Calculus And Analytical GEOMETRY

Mid Term Assignment

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Q1 (a) Differentiate $\dfrac{2x^3-3x^2+5}{x^2+1}$ with respect to x.

(a) Sol:. $\dfrac{d}{dx}\left(\dfrac{2x^3-3x^2+5}{x^2+1}\right)$

$$\Rightarrow \frac{(x^2+1)\frac{d}{dx}(2x^3-3x^2+5)-(2x^3-3x^2+5)\frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$\Rightarrow \frac{(x^2+1)(6x^2-6x)-(2x^3-3x^2+5)(2x)}{(x^2+1)^2}$$

$$\Rightarrow \frac{(x^2+1)\,6x(x^2-1)-(2x^3-3x^2+5)(2x)}{(x^2+1)^2}$$

$$\Rightarrow \frac{6x(x^2+1)(x^2-1)-(2x^3-3x^2+5)(2x)}{(x^2+1)^2}$$

$$\Rightarrow \frac{2x\left[3x(x^2+1)(x^2-1)-(2x^3-3x^2+5)\right]}{x^4+2x^2+1}$$

Q1 (b) Differentiate $\frac{(x^2+1)^2}{x^2-1}$ with respect to x.

(b) Sol:- $y = \frac{(x^2+1)^2}{x^2-1}$

$$\Rightarrow \frac{dy}{dx} = \frac{(x^2+1)^2}{x^2-1}$$

$$\Rightarrow \frac{(x^2-1)\frac{d}{dx}(x^2+1)^2 - (x^2+1)^2\frac{d}{dx}(x^2+1)}{(x^2-1)^2}$$

$$\Rightarrow \frac{(x^2-1)\,2(x^2+1)\frac{d}{dx}(x^2+1)\,(x^2+1)^2\,(2x)}{(x^2-1)^2}$$

$$\Rightarrow \frac{(x^2-1)\,[2(x^2+1)\,2x] - (x^2+1)^2(2x)}{(x^2-1)^2}$$

$$\Rightarrow \frac{(x^2-1)\,[4x(x^2+1) - (x^2+1)^2(2x)}{(x^2-1)^2}$$

$$\Rightarrow \frac{x^2+1\,[(x^2-1)4x) - (x^2+1)\,2x]}{(x^2-1)^2}$$

$$\Rightarrow \frac{(2x)(x^2+1)\,[2x(x^2-1) - x^2+1]}{x^4+2x^2+1}$$

Q2 (a) Find $\frac{dy}{dx}$ if $y = (1+2\sqrt{x})^3 \cdot x^{2/3}$ using chain rule:.

(a) Sol:. $\frac{dy}{dx} = (1+2\sqrt{x})^3 \cdot x^{2/3}$ let $x = u$

$$\Rightarrow \frac{dy}{dx} = (1+2\sqrt{u})^3 \frac{2}{3} u^{-1/3} + u^{2/3} \left[3(1+2\sqrt{u})^2 \cdot \frac{1}{u}\right]$$

$\Rightarrow \frac{dx}{du} = 1$ $\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\Rightarrow (1+2\sqrt{u})^3 \tfrac{2}{3} u^{-1/3} + u^{2/3} \left[\tfrac{3}{2}(1+2\sqrt{u})^2\right] \times 1$$

$$\Rightarrow \frac{dy}{dx} = (1+2\sqrt{u})^2 \left[(1+2\sqrt{u}) \tfrac{2}{3} u^{-1/3} + 3 \frac{u}{u}^{2/3}\right]$$

$$\Rightarrow (1+2\sqrt{u})^2 \left[(1+2\sqrt{u}) \frac{2}{6\sqrt{u}} + 3u^{-1/3}\right]$$

$$\Rightarrow (1+2\sqrt{u})^2 \left[(1+2\sqrt{u}) \frac{2}{6\sqrt{u}} + \frac{3}{3\sqrt{u}}\right]$$

use $u = x$

$$\Rightarrow (1+2\sqrt{x})^2 \left[(1+2\sqrt{x}) \frac{2}{6\sqrt{x}} + \frac{3}{3\sqrt{x}}\right]$$

Q2(b) Find $\frac{dy}{dx}$ if $y = \frac{\sqrt{1-x}}{1+x}$ using chain rule:

(b) Sol:- Let $\frac{1-x}{1+x} = u$

$$\Rightarrow \frac{du}{dx} = \frac{(1+x)(-1) - (1-x)(1)}{(1+x)^2}$$

$$\Rightarrow \frac{-(1+x) - (1-x)}{(1+x)^2}$$

$$\Rightarrow \frac{-1 - \cancel{x} - 1 + \cancel{x}}{(1+x)^2} \Rightarrow \frac{-2}{(1+x)^2}$$

$$\frac{dy}{du} = \sqrt{u} \Rightarrow \frac{1}{2}u^{-1/2} \Rightarrow \frac{1}{2\sqrt{u}}$$ using chain rule

$$\Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$\Rightarrow \frac{1}{2\sqrt{u}} \times \frac{-(1+x)^2}{2}$$

$$\Rightarrow \frac{-(1+x)^2}{2\sqrt{u}} \Rightarrow \frac{-(1+x)^2}{4\sqrt{\frac{1-x}{1+x}}}$$

$$\Rightarrow \frac{-(1+x)^2(1+x)^{1/2}}{4\sqrt{1-x}} \qquad \boxed{\Rightarrow \frac{-(1+x)^{3/2}}{4\sqrt{1-x}}}$$

Q3: (a) Find the integration of $\int \frac{1}{\sqrt{x^3}} dx$.

(a) Sol:. $\int \frac{1}{\sqrt{x^3}} dx$

$\Rightarrow \int \frac{1}{(x^3)^{1/2}} dx$

$\Rightarrow \int x^{-3/2} dx$

use formula $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$= \frac{x^{-3/2+1}}{-3/2+1} + c$ $\left(\frac{-3+2}{2} \right)$

$= \frac{x^{-1/2}}{-1/2} + c$

$\int \frac{1}{\sqrt{x^3}} dx = \frac{-2}{\sqrt{x}} + c$

CQ3(b) Find the Integration of $\int \frac{1}{(6x+7)^6} dx$

(b) Sol:. $\int \frac{1}{(6x+7)^6} dx$

$= \int (6x+7)^{-6} dx$

use again $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

$= \frac{1}{6} \int (6x+7)^{-6} (6) dx$ $\qquad (\frac{1}{\cancel{6}} \times \cancel{6}) = 1$

$= \frac{1}{6} \frac{(6x+7)^{-6+1}}{-6+1} + c$

$= \int \frac{1}{(6x+7)^6} dx = -\frac{1}{30} (6x+7)^{5} + c$

Submit by: Muhammad Farooque

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