BE(C) mid-exam 4th semester

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Section A

Subject:- MOS-II

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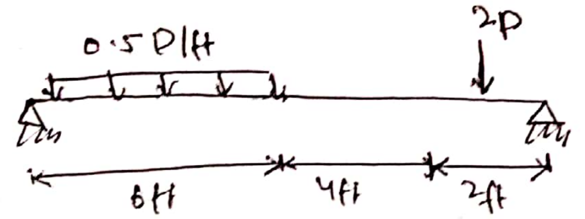
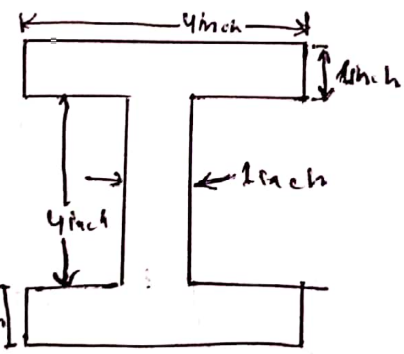
Submission date: 18/04/2020

Question:

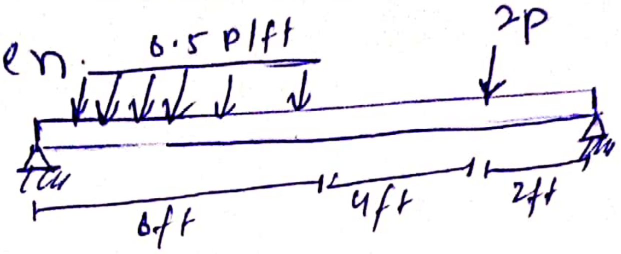
Construct the mohr's Circle diagram and find the principle stress and maximum ~~Stress~~ in plane shear stress for the stress state of a point 'C' ~~Located~~ located at the center of uniformly distributed load and 1inch below the top fiber of beam cross-section shown in figure. However, to construct mohr circle it is necessary to draw the shear stress and flexural stress variation diagram for maximum shear force and bending moment respectively. Campare the results obtained from the mohr's circle with the stress transformation equations.





where p is the last two digits of your class ~~room~~ registration number.

Answer:- Given.



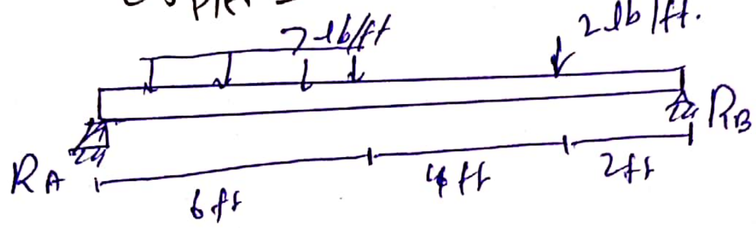
where 'p' is the last two digits of my registration ID (7914) that is 14.

Soi $P = 14$.

$\Rightarrow 2P = 14 \times 2 = 28\,lb$

$0.5\,p/ft = 0.5 \times 14 = 7\,lb/ft$.

Soi



Support reactions:- As we know.

$\Sigma F_y = 0 \quad (\uparrow^{+} \downarrow^{-})$

$\Rightarrow R_A + R_B = 35$

and $\Sigma M = 0$

$R_B \times 12 - 28 \times 10 - 42 \times 3 = 0$ ∴ UDL; $7 \times 6 = 42\,lb$ Pointload

$12 R_B = 280 + 126$

$12 R_B = 406$

$\boxed{R_B = 33.83\,lb}$

→

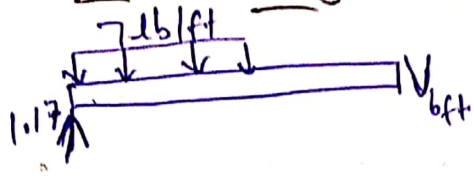
for R_A: $R_A + R_B = 35$

$R_A = 35 - 33.83$

$\Rightarrow \boxed{R_A = 1.17}$

Check
$R_A + R_B = 35$
$33.83 + 1.17 = 35$
$35 = 35$
hence in equalibrium

* **Shear force at change point:**



So shear force ~~from~~ at 6ft from left

$\Sigma F_y = 0$ $(\overset{-}{\uparrow} \overset{+}{\downarrow})$

$V_{6ft} - 1.17 + 7 \times 6 = 0$

$V_{6ft} + 43.17 = 0$

$\boxed{V_{6ft} = -43.17}$

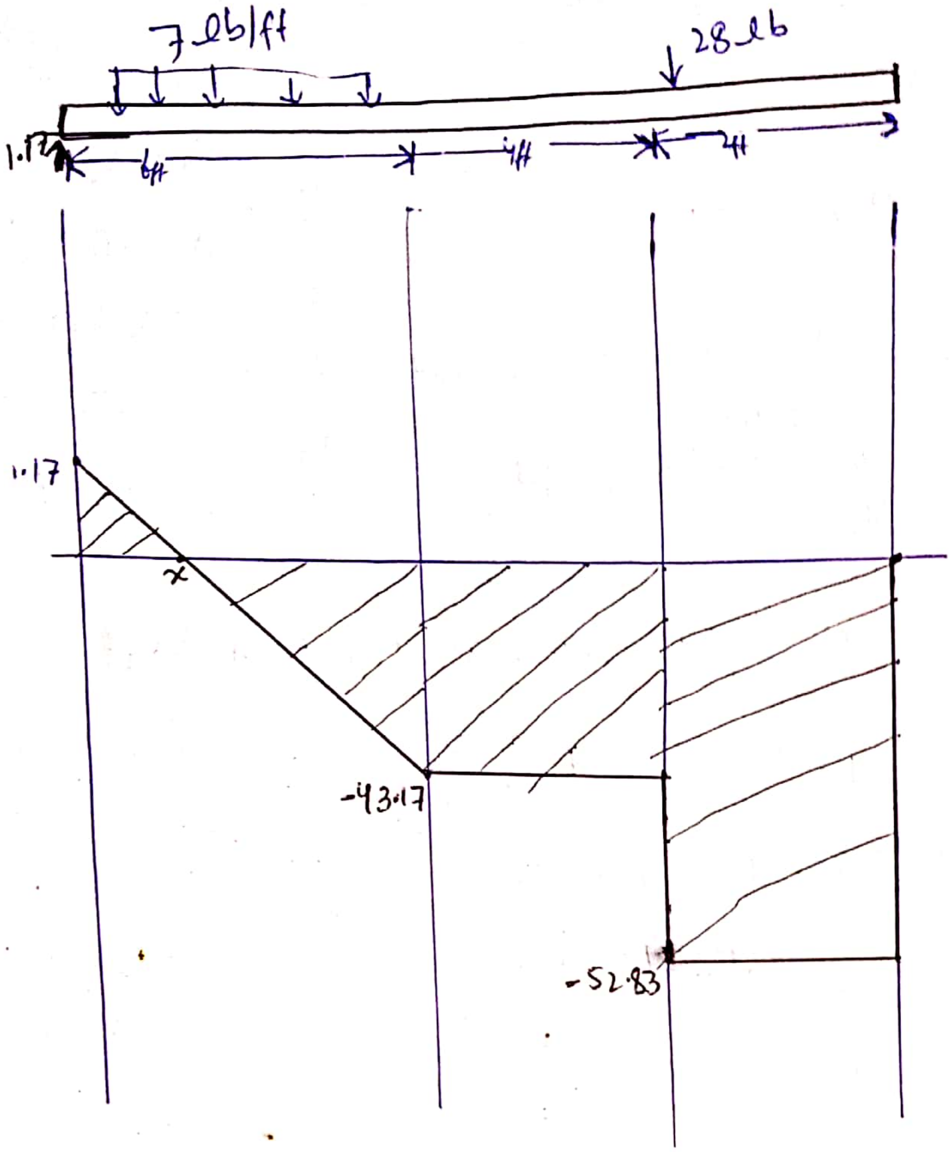
Shear force at V_{10}

$\Sigma F_y = 0$

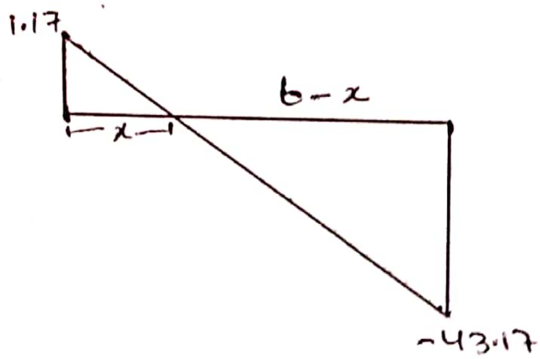
$-1.17 + 7 \times 6 + 12 + V_{10ft} = 0$

$\boxed{V_{10ft} = -52.83}$

→



Now to find moment at change point
to find zero shear point (x)



$$\frac{1.17}{x} = \frac{43.17}{6-x}$$

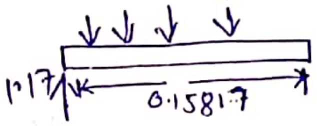
$$1.17(6-x) = x(43.17)$$

$$7.02 - 1.17x = 43.17x$$

$$7.02 = 44.38x$$

$$\Rightarrow \boxed{x = 0.15817 \text{ ft}}$$

• Now we will find moment at Section of 0.15817 from R_A.



$$\sum M_{0.16817} = 0$$

$$\Rightarrow M_{0.16817} + 1.17 \times 0.15817 - 42\left(\frac{0.15817}{2}\right) = 0$$

$$M_{0.16817} = 42\left(\frac{0.15817}{2}\right) - 1.17 \times 0.15817$$

$$\boxed{M_{0.16817} = 3.1365 \text{ lb.ft}}$$

$$\Rightarrow \sum M = 0$$

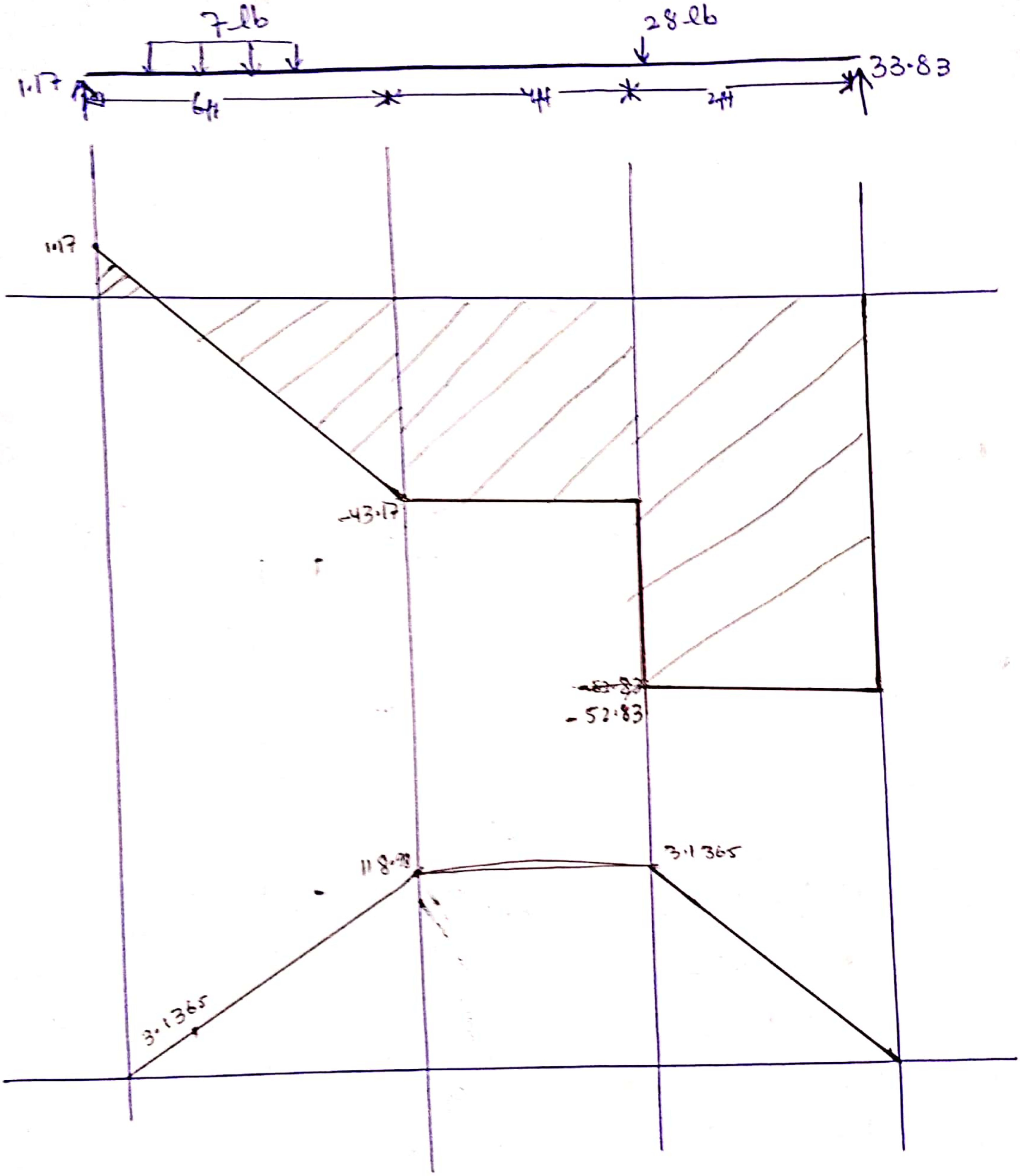
$$M_{6ft} + 1.17 \times 6 - 7 \times 6 \times 3 = 0$$

$$M_{6ft} = 126 - 7.02$$

$$\boxed{M_{6ft} = 118.98 \text{ lb.ft}}$$

$\longrightarrow$

Shear force and bending moment diagram



→

Shear Stress:-

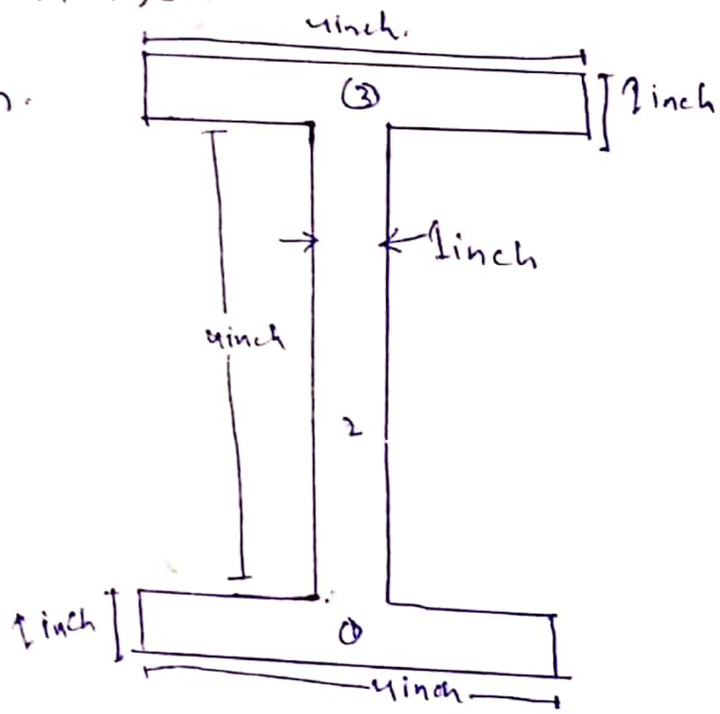
As per question the maximum shear Stress $\tau = \frac{VQ}{It}$ occur where the maximum Shear force is 33.83 lb

So to find the shear stress, we have the following formula.

$$\tau = \frac{VQ}{It}$$

• finding moment of inertia.

as given in question.



to find centriod.

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$

$A_1 = 4 \times 1 = 4 \text{ inch}^2$

$A_2 = 4 \times 1 = 4 \text{ inch}^2$

$A_3 = 4 \times 1 = 4 \text{ inch}^2$

$$\bar{y} = \frac{4 \times 0.5 + 4 \times 3 + 4 \times 5.5}{4+4+4}$$

$$\boxed{\bar{y} = 3''}$$

* Moment of inertia:-

No.	A (in²)	I_x (in)	$d = (\bar{y}-y_1)(\bar{y}-y_2)(\bar{y}-y_3)$
1	4	$\frac{4\times1^3}{12} = 0.333$	
2	4	$\frac{1\times4^3}{12} = 5.33$	
3	4	$\frac{4\times1^3}{12} = 0.333$	

for "d":

1. $d = \bar{y} - y_1 = 3 - 0.59 = 2.53$
2. $d = \bar{y} - y_2 = 3 - 3 = 0$
3. $d = 3 - 5.5 = -2.5$

"d^2"

1. $4 \times 2.5^2 = 25$
2. $4 \times 0^2 = 0$
3. $4 \times (2.5)^2 = 25$

Now $I_x = I_x + Ad^2$

1. $0.333 + 25 = 25.333$

~~2. 25.33 + 5.333 +~~

2. $5.333 + 0 = 5.333$
3. $0.333 + 25 = 25.333$

→

Total

$$I = I_{x_1} + I_{x_2} + I_{x_3}$$

$$I = 25.333 + 5.333 + 25.333$$

$$I = 55.999 \text{ in}^4$$

Now Shear stress

$$\tau = \frac{Va}{Ib}$$

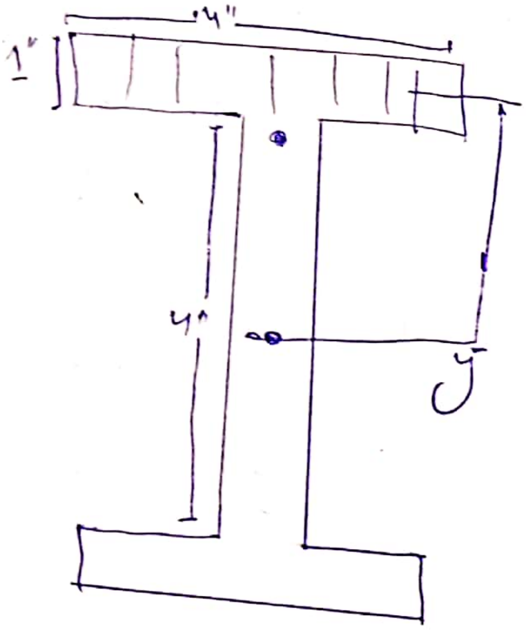
$$V_{max} = \cancel{1.17}\ 33.83$$

$(R_A = 1.17$, $R_B = 33.83)$

$a = \bar{y} A$

b = breadth of that fiber

→ Shear stress at point 'c' located at center of uniformly distributed load an 1inch below the top fibre.



$$\bar{y} = 2 + 0.5 = 2.5$$

$$A = 1 \times 4 = 4$$

$$a = 4 \times 2.5 = 10$$

As we know that

$$\tau = \frac{Va}{Ib}$$

$$\tau = \frac{(33.83)(10)}{(55.998)(9)}$$ →

→ $\boxed{\tau = 0.6712\ Psi}$

* Now flexural Stress analysis

$$\sigma = \frac{Mr}{I}$$

where 'm' is maximum moment in BMD

$$M = 118.98\ lb \cdot ft$$

$$\delta = \frac{118.98 \times 2}{55.996}$$

$$\delta = 4.249$$

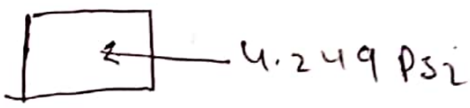
So; Shear stress at point 'c' is

$\boxed{\tau = 0.6712\ Psi}$

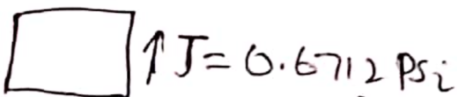
flexural Stress at point 'c' is

$\boxed{\delta = \cancel{+?}\ 4.249\ Psi}$

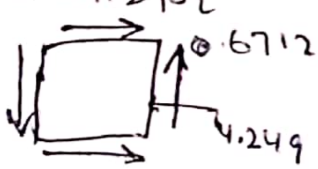
Consider 'c' is a planner element.



4.249 Psi is compressive because point 'c' lies in compressive zone of beam

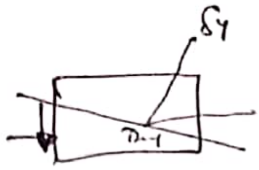


combine stress on 2D-element.



→

we can find the stress state condition of point 'c' at a degree of 20° clock-wise orientation.



Sol:- Given stress state.

$\sigma_x = -4.249$

$\sigma_y = 0$

$\tau_{xy} = 0.6712$

$\sigma_x' = ?$

$\sigma_y' = ?$

$\tau_{x'y'} = ?$

As we derived the following formula. equation for stress transformation.

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

For $\sigma_{x'}$

$$\sigma_x' = \frac{-4.249 + 0}{2} + \frac{-4.249 - 0}{2} \cos 2(-20°) + (0.6712) \sin 2(-20°)$$

$\boxed{\sigma_x' = -1.207 \text{ psi}}$ (Compression).

→

for $\sigma_{y'}$

$$\sigma_{y'} = \frac{-4.249+0}{2} - \frac{(-4.249)-0}{2} \cos(2(-20)) - 0.6712 \sin(2(-20))$$

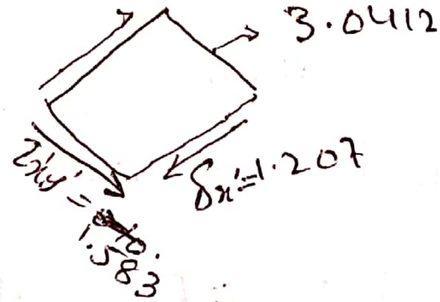
$$\boxed{\sigma_{y'} = +3.0412 \text{ psi}}$$ Tension

Simillarly;
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

$$= -\frac{(-4.249-0)}{2} \sin(2(-20))$$

$$\boxed{\tau_{x'y'} = 1.583 \text{ psi}}$$

So the new state of stress after 20° clock-wise rotation is



* To find the principle stress

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-4.249+0}{2} \pm \sqrt{\left(\frac{-4.249-0}{2}\right)^2 + (0.6712)^2}$$

$$\cancel{= -2.1245 \pm \sqrt{}}$$

$\cancel{\sigma_y =}$ $\sigma_1 = -2.1245 \pm 2.228$

$\sigma_2 =$

$\rightarrow$

$\Rightarrow \sigma_y = \sigma_1 = -2.1245 + 2.228 = 0.1035$

$\sigma_x = \sigma_2 = -2.1245 - 2.228 = -4.3525$

Now;

Maximum in plane Shear Stress

$$\tau_{xy} = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{-4.249-0}{2}\right)^2 + (0.6712)^2}$$

$\boxed{\tau_{xy} = 2.228}$ maximum Plane Shear Stress.

Mohr's Circle for the given problems

as we know;

To draw mohr's circle, we need the Coordinates of Circle as well as its radius.

The Co-ordinates of Circle can be find as;

$$\left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$$

$\longrightarrow$

Center coordinates:

$$(h,k) = \left(-\frac{4.249}{2}, 0\right)$$

$$(h,k) = (-2.1245, 0)$$

radius of mohrs circle.

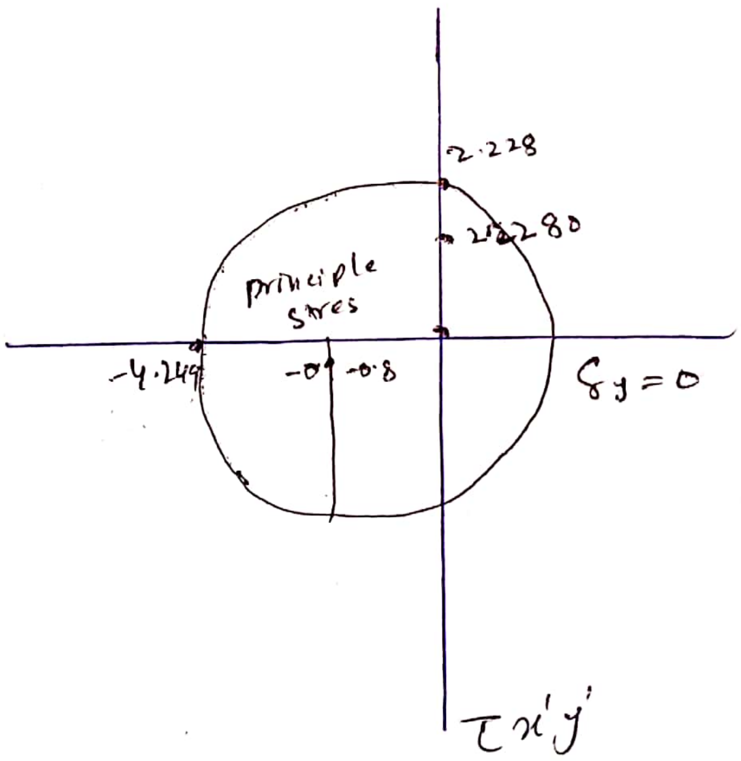
$$r = \sqrt{\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$r = \sqrt{\left(\frac{-4.249}{2}\right)^2 + (0.6712)^2}$$

$$r = \sqrt{4.5135 + 0.4505}$$

$$r = \sqrt{4.964}$$

$$\boxed{r = 2.2280}$$



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