Name = Shaheryar Khan

ID = 7891

Section = A.

Subject = Differential

Teacher = Mam shomaila

Assignment = 2.

Question (1)

(1) $x^3y''' + 2x^2y' + 2y - 10x + \frac{10}{x}$

Solution

$$x^3\frac{d^3y}{dx^3} + 2x^2\frac{dy}{dx} + 2y = 10x + 10x^{-1}$$

$$x^3 D^3y + 2x^2D^2 + 2y = 10x + 10x^{-1}$$

$$(x^3D^3 + 2x^2D + 2)y = 10x + 10x^{-1}$$

let $x = e^t \Rightarrow t = \ln x$.

$xD = D$.

$x^2D^2 = D(D-1) = D^2 - D$

$x^3D^3 = D(D-1)(D-2)$

Substituting into eq (1)

$$(D - 3D^2 + 2D + 2(D^2 - D) + 2)y = 10^x + 10x^{-1}$$

$$(D^3 - D^2 + 2)y = 10x + 10x^{-1}$$

$$(m^3 - m^2 + 2)y = 10e^t + \frac{10}{e^t}$$

Using Synthetic division.

-1	1	-1	0	2
		-1	2	-2
	1	-2	2	0

$D^2 - 2D + 2 = 0$

Now using Quadratic formula

$a = 1$ $b = -2, c = 2$

$$D = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$D = -(-2) \pm \sqrt{-2^2 - (4)(1)(2)}$$

$$A = \frac{2 \pm \sqrt{4-8}}{2}$$

$$A = \frac{2 \pm \sqrt{-4}}{2}$$

$$A = \frac{2 \pm \sqrt{-4}}{2}$$

$$A = \frac{2 \pm \sqrt{-1} \times \sqrt{4}}{2}$$

$$A = \frac{2 + 2i}{2}$$

$$A = \frac{2(1 \pm i)}{2}$$

$$A = 1 \pm i$$

Since roots are complex.

$$y_c = e^{-x}(c_1 \cos t + c_2 \sin t)$$

Now particular integration

$$y_p = \frac{1}{D^3 - D^2 + 2} \cdot 10e^t + \frac{1}{D^2 - d + 2} = 10/e^t$$

$$= \frac{10e^t}{(1)^3 - (1)^2 + 2} + \frac{10e^{-t}}{(1)^2 + (1)^2 + 2}$$

$$= \frac{\cancel{10}^{5} e^t}{\cancel{2}} + \frac{\cancel{10}^{5} e^{-t}}{\cancel{2}}$$

$$= 5e^t + 5e^{-t}$$

$$y_p = 5e^t + 5e^{-t}$$

General solution

$$y = y_c + y_p$$

$y = e^{-x}(c_1 \cos t + c_2 \sin t) + 5e^{t} + 5e^{-t}$

Put $e^{t} = x$ and $t = \ln x$.

$y' = e^{-x}(c_1 \ln x + c_2 \sin(x)) + 5e^{x} + 5e^{-x}$

Ans

Question # 02

2) $\dfrac{x^3 d^3y}{dx} + \dfrac{5x^2 d^2y}{dx} - \dfrac{5x\,dy}{dx} - 15y = x^4$

<u>Solution</u>:-

let $d/dx = D$.

$x^3D^3y + 9x^2D^2y - 5xDy - 15y = x^4$

$(x^3D^3 + 4x^2D^2 - 5xD - 15)y = x^4$

let $x = e^{t} \Rightarrow t = \ln x$.

$xD = \Delta$.

$x^2D^2 = \Delta(\Delta-1) = \Delta - \Delta$

$x^3D^3 = \Delta(\Delta-1)(\Delta-2) = \Delta^3 - 3\Delta^2 + 2\Delta$.

Now substituting..

$(x^3D^3 + 9x^2D^2 - 5xD - 15)y = x^4$

$(\Delta^3 - 3\Delta^2 + 2\Delta + 4(\Delta^2 - \Delta) - 5(\Delta) - 15)y = e^{4t}$

$(\Delta^3 + \Delta^2 - 7\Delta - 13)y + e^{4t}$

Sythetic division

5	1	+1	-7	-15
		3	12	15
1	1	4	5	0

$A^2+4A+5=0$

daodratic formula

$$A = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{4^2-4(1)(5)}}{2}$$

$$= \frac{-4 \pm \sqrt{16-20}}{2}$$

$$= \frac{-4 \pm 2i}{2}$$

$$D = \frac{\cancel{2}(-2 \pm 1)}{\cancel{2}}$$

$$y_c = e^{-2n}(c_1 \cos t + c_2 \sin t)$$

for $y_p = ?$

$$y_p = \frac{1}{D^3+D^2-7D-15} e^{4t}$$

$$= \frac{1}{(4)^3+(4)^2-7(4)-15} e^{4t}$$

$$= \frac{1}{64+16-28-15} e^{4t}$$

$$= \frac{1}{80-43} e^{4t}$$

$$y_p = \frac{1}{37} e^{4t}.$$

Hence

$y = y_c + y_p$

$y = (c_1 \cos t + c_2 \sin t) + \frac{1}{37} e^{4t}$

again put $t = \ln x$ and $x = \ln x$

$y = e^{-2x}(c_1 \cos \ln x + c_2 \sin \log x) + \frac{1}{37} e^{4x}.$

Question (3)

$x^2y'' + 2xy' - 6y = 10x^2$

Solution

$y(1) = 1$ and $y'(1) = -6$

$x^2 \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 6y = 10x^2$

$\Rightarrow (x^2 d^2/dx^2 + 2x\, d/dx - 6)y = 10x^2$

put $xD = D \Rightarrow x^2D^2 = D(D-1) = D^2 - D$

$x = e^t$ and $\log x = t$

$(D^2 - D + 2D - 6)y = 10e^{2t}$

$(D^2 + D - 6)y = 10e^{2t}$

The characteristic equation.

$D^2 + D - 6 = 0$

$D^2 + 3D - 2D - 6 = 0$

$\Rightarrow D(D+3) - 2(D+3) = 0$

$\Rightarrow (D+3)(D-2) = 0$

$D + 3 = 0,\ D - 2 = 0$

$D = 2,\ D = -3.$

Since roots are real and distinct.

for y_c = ?

$y_c = C_1e^{-3t} + C_2e^{2t}$

for y_p = ?

$y_p = \frac{1}{D^2 - D - 6} 10^{2t}$

$= 10 \frac{1}{0} e^{2t}$ fails

Now $\frac{1}{10\, d/dD\, (D^2 + D - 6)} e^{2t}$.

$$\Rightarrow 10 \frac{t}{2D+1} e^{2t}$$

$$= 10 \frac{1 \cdot t}{4+1} e^{2t}$$

$$y_p = 2te^{2t}$$

General solution

$$y = y_c + y_p$$

$$= c_1 e^{-3t} + c_2 e^{2t} + 2te^{2t}$$

$$y = c_1 x^{-3} + c_2 x^2 + 2(\log x) x^2 \text{ --- } (B)$$

Put $y(1) = 1$ i.e. $x = 1, y = 1$ in (B)

$$1 = c_1 (1)^{-3} + c_2 (1)^2 + 2 \log(1)$$

$$1 = c_1 + c_2 \longrightarrow (C)$$

Now differentiate eq (B) w.r.t x.

$y' = -3c_1 x^{-4} + 2c_2 x + \frac{2}{x}(x^2) + 4x\log x$

Now put $y'(1) = -6$ i-e $y' = -6$ and $x = -6$.

$-6 = -3c_1 + 2c_2 + 2 + 0$

$\Rightarrow -6 = -3c_1 + 2c_2 + 2.$

$\Rightarrow -6 - 2 = -3c_1 + 2c_2 + 2$

$-8 = -3c_1 + 2c_2$ — (1)

$\times$ing eq (1) with (2) and $-$ing from (1))

$2c_1 + 2\cancel{c_2} = 2$

$\mp 3c_1 \pm \cancel{2}c_2 = \mp 8$

$5c_1 = 10$

$c_1 = \frac{10}{5}$ $\boxed{c_1 = 2}$

$-8 = -3(2) + 2c_2$

$-8 = -6 + 2c_2$

$2c_2 = -8 + 6$

$2c_2 = -2.$

[illegible]

$y(1) = 2$ and $y'(1) = 2$.

$C_2 = -\frac{2}{2}\, 1$.

$C_2 = -1$.

Now put the values of c_1 and c_2 in eq (3)

$y = 2x^{-3} - x^2 + 2\ln x\,(x^2)$

$y = \frac{2}{x^3} - x^2 + 2x^2 \log x$ Ans

$\boxed{\text{Question \#04}}$ (1)

$x^2y'' + 7xy' + 5y = x^5$

$y(1) = 2$ and $y'(1) = 2$.

Solution

$$x^2\frac{d^2y}{dx^2} + 7x\frac{dy}{dx} + 5y = x^5$$

$$\Rightarrow \left(x^2\frac{d^2}{dx^2} + 7x\frac{d}{dx} + 5\right)y = x^5 \quad -(1)$$

Put $xD = \Delta \Rightarrow x^2D^2 = D(D-1) = D^2 - D$

$x = e^t \Rightarrow \log x = t$ in eq (2)

$$\Rightarrow (\Delta^2 - D - 7\Delta + 5)y = e^{5t}.$$

$$\Rightarrow (\Delta^2 + 6\Delta + 5)y = e^{5t}$$

$$\Rightarrow (\Delta^2 + 6\Delta + 5)y = e^{5t}.$$

By Quadratic formula

$$\Delta = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$D = \frac{-6 \pm \sqrt{6^2 - 4(1)(5)}}{2(1)}$$

$$= \frac{-6 \pm \sqrt{36-20}}{2}$$

$$= \frac{-6 \pm \sqrt{16}}{2}$$

$$= \frac{-6 \pm \sqrt{4^2}}{2}$$

$$= \frac{\cancel{2}(-3 \pm 2)}{\cancel{2}}$$

$D = -3 \pm 2$ Since roots are real and distinct

$$y_c = C_1 e^{-5t} + C_2 e^{-t}$$

For $y_p = ?$

$$y_p = \frac{1}{D^2+6D+5} e^{t}$$

$$= \frac{1}{(5)^2+6(5)+5} e^{5t}$$

$$= \frac{1}{60} e^{5t}$$

Now General solution is

$$y = y_c + y_p$$

$$y = C_1 e^{-5t} + C_2 e^{-t} + \frac{1}{60} e^{5t}$$

$$y = C_1 x^{-5} + C_2 x^{-1} + \frac{1}{60} x^5$$

$x = 0$ put in this equation

No in eq (13) $e^0 = 1$.

put $y(0) = 2$ i.e. $y = 2$ and $x = 2$

$$2 = C_1(2)^{-5} + C_2(2)^{-1} + \frac{1}{60}(2)^5$$

$$2 = -32c_1 - 2c_2 + \frac{1}{\cancel{60}_{15}}(3\cancel{2}^{8})$$

$$2 = -32c_1 - 2c_2 + \frac{8}{15}$$

$$2 - \frac{8}{15} = -32c_1 - 2c_2.$$

$$\frac{22}{15} = -32c_1 - 2c_2 \longrightarrow C$$

Now differentiate eq(B) w.r.t (x)

$$y' = -5c_1 x^{-6} - c_2 x^{-2} + \frac{1}{12}x^4 \longrightarrow$$

put $y'(1)=2$ i-e $y'=2$ and $x=2$ in above equation

$$2 = -5c_1 (2)^{-6} - c_2 (2)^{-2} + \frac{1}{12}(2)^4$$

$$2 = -5c_1(-64) - c_2(4) + \frac{1}{12}(16)$$

$$2 = 320c_1 + 4c_2 + 4/3$$

$\Rightarrow 2 - 4/3 = 320 C_1 + 4C_2$.

$\Rightarrow 2/3 = 320 C_1 + 4C_2$.

xing eq(1) with 2 and then - my eq(1) from.

$-\frac{44}{15} = 64 C_1 + 4C_2$.

$-\frac{44}{15} = 64 C_1 + 4C_1$

$+ 2/3 = +320 C_1 + 4C_2$

$\frac{34}{15} = -256 C_1$

$C_1 = \frac{34}{15} \times 256$

$C_1 = 580$.

put the values of C_1 in eq(1)

$\frac{22}{15} = -32(580 - 2C_2$.

$$\Rightarrow \frac{22}{15} = -13560 - 2C_2.$$

$$\Rightarrow \frac{22}{15} + 13560 = -2C_2$$

$$\Rightarrow \frac{13561}{-2} = C_2$$

$$\boxed{-9280 = C_2}$$

Now put the value of C_1 and C_2 in eq (B).

$$y = 580x^{-5} - 9280x^{-1} + \frac{1}{60}x^5$$

$$y = \frac{580}{x^5} - \frac{9280}{x} + \frac{1}{60}x^5$$

Ans

Question no 5.

$$(x+1)^2 y'' - 3(x+1)^1 y' + 4y = x^2$$

Solution:-

$$(x+1)^2 \frac{d^2y}{dx^2} - 3(x+1)\frac{dy}{dx} + 4y = x^2$$

$$\Rightarrow \left((x+1)^2 \frac{d^2}{dx^2} - 3(x+1)\frac{d}{dx} + 4\right) y = x^2$$

$$\left((x+1)^2 \Delta^2 - 3(x+1)\Delta + 4\right) y = x^2 \longrightarrow (A)$$

Put $(x+1)\Delta = \Delta \Longrightarrow (x+1)^2 \Delta^2 = \Delta$

$\Delta(\Delta - 1)$ $\Delta^2 - \Delta$.

$x = e^t$ in eq (A)

$$\Rightarrow (\Delta^2 - \Delta - 3\Delta + 4) y = e^{2t}$$

$$\Rightarrow (\Delta^2 - 4\Delta + 4) \quad y = e^{2t}$$

$$\Rightarrow (\Delta^2 - 4\Delta + 4) \quad 2 = e^{2t}$$

For y_c we find the roots.

$$\Delta^2 - 4\Delta + 4 = 0$$

$$\Delta^2 - 2\Delta - 2\Delta + 4 = 0$$

$$\Delta(\Delta - 2) - 2(\Delta - 2) = 0$$

$$\Delta - 2 = 0, \quad \Delta - 2.$$

$D - 2 = 0$, $D = 2$.

So the roots are real and repeat the general solution are.

$$y = (C_1 + C_2 x)^{2x}$$

$$y = (C_1 + C_2 x)^{2x}$$

For $y_p = ?$

$$y_p = \frac{1}{D^2 - 4D + 4}$$

$(2)^2 - 4(2) + 4$

$\Rightarrow 0$.

$$y_p = \frac{2}{2D - 4} e^{2t}$$

If we put "2"

$$2D - 4 \Rightarrow 2(2) - 4 = 0$$

we take again derivation.

$$y_p = \frac{\cancel{2}}{\cancel{2}} e^{2t}$$

General solution.

$$y_p = (C_1 + C_2 x)^{2t} + e^{2t}$$

Ans.