) = 24^{\text{th}} \text{ Value}$$

If we check the value of 24 in the table, the location tracks to

$$Q_3 = 3$$

Now Finding the semi Quartile range of  $Q_2$ .

$$\text{Formula, } \frac{Q_3 - Q_1}{2} = \frac{3-1}{2} = \frac{2}{2}$$

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The Semi, Inter Quartile R. of  $Q_2$   
is,  $\boxed{1}$

Finding semi Quartile range of  $Q_2$   
calculated as,

$$\frac{Q_3 + Q_1}{2}$$

$$\text{Mid or S-Q. Range} = \frac{3 + 1}{2} = \frac{4}{2} = 2$$

The Interquartile range means  
where the 50% the data is  
located in the table. i.e class (2)

Finding the variance and coefficient of the variance

$x$	$F$
1	4
2	13
3	9
4	4
5	1

→ Measure of dispersion

To find the variation  
First we've to calculate  
the general value.

1. Range = Max - Min  
Hence, the maximum value  
is 5 and the minimum  
is 1

$$R = 5 - 1 \Rightarrow 4$$

Calculating the variance so the data given

$$\text{St. Deviation} - s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{\sum F}}$$

$$s = \sqrt{\frac{\sum F (x_i - \bar{x})^2}{\sum F}}$$



$n$	$f$	$fx$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	$f(x_i - \bar{x})^2$
1	14	14	1-2.51	2.2801	$14 \times 2.2801 = 31.9213$
2	13	26	2-2.51	0.2601	$3 \times 3.813$
3	9	27	3-2.51	0.2401	$2 \times 1.609$
4	4	16	4-2.51	2.2201	$1 \times 8.884$
5	1	5	5-2.51	6.2001	6.2001
	$\Sigma f = 31$	$\Sigma fx = 88$			$\Sigma F = 52.544$

$$\bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{88}{31} \Rightarrow 2.83 \quad f = \frac{\Sigma f(x - \bar{x})^2}{\Sigma f}$$

Putting the values =  $f = \frac{52.544}{31}$

$$\text{Variance} = 1.694$$

To find the efficient we have to find the standard deviation first.

$$\begin{aligned} \text{S.D} &= \sqrt{\text{Variance}} \\ &= \sqrt{\frac{52.5441}{31}} = \sqrt{1.694} \end{aligned}$$

$$\text{Standard deviation} = 1.301$$

And finding co-efficient of variation,

Formula,

$$\text{C.V} = \frac{\text{Standard deviation} \times 100}{\text{Mean}}$$

$$\text{C.V} = \frac{1.301}{2.516} \times 100$$

$$\text{Coefficient of variation} = 51.70\%$$

### Question No: 04

Write down the short notes on the followings:

- Range
- Quartile Range
- Semi Inter Quartile Range
- Variance
- Standard Deviation
- Coefficient of Variation

**Solution :**

#### **Range:**

In statistics, the range is a measure of spread: it's the difference between the highest value and the lowest value in a data set.

Range is pretty simple when it comes to statistics. I will further explain this with the help of an example.

For instance, we are given the following data

13, 19, 30, 15, 19, 32, 17, 28, 33, 19, 30, 34

Therefore to find the range we subtract maximum value from the minimum value to find the range.

The largest value is denoted by "Xm", and smallest value by "Xo" (Xnot)

So in my set of data my maximum is 34, and my minimum value is 13. So when I subtract these two I get a value of 21.

$$\text{Range} = X_m - X_o$$

$$34 - 13 = 21 \text{ (Range)} \text{ Hence, 21 is the range for my data}$$

#### **Quartile Range:**

Quartiles in statistics are values that divide your data into quarters. However, quartiles aren't shaped like pizza slices; Instead they divide your data into four segments according to where the numbers fall on the number line. The four quarters that divide a data set into quartiles are:

The lowest 25% of numbers.

The next lowest 25% of numbers (up to the median).

The second highest 25% of numbers (above the median).

The highest 25% of numbers.

Quartile range is a measure of statistical dispersion being equal to the difference between 75<sup>th</sup> and 25<sup>th</sup> percentile. It is also called mid spread or H-spread.

#### **Semi-interquartile range**

The semi-interquartile range is half of the difference between the upper quartile and the lower quartile.

The semi-interquartile range is a measure of spread or dispersion. It is computed as one half the difference between the 75<sup>th</sup> percentile [often called (Q3)] and the 25<sup>th</sup> percentile (Q1). The formula for semi-interquartile range is therefore:  $(Q3 - Q1) / 2$ .

## Variance:

Variance in statistics is a measurement of the spread between numbers in a data set. That is, it measures how far each number in the set is from the mean and therefore from every other number in the set.

Variance is calculated by taking the differences between each number in the data set and the mean, then squaring the differences to make them positive, and finally dividing the sum of the squares by the number of values in the data set.

The Formula for Variance is  $(\sigma^2)$

$$\text{Variance } \sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

Where

- $x_i$  = the  $i^{\text{th}}$  data point
- $\bar{x}$  = the mean of all data points
- $n$  = The number of data points

## Standard Division :

The standard deviation is a statistic that measures the dispersion of a dataset relative to its mean and is calculated as the square root of the variance. It is calculated as the square root of variance by determining the variation between each data point relative to the mean. If the data points are further from the mean, there is a higher deviation within the data set; thus, the more spread out the data, the higher the standard deviation.

Standard deviation is a statistical measurement in finance that, when applied to the annual rate of return of an investment, sheds light on the historical volatility of that investment. The greater the standard deviation of securities, the greater the variance between each price and the mean, which shows a larger price range. For example, a volatile stock has a high standard deviation, while the deviation of a stable blue-chip stock is usually rather low.

## Formula for Standard Deviation

$$\text{Standard Deviation} = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}}$$

### **Coefficient of Variation:**

The coefficient of variation (CV) is the ratio of the standard deviation to the mean. The higher the coefficient of variation, the greater the level of dispersion around the mean. It is generally expressed as a percentage. Without units, it allows for comparison between distributions of values whose scales of measurement are not comparable.

When we are presented with estimated values, the CV relates the standard deviation of the estimate to the value of this estimate. The lower the value of the coefficient of variation, the more precise the estimate.

**End.**

