

Sol: $1:2:1, 2:1:1, 2:0:2$

40

P	E
A	E

B_1

50

P	P
A	E

B_2

P	P
A	A

B_3

Let x, y and z be the cost/kg of Pak, Egyptian, American cotton respectively. then according to the given conditions

$$\left. \begin{aligned} \frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z &= 40 \\ \frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z &= 50 \\ \frac{2}{4}x + \frac{2}{4}z &= 60 \end{aligned} \right\} \textcircled{A}$$

$$\left. \begin{aligned} 1x + 2y + 1z &= 160 \\ 2x + 1y + 1z &= 200 \\ 1x + 1z &= 120 \end{aligned} \right\} \textcircled{B'}$$

In matrix form, we can write as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$

$$AX = B$$

$$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix} \text{ Expand by } R_1$$

$$= 1 \begin{vmatrix} 1 & 200 \\ 0 & 120 \end{vmatrix} - 2 \begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix} + 160 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(120 - 0) - 2(240 - 200) + 160(0 - 1)$$

$$|A_3| = -120$$

Now according to Cramer's Rule

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{-40}{-2} = 20$$

$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = 60$$

$$(x, y, z) = (60, 20, 60)$$

Pakistani = \$ 60

Egyptian = 20

American = 60

Ans.

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$$A_1 = \begin{bmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix}$$

First $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$ Expand by R_1

$$= 1 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(1 \times 1 - 1 \times 0) - 2(2 \times 1 - 1 \times 1) + 1(2 \times 0 - 1 \times 1)$$

$$= -2$$

Next $|A_1| = \begin{vmatrix} 160 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix}$ Expand by R_1

$$= 160 \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1 \begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$$

$$= 160(1 - 0) - 2(200 - 120) + 1(0 - 120)$$

$$|A_1| = -120$$

Similarly $|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix}$ Expand by R_1

$$= 1 \begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} - 160 \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix}$$

$$= 1(200 - 120) - 160(2 - 1) + 1(240 - 200)$$

$$|A_2| = -40$$