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SECTION C

SUBJECT APPLIED CALCULAS

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FINAL TERM PAPER

Question No # 01

(1) $P(4,1,3) = 4\hat{i} + \hat{j} + 3\hat{k}$

$Q(1,2,4) = \hat{i} + 2\hat{j} + 4\hat{k}$

Now distance between PQ

So $|PQ| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 (z_2 - z_1)^2}$

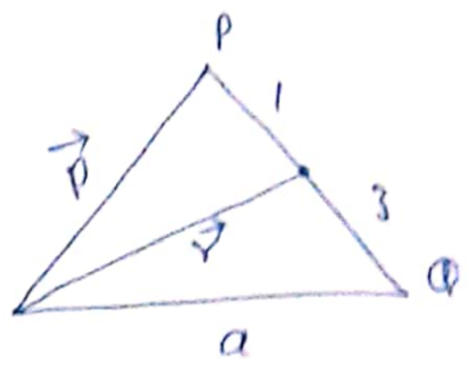
$= \sqrt{(4-1)^2 + (1-2)^2 + (3-4)^2}$

$= \sqrt{(3)^2 + (-1)^2 + (-1)^2}$

$= \sqrt{9+1+1}$ $\boxed{\Rightarrow \sqrt{11}}$

$\boxed{\cancel{\sqrt{11}}}$

Now find the position vector of the point dividing PQ in the ratio of 4 : 3



(2)

$a : b = 1 : 3$

$$r = \frac{bp + aq}{b + a}$$

$$= \frac{3(4i + j + 3k) + 1(\hat{i} + 2\hat{j} + 4\hat{k})}{3+1}$$

$$= \frac{12\hat{i} + 3\hat{j} + 9\hat{k} + \hat{i} + 2\hat{j} + 4\hat{k}}{4}$$

$$= \frac{13\hat{i} + 5\hat{j} + 13\hat{k}}{4}$$

$$r = \frac{13}{4}\hat{i} + \frac{5}{4}\hat{j} + \frac{13}{4}\hat{k}$$ Ans

Question No#2

ANSWER

(3)

$$\int \frac{4x^3+10x+4}{2x^2+x}\,dx$$

Rewrite/simplify

$$= \int \frac{2(2x^3+5x+2)}{x(2x+1)}\,dx$$

Applying linearity

$$= 2\int \frac{2x^3+5x+2}{x(2x+1)}\,dx$$

Now solving

$$\int \frac{2x^3+5x+2}{x(2x+1)}\,dx$$

perform polynomial long division

$$= \int \left(\frac{11x+4}{2x(2x+1)} + \frac{2x-1}{2} \right) dx$$

Applying linearity

$$= \frac{1}{2}\int \frac{11x+4}{x(2x+1)}\,dx + \int x\,dx - \frac{1}{2}\int 1\,dx$$

Now solving

(4)

$$\int \frac{11x+4}{x(2x+1)}\,dx$$

Perform partial fraction decomposition

$$= \int \left(\frac{3}{2x+1} + \frac{4}{x} \right) dx$$

Applying linearity

$$= 3\int \frac{1}{2x+1}\,dx + 4\int \frac{1}{x}\,dx$$

Now solving

$$\int \frac{1}{2x+1}\,dx$$

Substitute $u = 2x + 1 \rightarrow \frac{du}{dx} = 2$ (steps)

$\rightarrow dx = \frac{1}{2} du$:

$$= \frac{1}{2} \int \frac{1}{u} du$$

Now solving

(5)

$$\int \frac{1}{u} du$$

this is standard integral

$$= \ln(u)$$

Plug in solved integrals

$$\frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{\ln(u)}{2}$$

undo substitute $u = 2x + 1$:

Applying constant rule

$$= x$$

Plug in solved integral

$$\frac{1}{2}\int \frac{11x+4}{x(2x+1)}\,dx + \int x\,dx - \frac{1}{2}\int 1\,dx$$

$$= \frac{3\ln(2x+1)}{4} + 3\ln(x) + \frac{x^2}{2} - \frac{x}{2}$$

(6) Plug in solved integral

$$2\int \frac{2x^3+5x+2}{x(2x+1)}\,dx$$

$$= \frac{3\ln(2x+1)}{2} + 4\ln(x) + x^2 - x$$

The problem is solved. Apply the absolute value function to argument of logarithm function in order to extent the antiderivatives domain

$$\int \frac{2(2x^2+5x+2)}{x(2x+1)}\,dx$$

$$= \frac{3\ln(|2x+1|)}{2} + 4\ln(|x|) + x^2 - x + C$$

⑦ Rewrite/Simplify:

$$= \frac{3\ln(|2x+1|)}{2} + 4\ln(|x|) + (x-1)x + C$$

$$= \frac{\ln(2x+1)}{2}$$

= Now solving

$$\int \frac{1}{x}\,dx$$

$$= \ln(x)$$

Plug in solved integrals

$$3\int \frac{1}{2x+1}\,dx + 4\int \frac{1}{x}\,dx$$

$$= \frac{3\ln(2x+1)}{2} + 4\ln(x)$$

P.T.O →

Now solving

$$\int x\,dx$$

Applying power rule

$$\int x^n dx = \frac{x^{n+1}}{n+1} \quad \text{then } n=1:$$

(8)

$$= \frac{x^2}{2}$$

Now solving

$$\int 1\,dx$$

Applying constant rule

$$= 3$$

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Question No #03

PART A

$$\int_0^2 x^2 e^x \, dx$$

Integrating by parts

(9) $x^2 \int e^x - \int \left(e^x dx \cdot \frac{d}{dx} x^2 \right) dx$

$x^2 e^x - \int e^x (2x) \, dx$

$x^2 e^x - \int 2x e^x \, dx$

$x^2 e^x - 2 \int x e^x \, dx$

use integration by parts

$x^2 e^x - 2 \left[x \int e^x dx - \int \left(e^x dx \cdot \frac{d}{dx} x \right) dx \right.$

$x^2 e^x - 2 \left[x e^x - \int e^x \, dx \right]$

$x e^x - 2 x e^x + 2 e^x$

P.T.O →

$(x^2 - 2x + 2)e^x$

Applying limits

$(2^2 - 2(2) + 2)e^2 - (0^2 - 2(0) + 2)e^0$

10 $\boxed{2e^2 - 2}$ ANS

Question No # 03

PART # B

$$\int_1^0 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx \rightarrow (i)$$

let $u = \sqrt{x}$

(ii) $$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$$

$$dx = 2\sqrt{x}\, du$$

put in eq (i)

$$\int 2\sin u\, du$$

$$2\int \sin u\, du$$

$$-2\cos u$$

$$-2\cos\sqrt{x}$$

Applying limits

$$2\cos\sqrt{2} - 2\cos(1)$$

P.T.O →

-0.769

As area cannot be negative so

$\boxed{0.769}$

12

① Question No # 04

$$u(x,y,z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$$

SOLUTION

13 The laplace the equation in 3-d is

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \longrightarrow (A)$$

So $u(x,y,z) = \dfrac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x,y,z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{\cancel{2}}(x^2+y^2+z^2)^{-3/2}(\cancel{2}x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left[x\left(-3/2\right)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \rightarrow (1)$$

Now

$$\frac{\partial u}{\partial y} = \frac{-1}{2}(x^2+y^2+z^2)^{-3/2}(2y)$$

(14)

$$\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{-3/2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\left[y(-3/2)(x^2+y^2+z^2)^{-5/2}(2y) + (x^2+y^2+z^2)^{-3/2}\right]$$

$$\boxed{\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}} \rightarrow (2)$$

$$\frac{\partial u}{\partial z} = \frac{-1}{2}(x^2+y^2+z^2)^{-3/2}(2z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{-3/2}$$

$$\boxed{\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2) - (x^2+y^2+z^2)^{-3/2}} \rightarrow (3)$$

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(2)

putting eq (1), (1) & (2) in (A)

$$= 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3y^2(x^2+y^2+z^2)^{-5/2} -$$

$$(x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2}$$

(15) $$= (x^2+y^2+z^2)^{-5/2}\left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2)\right]$$

$$= (x^2+y^2+z^2)^{-5/2}\left[\cancel{3x^2} - \cancel{x^2} - \cancel{y^2} - z^2 + 3y^2 - \cancel{x^2} - \cancel{y^2} - z^2 + 3z^2 - \cancel{x^2} - \cancel{y^2} - z^2\right]$$

$$= (x^2+y^2+z^2)^{-5/2}(0) = 0$$

So the given $u(x,y,z)$ is solution of laplace equation