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Assignment 01

Digital Signal Processing

Date:

Q (a):

Determine the response $y(n)$ $n \geq 0$ of the system described by the Second-order difference equation

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

to the input $x(n) = 4^n u(n)$

Solution:

Consider the difference equation

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1)$$

The homogenous equation of the system is $y(n) - 3y(n-1) - 4y(n-2) = 0$

The Characteristic equation of the system

$$\lambda - 3\lambda^{-1} - 4\lambda^{-2} = 0$$

$$\lambda^2 - 3\lambda - 4 = 0$$

Determine the root of the Characteristic equation

$$\lambda^2 - 4\lambda + \lambda - 4 = 0$$

$$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$$

$$\lambda(-4)(\lambda + 1) = 0$$

$$\lambda = \{-1, 4\}$$

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The homogenous Solution is

$y_h(n) = c_1 (-1)^n u(n) + C_2 (4)^n u(n)$

Since 4 is a Characteristic root and the equation is

$x(n) = 4^n u(n)$

We assume a Particular Solution of the form

$y_p(n) = K n 4^n u(n)$

Then

$K n 4^n u(n) - 3K(n-1) 4^{n-1} u(n-1) - 4K(n-2) 4^{n-2} u(n-2)$

$= 4^n u(n) + 2(4)^{n-1} u(n-1)$

For $n = 2$

$K(32 - 12) = 4^2 + 8 = 24 \rightarrow K = \frac{6}{5}$

The total Solution is

$y(n) = y_p(n) + y_h(n)$

$= \left[\frac{6}{5} n 4^n + c_1 4^n + c_2 (-1)^n \right] u(n)$

The Solve for C_1 and C_2 we assume that

$y(-1) = y(-2) = 0$ then

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$y(0) = 1$ and

$y(1) = 3y(0) + 4 + 2 = 9$

Hence

$C_1 + C_2 = 1$ and

$\frac{24}{5} + 4C_1 - C_2 = 9$

$4C_1 - C_2 = \frac{21}{5}$

Therefore $C_1 = \frac{26}{25}$ and $C_2 = -\frac{1}{25}$

The total Solution is

$$y(n) = \left[\frac{6}{5} n 4^n + \frac{26}{25} 4^n - \frac{1}{25} (-1)^n\right] u(n)$$

Q (1) Part (B)

Determine the impulse response and unit step response of the System describe by the difference equation

$$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$$

Sol:

Consider the difference eq

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$y(n) = 0.6y(n-1) - 0.08y(n-2) + x(n)$

$y(n) - 0.6y(n-1) + 0.08y(n-2) = x(n)$

To obtain the homogenous equation

Set input

$x(n) = 0$

$y(n) - 0.6y(n-1) + 0.08y(n-2) = 0$

Determine the Solution of the homogenous equation

$y_h(n) = \lambda^n$

Substitute the Solution obtained in the homogenous equation

$\lambda^n \quad 0.6\lambda^{n-1} + 0.8\lambda^{n-2} = 0$

$\lambda^{n-2}(\lambda^2 - 0.6\lambda + 0.08) = 0$

$\lambda^2 - 0.6\lambda + 0.08 = 0$

$(\lambda - 0.2)(\lambda - 0.4) = 0$

there foo the root as:

$\lambda_1 = 0.2 \quad , \lambda - 0.4$

Those the genoral form of the Solution to the homogenous equation is

$y_h(n) = C_1(\lambda_1)^n + C_2(\lambda_2)^n$

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$y(n) = C_1(0.2)^n + C_2(0.4)^n$ ---- (i)

$\lambda = 0.2, \quad \lambda = 0.4$ hence

$y_h(n) = C_1 \frac{1}{5}^n + C_2 \frac{2}{5}^n$

With $x(n) = \delta(n)$ the initial condition are

$y(0) = 1$

$y(1) - 0.6\, y(0) = 0$

$y(1) = 0.6$

Hence $C_1 + C_2 = 1$ and

$\frac{1}{5} C_1 + \frac{2}{5} = 0.6$

$C_1 = -1, \quad C_2 = 3$

Therefore $h(n) = \left[-\left(\frac{1}{5}\right)^n + 2\left(\frac{2}{5}\right)^n\right] u(n)$

The step response is

$(n) = \sum_{k=0}^{n} h(n-k), \; n > 0$

$= \sum_{k=0}^{n} \left[2\left(\frac{2}{5}\right)^{n-k} - \left(\frac{1}{5}\right)^{n-k}\right]$

$= \left\{\frac{1}{0.12}\left[\frac{2}{5}^{n+1} - 1\right] - \frac{1}{0.16}\left[\left(\frac{1}{5}\right)^{n+1} - 1\right]\right\} u(n)$

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Q 2 Part (a)

Determine the causal signal $x(n)$ having z-transform

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Hint:- Take inverse z-Transform using Partial fraction method

Solution:-

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

By Partial fraction method

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-2z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A(1-z^{-1})^2+B(1-2z^{-1})(1-z^{-1})+Cz^{-1}(1-2z^{-1})}{(1-2z^{-1})(1-z^{-1})^2}$$

$$1 = A(1-z^{-1})^2+B(1-2z^{-1})(1-z^{-1})+Cz^{-1}(1-2z^{-1})$$

Put $z=1$

$$1 = (1-0)^2+B(-2)(1-1)+C(1)(1-2)$$

$$1 = 0+0-C$$

$$1 = -C$$

$$C = -1$$

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Put $z = 2$ in eq. ii

$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) + C\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$

$1 = A\left(\frac{1}{2}\right)^4 + B(1-1)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(1-1)$

$1 = \frac{A}{4} + B(0)\left(\frac{1}{2}\right) + C\frac{1}{2}(0)$

$1 = \frac{A}{4} + 0 + 0$

$A = 4$

Put $z = 3$ in eq ii

$1 = A\left(1-\frac{1}{3}\right)^2 + B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) + C\left(\frac{1}{3}\right)\left(1-\frac{2}{3}\right)$

$1 = A\left(\frac{4}{9}\right) + B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + C\left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$

$1 = A\frac{4}{9} + \frac{2}{9}B + \frac{1}{9}C$

$1 = \frac{4}{9}(4) + \frac{2}{9}B - \frac{1}{9}$

$= 1 + \frac{1}{9} - \frac{1}{16} = \frac{2}{9}B$

$-\frac{6}{9} \times \frac{9}{2} = B$

$B = -3$

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Hence $r(n) = [4(2)^n - 3 - n]\, u_n$

Q 2 Part (B)

Determine the Partial fraction expansion of the following

$$X(z) = \frac{1}{1 - 1.5z^{-1} + 0.5z^{-2}}$$

Sol:

Eliminate the -ve Power by multiplying both numerator and de-nominator by z^2

$$X(z) = \frac{z^2}{z^2 - 1.5z + 0.5}$$

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A}{z-1} + \frac{B}{z-0.5}$$

$z = (z-0.5)A + (z-1)B$ — L.C.M

Now Set $z = P_1 = 1$ in eq (1)

We eliminate the term involving

$1 = (1 - 0.5)A$

$A = 2$

Return to eq (1) $z = P_2 = 0.5$ these

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eleminating the term inviding A So We Have

$$0.5 = (0.5-1)B$$

$$B = -1$$

Q3 Part(1)

A two Pole Low Pass filter has the system response

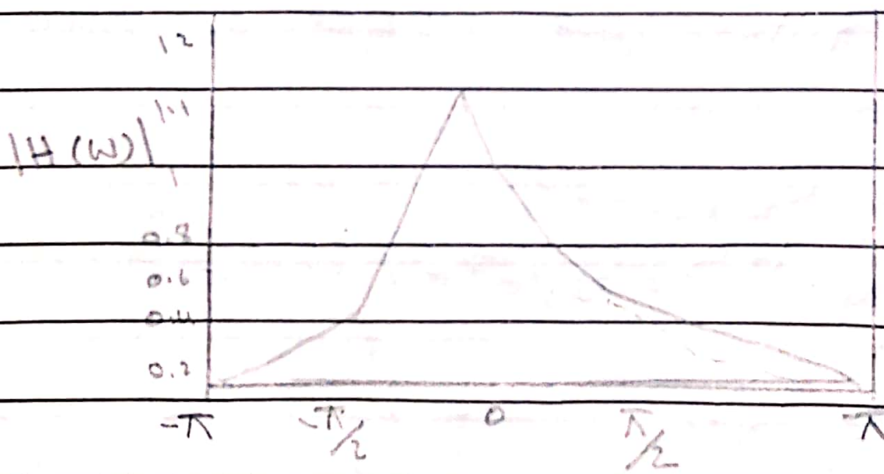
$$H(Z) = \frac{b_0}{(1-PZ^{-1})^2}$$

Determine the Value of b_0 and P Condition

$$H(0) = 1 \text{ and } |H(\frac{\pi}{4})|^2 = \frac{1}{2}$$

Sol:

Linear time-invariant System as frequency



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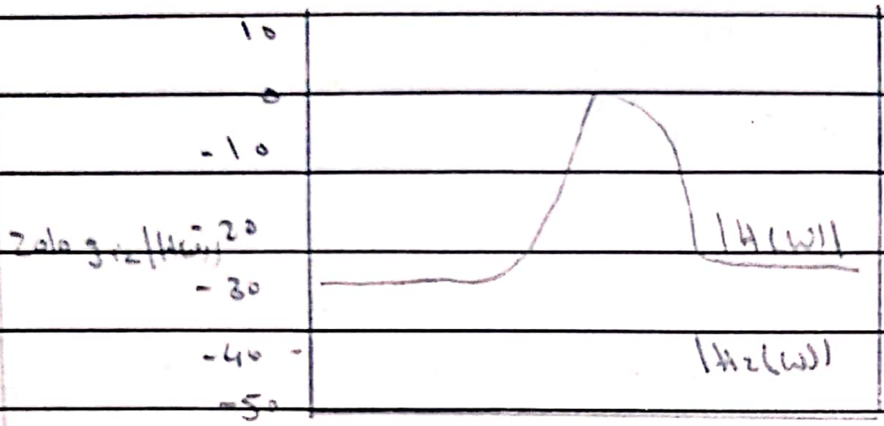

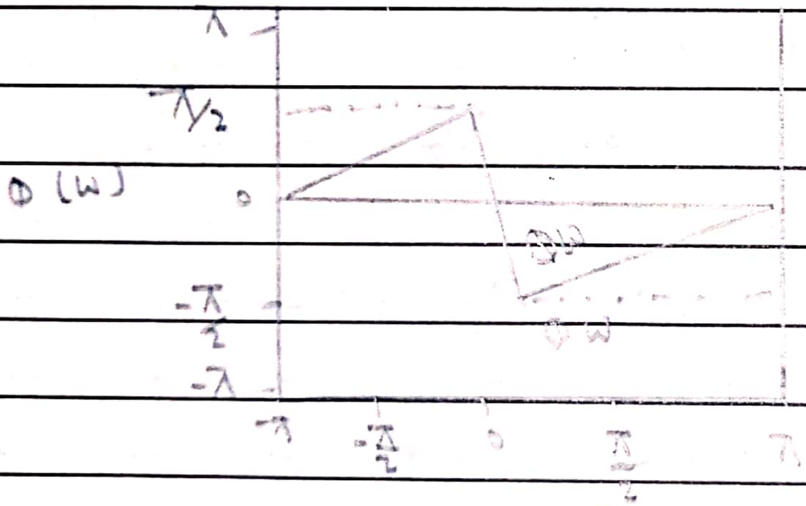

Magnitude of and Phase response of (1) Single Pole filter and (2) a one-Pole One, Zero filter

$H_1(z) = (1-a)/(1-az^{-1})$

$H_2(z) = [(1-a)/2][1+z^{-1}/(1-az^{-1})]$

$a = 0.9$

Now We have determine the Value of b_0 and P

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Such that the frequency response $H(w)$ satisfies the condition

$$H(0) = 1 \quad \text{and} \quad \left|H\left(\frac{\pi}{4}\right)\right|^2 = \frac{1}{2}$$

Sol: At

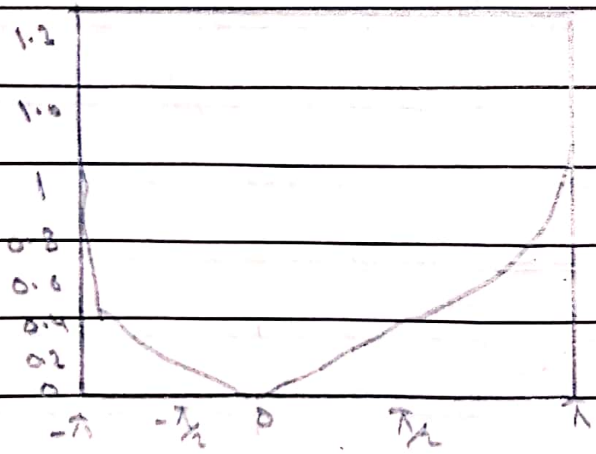
$$w = 0$$

$$H(0) = \frac{b_0}{(1-p^2)} = 1$$

both side multiply by $(1-p)^2$

$$\Rightarrow \frac{b_0}{(1-p)^2} \times (1-p)^2 = 1 \times (1-p)^2$$

$$b_0 = (1-p)^2$$



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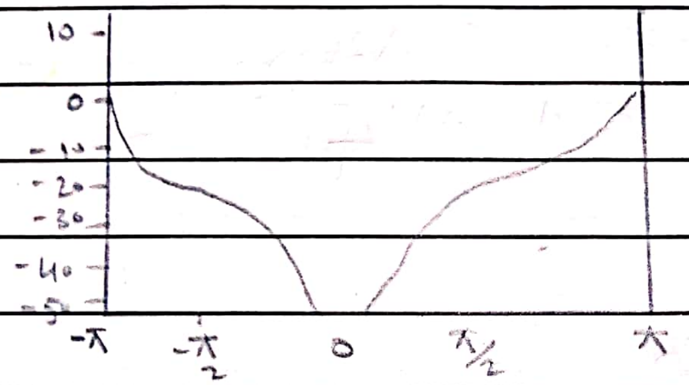

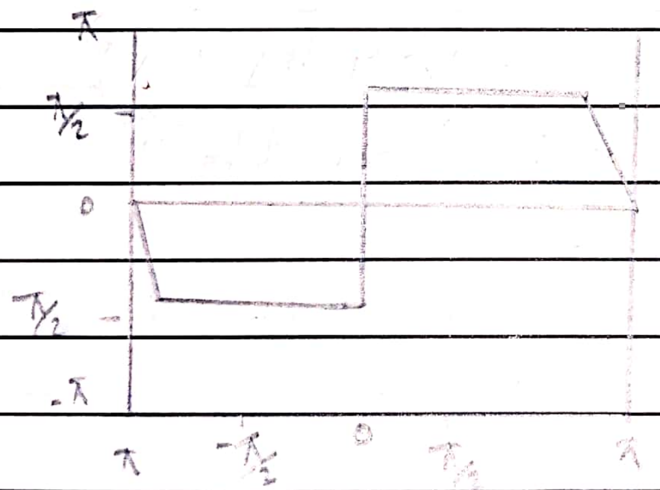

Magnitude and Phase response of a Simple highpass filter

$$H(z) = \frac{(1-a)]}{2} \frac{[1-z^{-1}]}{1+az^{-1}}$$

$$a = 0.9$$

at $\omega = \frac{\pi}{4}$

$$H\left(\frac{\pi}{4}\right) = \frac{(1-P)^2}{(1-Pe^{3\pi/4})^2}$$

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$$= \frac{(1-P)^2}{(1-P(\cos(\pi/4)+jP\sin(\pi/4))^2}$$

$$= \frac{(1-P^2)}{\left(\frac{1-P}{\sqrt{2}}+\frac{jP}{\sqrt{2}}\right)^2}$$

take square on above eq

$$\frac{([1-P]^2)}{\left(\left[1-\frac{P}{\sqrt{2}}+\frac{jP}{\sqrt{2}}\right]^2\right)^2}$$

$$H(z) = \frac{[1-P]^4}{[(1-P/\sqrt{2})^2+P^2/2]^2} = \frac{1}{2}$$

$$= \sqrt{2}(1-P)^2 \div 1+P^2-\sqrt{2}P$$

the value of $P=0.32$ satisfies this equation desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

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Q 4 Part (a) $\begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases}$

Sol:

The Fourier transform of this sequence is

$$X(w) = \sum_{n=0}^{L-1} x(n) e^{-jwn}$$

$$= \sum_{n=0}^{L-1} e^{-jwn} = \frac{1 - e^{-jwL}}{1 - e^{-jw}} = \frac{\sin(wL/2)}{\sin(w/2)} e^{-jw(L-1)/2}$$

The magnitude and phase of $X(w)$ are illustration $L=10$. The N-point DFT of $x(n)$ is simply $X(w)$ evaluated at the set of N equally spaced frequency $w_k = \frac{2\pi k}{N}$

$k = 0, 1 \ldots N-1$

$$X(k) = \frac{1 - e^{-j2\pi kL/N}}{1 - e^{-j2\pi k/N}} \qquad k = 0, 1, N-1$$

$$= \left(\frac{\sin(\pi kL/N)}{\sin(\pi k/N)} \right) e^{-j\pi k(L-1)/N}$$

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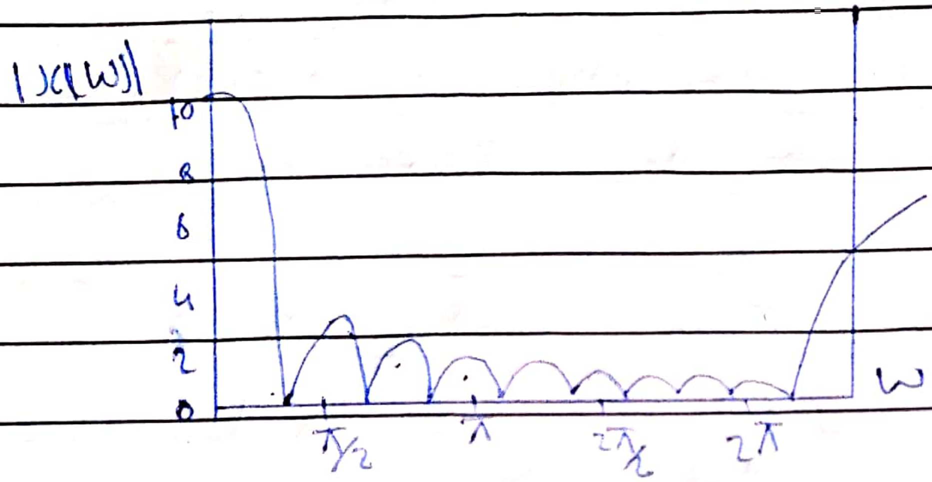

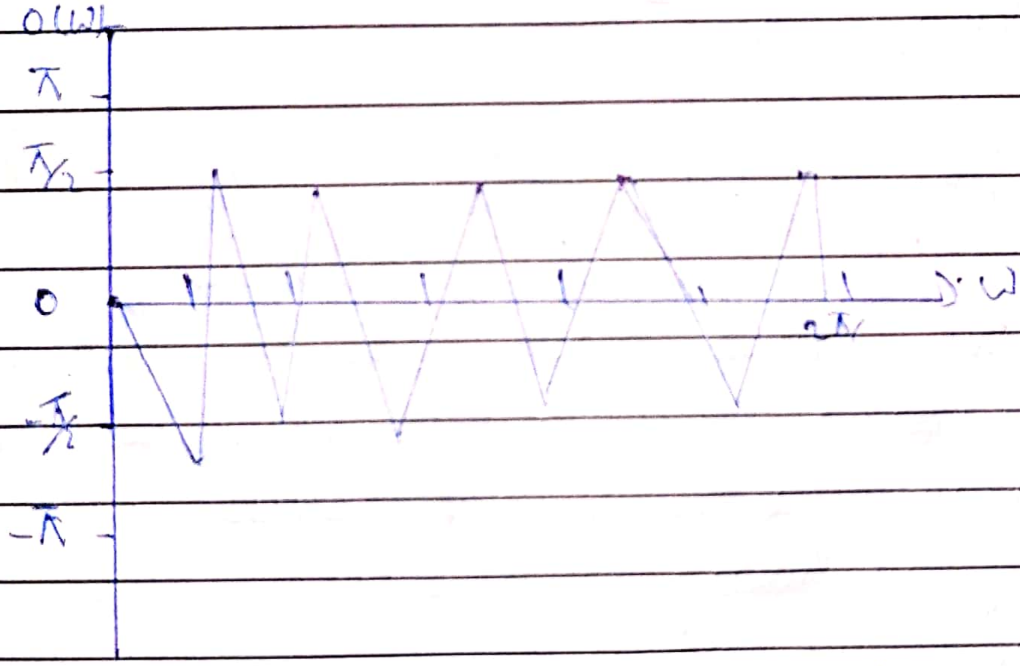

If N is Selected Such that N=L then the DFT become

$$X(K) = \begin{cases} L, & K=0 \\ 0, & K=1,2,\ldots,L-1 \end{cases}$$

Thus there is only one non-zero

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non zero value in DFT. This is apparent from observation of $X(W)$ since $X(W)=0$ at the frequencies $W_K = 2\pi K/2$

$K \neq 0$ The reader should verify that $x(n)$ can recovered from $X(K)$ by performing an 1-Point DFT

QNo 4 Part (B)

Sol:.

The first step is to determine the matrix W_4 By exploting the periodicity property of W_4 and the summetry property

$$W_N^{K+N/2} = W_N^K$$

The matrix W_4 may be expressed as

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$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^4 \\ W_4^0 & W_4^3 & W_4^6 & W_4^9 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W^2 & W^3 \\ 1 & W_4^2 & W^0 & W^2 \\ 1 & W_4^3 & W^4 & W^1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & 1 \\ 1 & j & -1 & -j \end{bmatrix}$$

Then $Y_4 = W_4 X_4 = \begin{bmatrix} -2+2j \\ -2 \\ -2-2j \end{bmatrix}$

The DFT of X_4 ~~may~~ may by determined by Conjudating the element in W_4 to obtain W_4^+ and then applying the formula