Date 28/09/2020 (1)

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ID: 13794

Subject: Differential Equations

Q3

A. Find the laplace transforms of the given functions.

1. $f(t) = 6e\wedge(-5t) + e\wedge 3t + 5(t\wedge 3) - 9$

Sol: $F(s) = L\{f(t)\} = 6L\{e^{3t}\} + 5L\{t^3\} - 9L\{t\}$

$$= 6\frac{1}{s-(-5)} + \frac{1}{s-3} + 5\frac{3!}{s^{3+1}} - 9\frac{1}{s}$$

$$= \frac{6}{s+5} + \frac{1}{s-3} + \frac{30}{s^4} - \frac{9}{s} \text{ Ans.}$$

Q3) Define laplace transform along with two examples.

Ans: In Mathematics the laplace transform, named after its inventor, is an integral transforms that converts a function of a real variable t to a function of a complex variable s. The transform has many applications in science & engineering.

Darsi Notes

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ex① $f''(t)+3f'(t)+2f(t)=0$ where $f(0)=1$ &
$f'(0)=0$

$L[f''(t)+3f'(t)+2f(t)] = L[f''(t)]+3L[f'(t)]+2Lf(t)$

$(s^2+3s+2)L[f(t)] = s+3$

÷ by (s^2+3s+2)

$L[f(t)] = \dfrac{s+3}{s^2+3s+2}$

$\dfrac{s+3}{s^2+3s+2} = \dfrac{s+3}{(s+1)(s+2)}$

$\dfrac{s+3}{(s+1)(s+2)} = \dfrac{A}{s+1} + \dfrac{B}{s+2}$

cross X

$s+3 = A(s+2)+B(s+1)$

coeff of A & B

$s=-1,\ 2=A(1),\ A=2,\ s=-2,\ 1=B(-1),\ B=-1$

subst the eq $\dfrac{2}{s+1}+\dfrac{-1}{s+2}$

ex2) $L\{c_1f(t)+c_2g(t)\} = c_1L\{f(t)\}+c_2L\{g(t)\}$

Q 1a) Define 2nd order linear homogenous/non-homogenous differential equations along with examples.

Ans: 2nd Order linear Homogenous :-

(1) The general solution of homogenous differential equation depends on the root of the Characteristics quadratic equation

example $y'' - 6y' + 5y = 0$, $y' - 6ay + a = 0$

(2) The non homogenous differential equation are same homogenous diff equation except they can have terms including only x (and constant) on the right side as in this equation

$$\frac{d^4y}{dx^4} + x\frac{d^2y}{dx^2} + y^2 = 6x+3$$

Q3 A

ii)

Sol: $G(s) = \mathcal{L}\{g(t)\} = 4\mathcal{L}\{\cos(4t)\} - 9\mathcal{L}\{\sin(4t)\}$

$+2\mathcal{L}\{\cos(10t)\}$

$$= 4\frac{s}{s^2+4^2} - 9\frac{4}{s^2+4^2} + 2\frac{s}{s^2+10^2}$$

$$= \frac{4s-36}{s^2+16} + \frac{2s}{s^2+100} \quad \text{Ans.}$$

iii)

Sol: $H(t) = \mathcal{L}\{h(t)\} = \mathcal{L}\{e^{3t}\} + \mathcal{L}\{\cos(6t)\} -$

$\mathcal{L}\{e^{3t}\cos(6t)\}$

$$= \frac{1}{s-3} + \frac{s}{s^2+6^2} - \frac{s-3}{(s-3)^2+3^2}$$

$$= \frac{1}{s-3} + \frac{2}{s^2+9} - \frac{s-3}{(s-3)^2+9} \quad \text{Ans.}$$

Q 4)

b) $y'' + 3y' + 2y = e^{\wedge}(-t)$, $y(0)$ $y'(0) = 0$

Sol: $y + 3y' - 2y = e - t$ $y(0) = 0, y(0) = 0$

taking laplace

$L[y''] + 3L[y'] + 2L[y] = L[e^{-t}]$

$s^2 y(s) = sy(0) - y'(0) + 3\{sY(s) - y(0)\}$
$+2Y(s)$

put the initial value

$s^2 Y(s) + 3\{sY(s)\} + 2Y(s) = \frac{1}{s+1}$

$\Rightarrow Y(s)[s^2 + 3s + 2] = 1/s+1$

$= Y(s) = \frac{1}{(s+1)^2(s^2+3s+2)}$

$\Rightarrow \frac{1}{(s+1)^2(s+2)}$

Now $Y(s) = -\frac{1}{s+1} + \frac{1}{(s+1)^2} + \frac{1}{s+2}$

Now $y(t) = -e^{-t} + te^{-1} + e^{-2t}$ Ans.

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Q 4 a)

Sol: $y(t) \to y(s)$

$y'(t) \to sy(s) - y(0)$

$y''(t) \to s^2y(s) - sy(0) - y'(0)$

Now taking Laplace

$s^2y(s) - sy(0) - y'(0) - 4sy(s) + 4y(0)$

$= \frac{1}{s-3}$

$\Rightarrow s^2y(s) - s\times 0 - 0 - 4sy(s) = 4\times 0 = \frac{1}{s-3}$

$\Rightarrow s^2y(s) - 4sy = \frac{1}{s-3}$

$\Rightarrow \quad y(t)(s^2 - 4s) - 1/s-3$

$\Rightarrow y(s) = \frac{1}{s(s-4)(s-3)}$

so

$\frac{1}{s(s-4)(s-3)} = \frac{A}{s} + \frac{B}{s-4} + \frac{C}{s-3}$

$\Rightarrow 1 = A(s-4)(s-3) + B(s)(s-3) + C(s)(s-4)$

Take $s=0$

$1 = A(-4)(-3)$

$\therefore \quad A = \frac{1}{12}$ take $s=4$

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$1 = B \times 4 \times 1$

$\therefore B = 1/4$

Take $s = 3$

$1 = C \times 3 \times -1$

$\therefore C = -1/3$

Hense $Y(s) = \frac{1}{12}\frac{1}{s} + \frac{1}{4}\frac{1}{s-4} - \frac{1}{3}\frac{1}{s-3}$

Q 1 b)

i) $16y'' + 24y' + 9y = 0$

Sol: replacing y'' with r^2, y' with r

& y with 1

Now $16r^2 + 24r + 9 = 0$

$\therefore \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

roots

$$r = \frac{-24 \pm \sqrt{(24)^2 - 4(16)(9)}}{2(16)}$$

$$\Rightarrow \frac{-3}{4}$$

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now $y(t) = c_1 e^{rt} + c_2 t e^{rt}$

with r the double root

since

$r = -3/4$ solution

$y(t) = c_1 e^{-3t/4} + c_2 t e^{-3t/4}$ Ans.

Q2 (c) iii

Sol. $m = \frac{d}{dx}$

$\Rightarrow m^2 - 4m + a = 0$

$ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Now $m^2 - 4m + a = 0$

$$m = \frac{4 \pm \sqrt{16 - 36}}{2a}$$

$$\Rightarrow \frac{4 \pm \sqrt{20}}{2}$$

$\Rightarrow 2 \pm i\sqrt{5}$

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$y = e^{2t}(A \cos\sqrt{5}t + B \sin\sqrt{5}t)$

we know

$y(0) = 0$

$0 = e^{0}(A)$

$\Rightarrow A = 0$

Hense $y = e^{2t}(B \sin\sqrt{5}t)$

$y'(0) = 8$

$y' = 2\sqrt{5}e^{2t} B \cos\sqrt{5}t$

$y'(0) = 2\sqrt{5} B$

$\Rightarrow -8 = 2\sqrt{5}B$

$\Rightarrow B = -4/\sqrt{5}$ $y = -4/\sqrt{5}\, e^{2t} \cos\sqrt{5}t$ Ans.

Q 2 ii

Sol: Solution

$2m^2 + 5m - 3 = 0$

$\Rightarrow 2m^2 + 5m - 3 = 0$

$2m^2 + 6m - m - 3 = 0$

$\Rightarrow 2m(m+3) - 1(m+3) = 0$

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$\Rightarrow 2m(m+3) - 1(m+3) = 0$

$\Rightarrow (2m-1)(m+3) = 0$

$\Rightarrow m_1 = -3 \quad m_2 = 1/2$

Now

$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$

$y = c_1 e^{-3x} + c_2 e^{m_2 x}$

$y = c_1 e^{-3x} + c_2 e^{1/2 x}$

applying initial condition

$y(0) = 3$

$c_1 \; c_2 = 3$ —— (1)

$\Rightarrow y' = -3ce^{-3x} + 1/2 \; c_2 \; {}^{1/2} e^{x}$

$y'(0) = 4$

$\Rightarrow -3c + 1/2 \; c_2 = 4$ —— "

Now

$3 \times (1) + (2)$ implies

$\Rightarrow (3 + 1/2) c_2 = 13$

$c_2 = 26/7$

$c_1 = -5/7$ so $y = 5/7 e^{1/2 x}$ $26/7 e^{-3x}$ –