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Digital Signal Processing.

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Abdul Wahab Paper

Question 1(a)

$$y(n)-4y(n-1)+4(n-2)=x(n)-x(n-1)$$

The characteristic equation is

$$x^2-4\lambda+4=0$$

$$\lambda = 2,2 \text{ Hence}$$

$$y_h(n) = c_1 2^n + c_2 n 2^n$$

The particular Solution is

$$y_p(n) = K(-1)^m u(n)$$

Subsitituting the solution into the difference equ, we obtain

$$K(-1)^n u(n) - 4K(-1)^{m-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^m u(n) - (-1)^{n-1} u(n-1)$$

For $n=2, K(1+4+4) = 2 \Rightarrow K = 2/9$

$$y(n) = \left[c_1 2^m + c_2 n 2^n + 2/9(-1)^n\right] u[n]$$

~~y=(0)~~

$y(0)=1$ $y(1)=2$ then

$$c_1 + 2/9 = 1$$

$$\Rightarrow c_1 = 7/9$$

$$2c_1 + 2c_2 - 2/9 = 2$$

$$c_2 = 1/3$$

Part (B)

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

The characteristic solution:-

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = 1/2, 1/3$ Hence

$$y_h(n) = c_1 \left(1/2\right)^n + c_2 \left(1/5\right)^n$$

with

$x(n) = \delta(n)$, we have

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0 \Rightarrow y(1) = 1.4$$

Hene $c_1 + c_2 = 2$ and

$$\tfrac{1}{2} c_1 + \tfrac{1}{5} = 1.4 = 7/5$$

$$c_1 + \tfrac{2}{5} c_2 = 14/5$$

The equ yield

$$c_1 = 10/3 \qquad c_2 = 4/3$$

$$h(n) = \left[\tfrac{10}{3}\left(1/2\right)^n - \tfrac{4}{3}\left(1/5\right)^n\right] u(n)$$

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The step responce

$$s(n) = \sum_{k=0}^{n} h(n-k),$$

$$= 10/3 \sum_{k=0}^{n} (1/2)^{n-k} - 4/3 \sum_{k=0}^{n} (1/5)^{n-k})$$

$$= \frac{10}{3} (1/2)^n \sum_{k=0}^{n} 2^k - 4/3 (1/5)^n \sum_{k=0}^{n} 5^k$$

$$= 10/3 (1/2^n (2^{m+1} - 1) u(n) - 1/3 (1/5^n (5^{n+1} - 1) u(n)$$

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Question 2 (a).

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4 \quad B = -3 \; , \; C = -1$

Hence $x(n) = [4(2)^n - 3 - n]\,u(n)$.

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Question (2 B)

Solution :-

we have.

$$x(n) = \frac{1}{2\pi j}\oint \frac{z^{n-1}}{1-az^{-1}} = \frac{1}{2\pi j}\oint \frac{z^n dz}{z-a}$$

where 'c' is a circle at radius greater than $|a|$ we shall avaluate this integral using $f(z) = z^n$ we distinguish two cases.

→ If $n>0$, $f(z)$ has only zero and hence no pole inside 'c'. The only pole 'c' is $z = a$ Hene

$$x(n) = f(z) = a^n \quad n \geq 0$$

→ If $n<0$ $f(z) = z^n$ has n^{++} order pole at $z = 0$ which is also inside 'c'.

Thus there are condributions from both poles for $n = -1$

we have

$$x(-1) = \frac{1}{2\pi j}\oint_c \frac{1}{z(z-a)} dz = \frac{1}{z-a}\Bigg|_{z=0} + \frac{1}{z^2}\Bigg|_{z=a} = 0$$

By continuatng in the same way we can show that

$x(n) = 0$ for $n<0$

$x(n) = an(n).$

Question No 3(A)

Sol:-

At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$

At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^3}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{(1-p\cos(\pi/4) + jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2} + jp/\sqrt{2})^2}$$

Hence

$$\frac{(1-p)^n}{[(1-p/\sqrt{2})^2 + p^2/2]^2} = 1/2.$$

as equivalently

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

The value of $P = 0.32$ satisfied this equation - consequently the system function for the desired filter is

$$H(z) = 0.46/(1-0.32z^{-1})^2$$

The same principle can be applied for the design of bandpass filter.

Question No 3(b).

Sol:.

Clearly the filter must have pass at.

$$P_{b_2} = re^{\pm j\pi/2}$$

and zeros at $z=1$ and $z=-1$

Consequently the System function

$$H(z) = G \frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G \frac{z^2-1}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency response $H(\omega)$ at the filter at $\omega = \pi/2$ thus.

$$H(\pi/2) = G\, 2/1-r^2 = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at $\omega = 4\pi/9$

Thus

$$|H(4\pi/9)|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)} = 1/2$$

or equivalently

$$1.94(1-r^2) = 1-1.88r^2+r^4$$

The value of $r^2 = 0.7$ satisfied this equation. Therefore the system function desired filter is

$$H(z) = 0.15 \frac{(1-z^{-2})}{1+0.7z^{-2}}$$

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Question No 4 (A).

A filter duration sequence of length 1 is given as.

$$x(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Solution:→

The fourier transform of this sequence

$$X(\omega) = \sum_{n=0}^{1-1} x(n) e^{-j\omega n}$$

$$= \sum_{n=0}^{1-1} e j^{-\omega n} = \frac{1 - e^{-j\omega L}}{1 - e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega L/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated $L=10$ The N-Point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally $K = 0, 1, --- N-1$ Hence.

$$X(K) = \frac{1 - e^{-j2\pi K L/N}}{1 - e^{-j2\pi K/N}} \quad K = 0, 1, --- N-1$$

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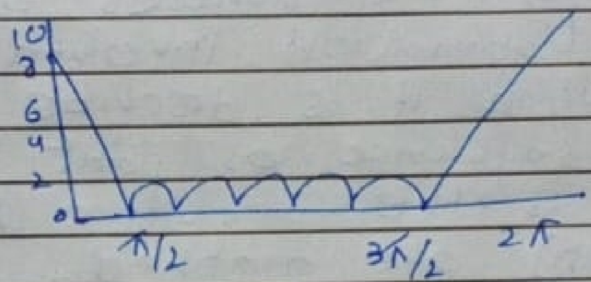
$$= \frac{\sin(\pi K L/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$

If N is selected such that $N = L$ than DFT

$$X(K) = \begin{cases} L & K = 0 \\ 0 & K = 1, 2, \ldots L-1 \end{cases}$$



Thus there is only one non zero value in DFT this apperent from observation of $X(\omega)$ sine $X(\omega) = 0$ at the frequency $\omega_K = 2\pi K/L$

$K = 0$.

Question No 4(b).

$$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$

$$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$

Sol:→

Each sequence consists of single points for the purposes of illustrating the operation involved in circular convolution it is describe to graph each sequence as point on a circle. This is the sequence $x_1(n)$ and $x_2(n)$ are graphed.

Now $x_3(m)$ is obtained by circulating value $x_1(n)$ and $x_2(n)$

$$x_3(0) = \sum_{n}^{3} x_1(n)\, x_2(1-n)_n$$

$x^2(1-n)$ is simply sequence $x(n)$ folded and graph in a circle

$$x_3(0) = 14$$

For $m = 1$ we have.

$$x_2(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

it is easy verified that $x^2((1-n))_4$ is simply the sequence.

$x_2((1-n))_4$ rotated by one unit in time as illustrated.

for $m=2$

we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded sequence.