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NAZISH

ID # 6861

Degree # BS(SE)

Subject # Linear Algebra

mid - Term

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Question # 2
part # b

Find an echelon from for the below matrix using row operations.

$$\begin{bmatrix} 1 & ID3 & 8 \\ 2 & ID4 & -1 \\ -3 & 0 & 0 \\ 1 & -ID3 & 16 \end{bmatrix}$$

Solution:-

$$A = \begin{bmatrix} 6 & -1 & 0 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{bmatrix} 1 & 6 \\ 1 & 0 \end{bmatrix}$$

$$A_{11} = (1)(0-6) \Rightarrow (1)(-6) \Rightarrow -6$$

$$A_{12} = (-1)^{1+2} \begin{bmatrix} 0 & 6 \\ 1 & 0 \end{bmatrix}$$

$$A_{12} = (-1)(0-6) \Rightarrow (-1)(-6) \Rightarrow 6$$

$$A_{13} = (-1)^{1+3} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A_{13} = (1)(0-1) \Rightarrow (1)(-1) \Rightarrow -1$$

$$A_{21} = (-1)^{2+1} \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$A_{21} = (-1)(0-0) \Rightarrow (-1)(0) \Rightarrow 0$$

$$A_{22} = (-1)^{2+2} \begin{bmatrix} 6 & 0 \\ 1 & 0 \end{bmatrix}$$

$$A_{22} = (1)(0-0) \Rightarrow (1)(0) \Rightarrow 0$$

$$A_{23} = (-1)^{2+3} \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A_{23} = (-1)(6+1) \Rightarrow (-1)(7) \Rightarrow -7$$

$$A_{31} = (-1)^{3+1} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A_{31} = (1)(0-1) \Rightarrow (1)(-1) \Rightarrow a -1$$

$$A_{32} = (-1)^{3+2} \begin{bmatrix} 6 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A_{32} = (-1)(6+1) \Rightarrow (-1)(7) \Rightarrow -7$$

$$\cancel{A_{33} = (A1)(6+0) \Rightarrow (1)(6)}$$

$$A_{33} = (-1)^{3+3} \begin{bmatrix} 6 & -1 \\ 0 & 1 \end{bmatrix}$$

$$A_{33} = (+1)(6+0) \Rightarrow (1)(6) \Rightarrow 6$$

$$[Adj]_{3\times3} = \begin{bmatrix} -6 & 6 & -1 \\ 0 & 0 & -7 \\ -1 & -7 & 6 \end{bmatrix}$$

$$Adj[A_{ij}]_{3\times3} = \begin{bmatrix} -6 & 0 & 1 \\ 6 & 0 & -7 \\ -1 & -7 & 6 \end{bmatrix}$$

$$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$|A| = 6(-6) + (-1)(0) + (0)(-1)$$

$$|A| = -36 - 0 - 0$$

$$|A| = -36$$

$$A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

$$A^{-1} = \frac{1}{-36}\begin{bmatrix} -6 & 0 & -1 \\ 6 & 0 & -7 \\ -1 & -7 & 6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{-6}{-36} & \frac{0}{-36} & \frac{-1}{-36} \\ \frac{6}{36} & \frac{0}{-36} & \frac{-7}{-36} \\ \frac{-1}{-36} & \frac{-7}{-36} & \frac{6}{-36} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{6} & 0 & \frac{1}{36} \\ -1/6 & 0 & 7/36 \\ 1/36 & 7/36 & -1/6 \end{bmatrix}$$

Question # 02
Part # a

Find Inverse of a Matrix

$$\begin{bmatrix} ID3 & -1 & 0 \\ 0 & 1 & ID3 \\ 1 & 1 & 0 \end{bmatrix}$$

Solution:

Let

$$A = \begin{bmatrix} 6 & -1 & 0 \\ 0 & 1 & 6 \\ 1 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$A_{11} = (-1)^{1+1} \begin{bmatrix} 1 & 6 \\ 1 & 0 \end{bmatrix}$$

$$A_{11} = (1)(0-6) \Rightarrow (1)(-6) \Rightarrow -6$$

$$A_{12} = (-1)^{1+2} \begin{bmatrix} 0 & 6 \\ 1 & 0 \end{bmatrix}$$

$$A_{12} = (-1)(0-6) \Rightarrow (-1)(-6) \Rightarrow 6$$

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$$A_{13} = (-1)^{1+3} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A_{13} = (1)(0-1) \Rightarrow (1)(-1) \Rightarrow -1$$

$$A_{21} = (-1)^{2+1} \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$A_{21} = (-1)(0-1) \Rightarrow (-1)(0) \Rightarrow 0$$

$$A_{22} = (-1)^{2+2} \begin{bmatrix} 6 & 0 \\ 1 & 0 \end{bmatrix}$$

$$A_{22} = (1)(0-0) \Rightarrow (1)(0) \Rightarrow 0$$

$$A_{23} = (-1)^{2+3} \begin{bmatrix} 6 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A_{23} = (-1)(6+1) \rightarrow (-1)(7) \Rightarrow 7$$

$$A_{31} = (-1)^{3+1} \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$A_{31} = (1)(0-1) \Rightarrow (-1)(-1) \Rightarrow -1$$

$$A_{32} = (-1)^{3+2} \begin{bmatrix} 6 & -1 \\ 1 & 1 \end{bmatrix}$$

$A_{32} = (-1)(6+1) \Rightarrow (-1)(7) \Rightarrow -7$

$A_{33} = (-1)^{3+3} \begin{bmatrix} 6 & -1 \\ 0 & 1 \end{bmatrix}$

$A_{33} = (+1)(6+0) \Rightarrow (1)(6) \Rightarrow 6$

$$[A_{ij}]_{3\times3} = \begin{bmatrix} -6 & 6 & -1 \\ 0 & 0 & -7 \\ -1 & -7 & 6 \end{bmatrix}$$

$$Adj\,[A_{ij}]_{3\times3} = \begin{bmatrix} -6 & 0 & -1 \\ 6 & 0 & -7 \\ -1 & -7 & 6 \end{bmatrix}$$

$|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$|A| = 6(-6) + (-1)(0) + (0)(-1)$

$|A| = -36 - 0 - 0$

$|A| = -36$

$$A^{-1} = \frac{1}{|A|} \, adj A$$

$$A^{-1} = \frac{1}{-36} \begin{bmatrix} -6 & 0 & -1 \\ 6 & 0 & -7 \\ -1 & -7 & 6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{-6}{36} & \frac{0}{-36} & \frac{+1}{+36} \\ \frac{6}{-36} & \frac{0}{-36} & \frac{+7}{+36} \\ \frac{+1}{+36} & \frac{+7}{+36} & \frac{6}{-36} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{6} & 0 & \frac{1}{36} \\ \frac{1}{-6} & 0 & \frac{7}{36} \\ \frac{1}{36} & \frac{7}{36} & \frac{1}{-6} \end{bmatrix}$$

Question # 1

Solve the system of equations that corresponds to this augmented matrix

$$\begin{bmatrix} 1 & -3 & -ID2 \\ 3 & -7 & ID4 \\ -4 & 6 & ID3 \end{bmatrix}$$

Solution:

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -8 \\ 0 & -16 & 19 & -25 \\ 0 & -6 & 15 & -26 \end{array}\right] \begin{array}{l} R_2 + 3R_1 \\ R_3 + 4R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & 4 & -8 \\ 0 & -16 & 19 & -25 \\ 0 & 0 & 126 & -266 \end{array}\right] \quad 16R_3 - 6R_{12}$$

So in equation

$126x_3 - 266$

$\boxed{x_3 = -2.11}$

$-16x_2 + 19x_3 = -25$

$-16x_2 = -25 - 19(-2.11)$

$\boxed{x_2 = 0.94}$

Now

$x_1 - 3x_2 + 4x_3 = -8$

$x_1 = 3(0.94) - 4(-2.11) - 8$

$\boxed{x_1 = 3.26}$

Question # 03

Find the Eigen values and Eigen vectors of the below matrix

$$\begin{bmatrix} ID3 & -6 & 2 \\ -6 & ID2 & -4 \\ 2 & -4 & ID4 \end{bmatrix}$$

Solution:-

$$\begin{bmatrix} -6 & -6 & 2 \\ -6 & 8 & -4 \\ 2 & -4 & 1 \end{bmatrix}$$

Eigen values

$$\det(A - \lambda I) = 0$$

I is 3×3 identify matrix from $A - \lambda I$

$$A - \lambda I = \begin{bmatrix} 6 & -6 & 2 \\ -6 & 8 & -4 \\ 2 & 4 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

(11)

$$= \begin{pmatrix} 6-\lambda & -6 & 2 \\ -6 & 2-\lambda & -4 \\ 2 & -4 & 1-\lambda \end{pmatrix}$$

Calculate ~~det~~ $(A - \lambda I)$

$$\det (A - \lambda I) = (6-\lambda) \begin{vmatrix} 8-\lambda & -4 \\ -4 & 1-\lambda \end{vmatrix}$$

$$-(-6) \begin{vmatrix} -6 & 2 \\ -4 & 1-\lambda \end{vmatrix} + 3 \begin{vmatrix} -6 & 2 \\ 8-\lambda & -4 \end{vmatrix}$$

$$= (6-\lambda)(8-\lambda)(1-\lambda) - (-4)(-4) + 6$$
$$(-6)(1-\lambda) - (-4)(2) + 3(-6)(-4)$$
$$-(8-\lambda)(2)$$

$$= (6-\lambda)(8-8\lambda-\lambda-\lambda^2) - (16) + (-36)$$
$$(1-\lambda) - (-8) + 36\lambda - 8 - 18$$

$$= (6-\lambda)(8-9\lambda+\lambda^2) - 52 + 36\lambda$$
$$+26.$$

$$= 48 - 54\lambda + 6\lambda^2 - 8\lambda + 9\lambda^2 + \lambda^3$$
$$-52 + 36 + 26$$

$$= \lambda^3 + 6\lambda^2 + 9\lambda^2 - 54\lambda - 8\lambda + 48$$
$$-52 + 36 + 26$$

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$= \lambda^3 + 15\lambda^2 - 63\lambda + 58$

$\det (A - \lambda I) = 0$

$\lambda^3 + 15\lambda^2 - 63\lambda + 58 = 0$

Factors of 58 is $\pm 1, \pm 2, \pm 29,$
± 58

the eigen value is $2, -2$.

$\lambda - 20583 \begin{vmatrix} 13722 - \lambda & -4 \\ -4 & 27444\lambda \end{vmatrix} - (-6) \begin{vmatrix} -6 & -4 \\ 2 & \lambda - 27444 \end{vmatrix}$

$+ 2 \begin{vmatrix} -6 & 13722 - \lambda \\ 2 & -4 \end{vmatrix} = 0$

$= \left[(13722 - \lambda)(27444 - \lambda) - (-4 \times -4) \right]$
$20583 - \lambda +$

$6 \left[(-6)(27444 - \lambda) + (-4)(2) \right] + 2$
$\left[(-6)(-4) - (13722 - \lambda)(2) \right] =$
6

$$= \left[(13722-\lambda)(27444-\lambda)-(-4\times-4)\right]$$

$$20583-\lambda$$

$$+6\left[(-6)(27444-\lambda)+(-4)(2)\right]+$$

$$2\left[(-6)(-4)-(13722-\lambda)(2)\right]=6$$

$$= (376586568)-27444\lambda-13722\lambda+\lambda^2-$$

$$329328+16\lambda+987984+36\lambda-48+$$

$$48-54888+4\lambda=0$$