IQRA NATIONAL UNIVERSITY

Intro To Structural Dynamics And Earthquake Engineering

Final Term Assignment

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Section = (A)

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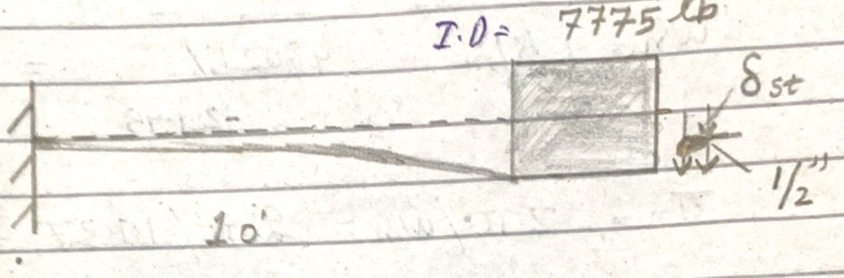
Q 1:- A beam shown - - - - - - - - - - - - - force with time.

Sol



Sol.

The general E.O.M for SDOF system is

$ku + c\dot{u} + m\ddot{u} = p(t)$

In our case system is undamped ($c=0$) undergoing free vibration ($p(t)=0$).

Hence general EOM becomes

$ku + m\ddot{u} = 0$ - - - - (1)

$k = 3EI/L^3$

$$= \frac{3 \times 29000 \frac{k}{in^2} \times 150\ in^4}{(10 \times 12\ in)^3}$$

$= 7.55\ k/in$

In order to eliminate the chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or kg, m, sec.

$k = 7.55 k/in = 90625\ lb/ft$

$m = \frac{7775\ lb sec^2}{32.2 ft} = 241.45\ slug$

$w_n = \sqrt{k/m} = \sqrt{90625/241.45} = 19.37\ rad/sec$

$T_n = 2\pi/w_n = 2\pi/19.37 = 0.32\ sec$

Where 'k' is in lb/ft and 'm' is in lb sec/ft^2

General solution to the EOM for undamped free vibration is,

$u(t) = u(0)\cos(w_n t) + \dot{u}(0)/w_n \sin(w_n t)$

$u(0) = 1/2'' = 1/24\ ft$ and $\dot{u}(0) = 0$

equivalent static force at any time "t" is

$f_s(t) = k \cdot u(t) = \frac{90625 \times \cos(19.37t)}{24}$

$f_s(t) = 3776\ (19.37t)$

Amplitude of dynamic displacement, u_o undamped free vibration is

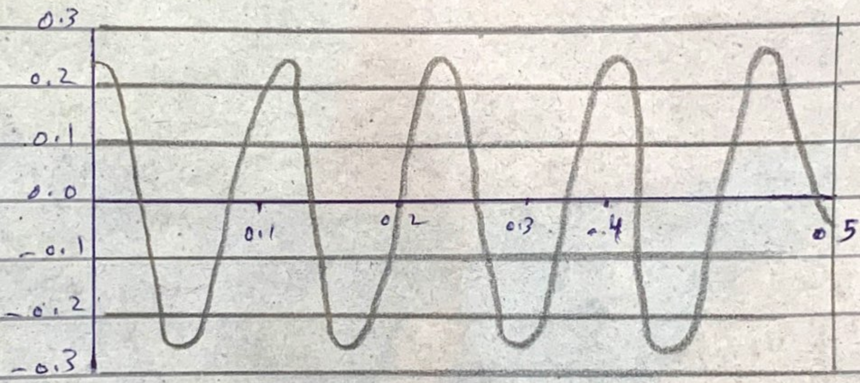
$$\sqrt{\left[(u(0))^2 + (\dot{u}(0)^2)^2\right]} = \sqrt{\left(\frac{1}{24}\right)^2 + 0}$$

$$u_0 = \frac{1}{24}\ ft$$

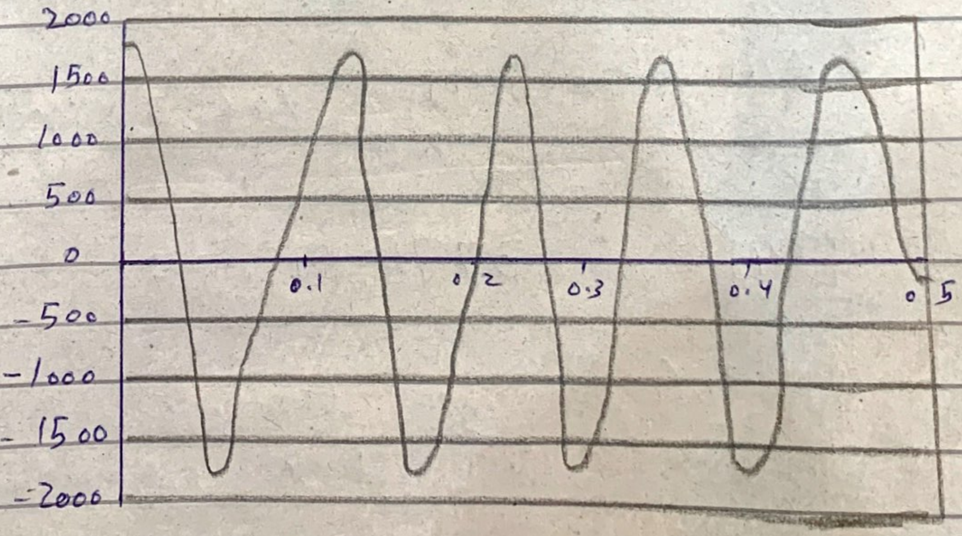
Amplitude of equivalent static force f_{s0}

$kU_0 = 90625 \times \frac{1}{24} = 3776\ lb$

Undamped Free vibration



Undamped Free vibration



Q2:- For the beam's data ---- ------ With time.

Given:

ζ (Damping ratio) of Reinforced concrete with considerable cracking = 3-5%

= 3 %.

Using data of beam given in Q# 1

Required

-> Develope and solve the equation showing variation in equivalent static force with time.

-> Draw graph to show the variation of displacement with time and the variation of equivalent static force with time.

Sol:-

E.OM for damped free vibrations is:

$Ku + c\dot{u} + m\ddot{u} = 0$ ——— (1)

from Q# 1

$K \approx 90625$ lb/ft & $m = 241.45$ lb sec²/ft

$\omega_n = 19.37$ rad/sec

$C = \zeta \times 2m\omega_n$

$C = (0.03) \times 2 (241.45)(19.37)$

$C = 280.61$ lb/sec/ft

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Put values in eq(1)

$90625\ddot{u} + 280.61\dot{u} + 241.45u = 0$

Solution to the E.O.M for damped free vibration is

$$u(t) = e^{-\xi w_n t}\left[u(0)\cos(w_D t) + \frac{1}{w_D}\left[\dot{u}(0) + u(0)\xi w_n\right]\cdot \sin w_D t\right]$$

$w_D = 19.37$ rad/sec

$$u(t) = e^{-0.03 \times 19.37t}\left[\frac{1}{24} \times \cos(19.37t) + \frac{1}{19.37} \times \left[0 + \frac{1}{24} \times 0.03 \times 19.37 \times \sin(19.37t)\right]\right]$$

$$u(t) = e^{-0.581t}\left[0.041 \times \cos(19.37t) + 0.00125 \times \sin(19.37t)\right]$$

$f_s(t) = k \cdot u(t) \Rightarrow 90625 \times u t$

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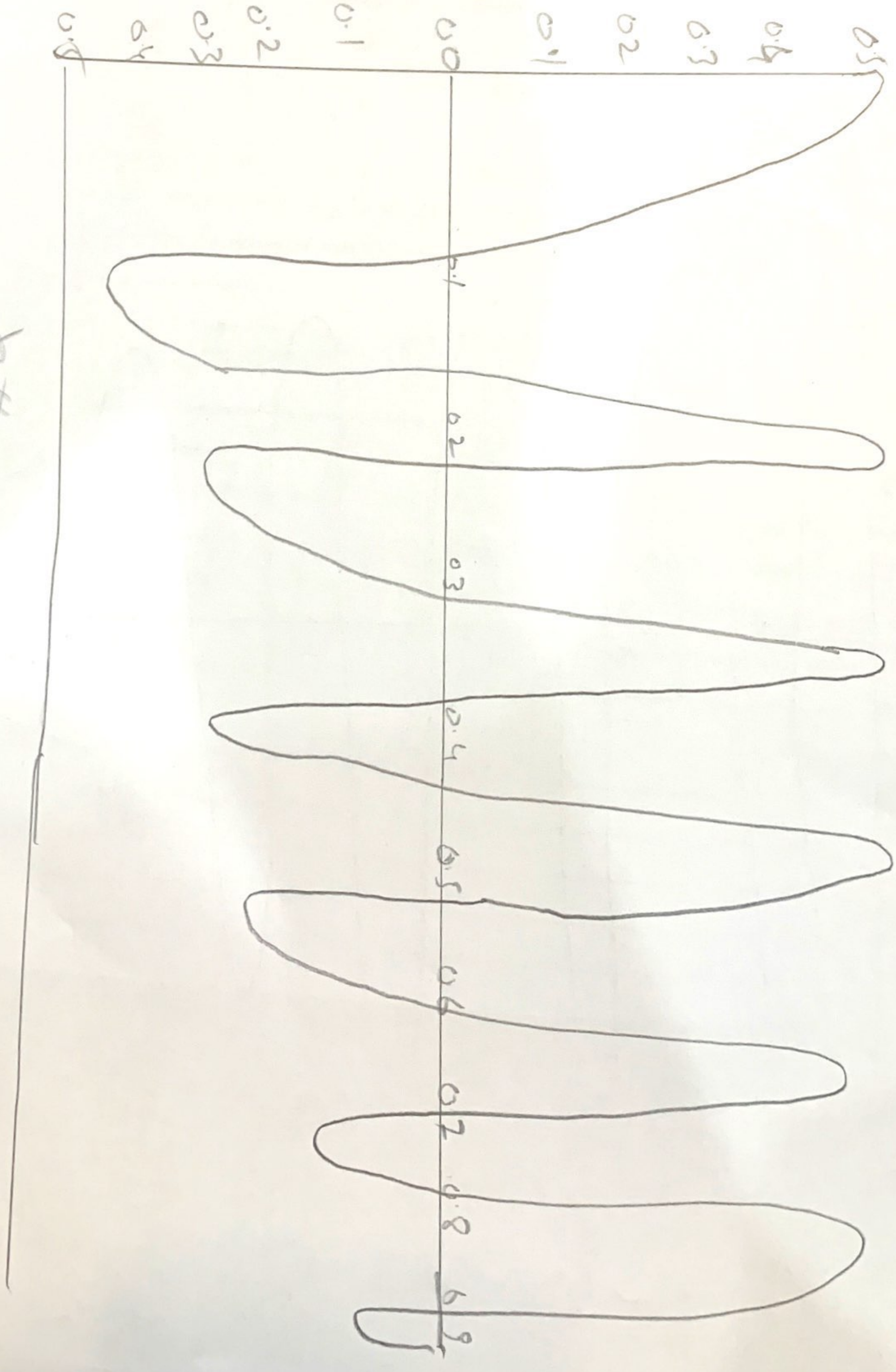
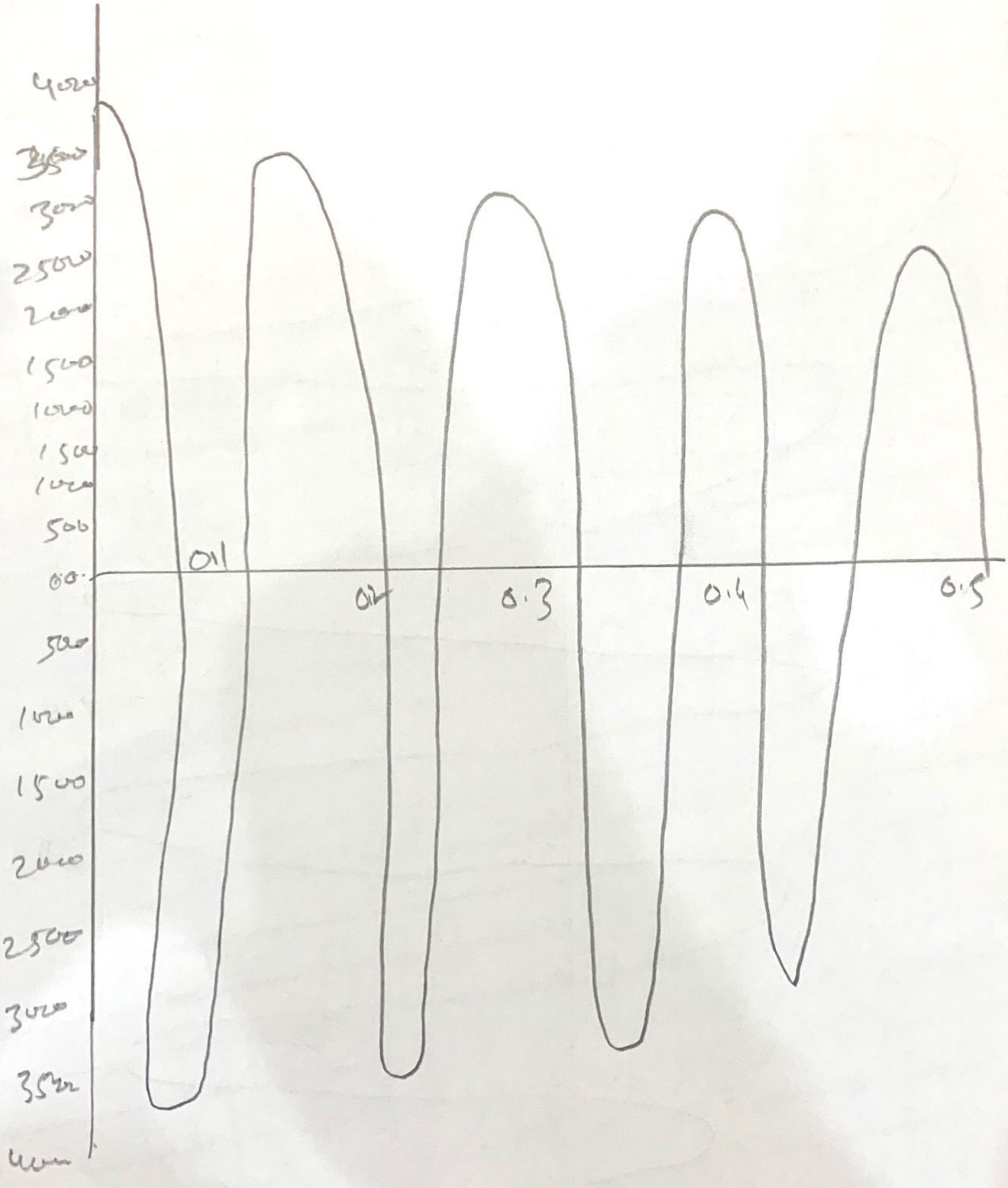
$$f_s(t) = e^{-0.581t}\left[(90625 \times 0.041)\cos(19.37t) + (90625 \times 0.00125)\sin(19.37t)\right]$$

$$f_s(t) = e^{-0.581t}\left[3715.62\cos(19.37t) + 113.28\sin(19.37t)\right]$$



Q# 2


4000
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1500
2000
2500
3000
3500

Q3:- A free vibration — — — — —
— — — — — is 2.286cm.

Given data:

→ Amplitude cable force = 60 kips

→ Horizental displacement of tank
$= \frac{7775}{1000} = 7.775in$

→ Cycles = 7
→ Cycle completion time = 3.57 sec
→ Amplitude of displacement ~~of~~
$= 2.286cm = 0.9in.$

Required data:

→ Damping ratios
→ Natural period of undamped vibration
→ Stiffness of structures
→ weight of tank
→ Damping co-efficient
→ Number of cycles to reduce the displacement amplitude to 0.5".

Sol:

As given in equation.

$u_1 = 7.775$ in

After $J = 7$

$U_{J+1} = U_8 = 2.286$ cm $= 0.9$ in

(a) δ = Damping ratio = ?

$$\delta = \frac{1}{2\pi \zeta} \ln\left[\frac{u_1}{u_{J+1}}\right]$$

By putting values we get:

~~$7 = \frac{1}{2\pi\delta}$~~ $$7 = \frac{1}{2\pi \delta} \ln\left[\frac{7.775}{0.9}\right]$$

$$\delta = \frac{1}{2\pi 7} \ln\left[\frac{7.775}{0.9}\right]$$

$$\delta = 0.0490 = 4.90\%$$ Ans

~~$\delta = 4.8$~~ $$\boxed{\delta = 4.90\%}$$

(b) Natural period of undamped vibration $= T_n = ?$

As, the 7 cycles of vibrations are completed in 3.57 sec

=> Time required to complete one cycle, $T_D = \frac{3.57}{7} = 0.51$ sec

$$T_D = 0.51 \text{ sec}$$

Now,

$$\omega_D = \omega_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{\omega_D} = \frac{2\pi}{(\omega_n\sqrt{1-\zeta^2}}$$

$$\Rightarrow T_D = \frac{T_n}{(1-\beta)^2}$$

$\Rightarrow T_n = T_0 \times \sqrt{1-\beta^2}$

$\Rightarrow T_n = 0.51 \times \sqrt{1-(0.0049)^2}$

$T_n = 0.5093 = 0.51$ sec

$\boxed{T_n = 0.51 \text{ sec}}$

(c) Stiffness of structure, $k = ?$

$$k = \frac{60 \times \cos 60^\circ}{7.775} = 3.85 \text{ k/in}$$

$$\boxed{k = 3.85 \text{ k/in} = 46200 \text{ lb/ft}}$$

(d) Weight of tank $W = ?$

$$w_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{W/g}} = \sqrt{\frac{k \cdot g}{W}}$$

$$\Rightarrow w_n^2 = \frac{k \times g}{W}$$

$$W = \frac{k \times g}{w_n^2}$$

Also, $w_n = \frac{2\pi}{T_n}$

$$W = \frac{kg}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$= kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \left[\frac{46900\ lb}{ft} \times \frac{32.2\ ft}{Sec^2}\right] \times \frac{(0.51\ sec)^2}{4\pi^2}$$

9801.93 lb = 9.801 k

9801.93 lb = 9.801 k

(e) Damping co-efficient, c = ?

It is known that

$$\xi = \frac{c}{2m\omega_n}$$

$$\Rightarrow c = \xi \times 2m\omega_n$$

$$= \xi \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$\Rightarrow c = \frac{(0.0490) \times 4 \times \pi \times \left(\frac{9801.93}{32.2}\right)}{0.51}$$

c = 371.71 lb·sec/ft.

(f) Number of cycles to reduce the displacement amplitude to 0.5″, j = ?

$$j = \frac{1}{2\pi\xi} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$\Rightarrow j = \frac{1}{2\pi \times 0.0490} \ln\left[\frac{7.775}{0.5}\right]$$

j = 8.91 or 9 cycles