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Question: 1 (a)

Let $A = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 2 & 1 \\ 0 & 1 & -2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 4 \\ 3 & -1 \\ -2 & 2 \end{bmatrix}$

Identify the (3,2) entry of AB

Solution:

$$A \cdot B = \begin{bmatrix} 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$$

$$= (0 + (-1) + (-4))$$

$$= 6 - 1 - 4$$

$$= -5 \quad \text{Ans}$$

Question : 3

Find A^{-1} where

$$A = \begin{bmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{bmatrix}$$

Solution :

$$|A| = \begin{vmatrix} 3 & -2 & 1 \\ 5 & 6 & 2 \\ 1 & 0 & -3 \end{vmatrix} \quad |A|$$

Expand by Row 1

$$= 3\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} - (-2)\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} + 2\begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} + 1\begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix}$$

$$= 3(-18-0) + 2(-15-2) + 1(0-6)$$

$$= 3(-18) + 2(-17) + 1(-6)$$

$$= -54 - 34 - 6$$

$$= -94$$

$$|A| = -94$$

$$A_{12} = (-1)^{1+1}\begin{vmatrix} 6 & 2 \\ 0 & -3 \end{vmatrix} = -18$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} = 17$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 6 \\ 1 & 0 \end{vmatrix} = -6$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 1 \\ 0 & -3 \end{vmatrix} = -6$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 1 \\ 1 & -3 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix} = 2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 1 \\ 6 & 2 \end{vmatrix} = -10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} = -1$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -2 \\ 5 & 6 \end{vmatrix} = 28$$

$$\text{adj } A = \begin{vmatrix} A_{11} & A_{12} & A_{13} \\ A_{12} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{vmatrix}^t$$

$$= \begin{vmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{vmatrix}$$

As

$$A^{-1} = \frac{1}{|A|} \text{adj } A$$

$$A^{-1} = \frac{1}{-94} \begin{vmatrix} 18 & 6 & 10 \\ 17 & 10 & 1 \\ 6 & 2 & -28 \end{vmatrix}$$

Question: 2 (a)

If A & B are nxn matrices where $|A| = 2$ & $|B| = -3$, calculate $|A^{-1} B^T|$

Solution:

Since $|A^{-1} B^t| = |A^{-1}| |B^t|$

$\Rightarrow \frac{1}{|A|} |B|$ become $|B^t| = |B|$

So $|A^{-1} B^t| = \frac{1}{|A|} |B|$

$= \frac{1}{2} \cdot 3$

$= \frac{3}{2}$ Ans

Question 1 (b):

Find the quadratic polynomial that interpolates the point (1, 3), (2, 4) (3, 7)

Solution:

$$a_2 x_1^2 + a_1 x_1 + a_0 = y_1$$
$$a_2 x_2^2 + a_1 x_2 + a_0 = y_2$$
$$a_2 x_3^2 + a_1 x_3 + a_0 = y_3$$

Now $(x_1, y_1) = (1, 3)$

$(x_2, y_2) = (2, 4)$

$(x_3, y_3) = (3, 7)$

$a_2 + a_1 + a_0 = 3$

$4a_2 + 2a_1 + a_0 = 4$

$9a_2 + 3a_1 + a_0 = 7$

$$A_b \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 4 & 2 & 1 & : & 4 \\ 9 & 3 & 1 & : & 7 \end{bmatrix}$$

$$R_1 \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & -2 & -3 & : & -8 \\ 0 & -6 & -8 & : & -20 \end{bmatrix} \begin{matrix} R_2 - 4R_1 \\ R_3 - 9R_1 \end{matrix}$$

$$R_1 \begin{bmatrix} 1 & 1 & 1 & : & 3 \\ 0 & -2 & -3 & : & -8 \\ 0 & 0 & 1 & : & 4 \end{bmatrix} R_3 - 3R_2$$

So

$a_2 + a_1 + a_0 = 3 \rightarrow$ (i)

$-2a_1 - 3a_0 = -8 \rightarrow$ (ii)

$a_0 = 4$ put in (ii)

$-2a_1 - 12 = 8$

$a_1 = \frac{4}{-2} = -2$

put in (i)

$a_2 = 2 + 4 = 3$

$a_2 = 1$

So

$a_0 = 4$

$a_1 = -2$

$a_2 = 1$ Ans

Question 2(b):

Estimate the linear system of equation

$$x + y + 2z = 1$$
$$x - 2y + z = -5$$
$$3x + y + z = 3$$

Solution:

$$x + y + 2z = 1$$
$$x - 2y + z = -5$$
$$3x + y + z = 3$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ 1 & -2 & 1 & -5 \\ 3 & 1 & 1 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ 0 & -3 & -1 & -6 \\ 0 & -2 & -5 & 0 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & -2 & -5 & 0 \end{array}\right] \frac{R_2}{-3}$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & 0 & -13/2 & 4 \end{array}\right] R_3 + 2R_2$$

$$\left[\begin{array}{ccc:c} 1 & 1 & 2 & 1 \\ 0 & 1 & 1/3 & 2 \\ 0 & 0 & 1 & -8/13 \end{array}\right] R_3 \times \frac{2}{-13}$$

$$x + y + 2z = 1 \quad \rightarrow (i)$$

$$y + \frac{1}{3}z = 2 \quad \rightarrow (ii)$$

$$z = \frac{-8}{13} \quad \rightarrow (iii)$$

Put eq (iii) in eq (i)

$$y + \frac{1}{3}\left(\frac{-8}{13}\right) = 2$$

$$y - \frac{8}{39} = 2$$

$$y = 2 + \frac{8}{39}$$

$$y = \frac{78 + 8}{39}$$

$$y = \frac{86}{39} \quad \rightarrow (a)$$

Put (a) in (i)

$$x + \frac{86}{39} + 2\left(\frac{-8}{13}\right) = 1$$

$$x + \frac{86}{39} - \frac{16}{13} = 1$$

$$x = 1 - \frac{38}{39}$$

$$x = \frac{1}{39}$$

So

$$x = \frac{1}{39}$$

$$y = \frac{86}{39}$$

$$Z = \frac{-8}{13}$$ Ans