Date: / /

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IQRA NATIONAL UNIVERSITY

Submit To : Sir SHAKEEL

Subject : Linear Algebra

Program : (BS SE) SEC: (B)

Final Term paper

Date: 30 June 2020.

Q No 1

ID = 16764

$ID_3 = 7$

$$\left[\begin{array}{ccc|c} 1 & -7 & 1 & 0 \\ 0 & 2 & -8 & 8 \\ 5 & 0 & -5 & 10 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -7 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -10 & 5 \end{array}\right] \begin{array}{l} \frac{1}{2}R_3 \\ \\ R_3+3R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -7 & 1 & 0 \\ 0 & 1 & -4 & 4 \\ 0 & 0 & -10 & 6 \end{array}\right] \quad -1R_2$$

There is unique and consistent because

$x_1 = 1$

$x_2 = 1$

$x_3 = 6$

Q2:

Ans:

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$ID_4 = 6$ putting

$$A = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 6 \\ 5 & -2 & 7 \end{bmatrix}$$

$$A^{-1} = \frac{Adj\ (A)}{|A|}$$

$$|A| = 3\begin{vmatrix} -1 & 6 \\ -2 & 7 \end{vmatrix} - 4\begin{vmatrix} 2 & 6 \\ 5 & 7 \end{vmatrix} + 5\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix}$$

$3(5) - 4(-22) + 5(1)$

$15 + 48 + 5 = 68$

$|A| = 68$

$$A_{11} = (-1)^2 \begin{vmatrix} 2 & 5 \\ 5 & 7 \end{vmatrix} = 3$$

$A_{12} = 11$

$a_{13} = 1$

$a_{21} = -3$, $a_{22} = -4$, $a_{23} = 20$

$a_{31} = 22$, $a_{32} = -8$, $a_{33} = -10$

$$\text{Adj } A = \begin{bmatrix} 3 & 1 & 11 \\ -3 & -4 & 20 \\ 22 & -8 & -10 \end{bmatrix}$$

$a_{13} = 1$

$a_{21} = -3$, $a_{22} = -4$ $a_{23} = 20$

$a_{31} = 22$, $a_{32} = -8$, $a_{33} = -10$

$$\text{Adj } A = \begin{bmatrix} 3 & 1 & 11 \\ -3 & -4 & 20 \\ 22 & -8 & -10 \end{bmatrix}$$

Take transpose

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$$\begin{bmatrix} 3 & -3 & 22 \\ 1 & -4 & -8 \\ 11 & 20 & -10 \end{bmatrix}$$

$$A^{-1} = \frac{1}{68} \begin{bmatrix} 3 & -3 & 22 \\ 1 & -4 & -8 \\ 11 & 20 & -10 \end{bmatrix}$$

Ans

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Q 3:

$$2x + 2y + 4z = 18$$
$$x + 3y + 2z = 13$$
$$3x + y + 3z = 14$$

Solution

$$\begin{bmatrix} 2 & 2 & 4 & 18 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix} \xrightarrow{R_1 = 1/2\, R_1}$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \begin{matrix} R_2 = R_2 - R_1 \\ R_3 = R_3 - 3R_1 \end{matrix} \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix} \xrightarrow{R_2 = 1/2\, R_2}$$

$$\begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} R_3 = R_3 + 2R_2 \begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \begin{matrix} \xrightarrow{R_1 = R_1 - R_2} \\ R_3 = -1/3\, R_3 \end{matrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} R_1 = R_1 - 2R_3$$

So the solution is

$$x = 1$$
$$y = 2$$
$$z = 3$$

So $z = 3$

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Q (4)

: Answer

$$\begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 4-\lambda & 2 & -2 \\ -5 & 3-\lambda & 2 \\ -2 & 4 & 1-\lambda \end{vmatrix} = 0$$

First we find eigan value

$(4-\lambda)(3-\lambda)(1-\lambda)$

$4-\lambda \Rightarrow \lambda = 4$
$3-\lambda \Rightarrow \lambda = 3$
$\lambda = 1$

$\lambda = 4, 3, 1$

Now we find eigan vactor

; when $\lambda = 4$

Let $x_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ be ibi eigen vector

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$[A-4I]x_1 = 0$

"I" mean identity.

$$\begin{bmatrix} 4-4 & 2 & -2 \\ -5 & 3-4 & 2 \\ -2 & 4 & 1-4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & -2 \\ 5 & -1 & 2 \\ -2 & 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5 & -1 & 2 \\ 0 & 2 & -2 \\ -2 & 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

change R_1 to R_2

$$\begin{bmatrix} 1 & -\frac{1}{5} & \frac{2}{5} \\ 0 & 2 & -2 \\ -2 & 4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -\frac{1}{5} & \frac{2}{5} \\ 0 & -2 & -2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\frac{1}{5}R_1 + R_3$

$\frac{1}{2}R_2 + 3R_2$

Hence Rank of the matrix is 2

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determinate = 2

Matrix => 3 X 3

Linear independed = 3 - 2

$\boxed{L-1 = 1}$

Hence Linear independing value & eigen vector is equal (1)

$x + y - 3z = 0$

$-4y + 8z = 0$

$y = 2z$ putting in above equation

$x + 2z - 3z = 0$

$\boxed{x = 2}$

eigen $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

$\lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$R_2 = R_2 - 2R_1$, $R_3 = R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$ vector $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

Rank = 1

matrix = 3

L.I = 3 − 1 = (2)

$x + y + z = 0$

Let $z = 0$, $y = 1$, $x = -1$

vector $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$$P = \begin{bmatrix} 1 & -1 & -1 \\ 2 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

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Question No (5)

Solution

$3x_1 + 5x_2 - 4x_3 = 0$
$-3x_1 - 25x_2 + 4x_3 = 0$
$6x_1 + x_2 - 8x_3 = 0$

first we change into an augmented matrix

$$\left[\begin{array}{ccc|c} 3 & 5 & -4 & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & \frac{-4}{3} & 0 \\ -3 & -25 & 4 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] \frac{1}{3}R_1$$

$$\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & \frac{-4}{3} & 0 \\ 0 & -22 & 7 & 0 \\ 6 & 1 & -8 & 0 \end{array}\right] +3R_{1,2}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & -\frac{4}{3} & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & -13 & 0 \end{array}\right] \begin{array}{l} \frac{1}{4}R_2 + 3R_3 \\ R_3 + 3R_2 \end{array}$$

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$$\left[\begin{array}{ccc|c} 1 & \frac{5}{3} & -4 & 0 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] +13R_3$$

→ free variable

$x_1 + \frac{5}{3}x_2 - 4x_3 = 0$

$0x \quad x_2 \quad 4x_3 = 0$

$0x_3 = 0$

$x_1 - 4x_3 = 0$

$x_2 + 4x_3 = 0$

$x_3 = x_3$

$x_1 = 4x_3$

$x_2 = 4x_3$

$x_3 = x_3$

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Question No (6)

Answer

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 3 & 9 & 12 & 3 \\ 1 & 3 & 4 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 6 & 9 & 0 \\ 1 & 3 & 4 & 0 \end{bmatrix} \quad -3R_2$$

$$A = \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 6 & 9 & 0 \\ 0 & 2 & 3 & 0 \end{bmatrix} \quad -1R_3$$

$$= \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 2 & 3 & 0 \\ 0 & 6 & 9 & 0 \end{bmatrix} \quad \text{changing } A_2 \text{ to } R_3$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 1 & \frac{3}{2} & 0 \\ 0 & 6 & 9 & 0 \end{bmatrix} \quad \frac{1}{2} B_2$$

$$\begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix} \quad -6R_3$$

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$$\begin{bmatrix} 1 & 0 & 4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{array}{l} -\frac{1}{2}R_1 + 3R_{13} \\ R_1 + 3R_{12} \\ \frac{1}{3}R_3 \end{array}$$

So

$$\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$$ normal form.

Hence Rank of this matrix is 2 b/c 2 is minimum matrix which value are non zero.

end