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Sub :- Differential Equations

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Q1: (i) $W = \sin(x+ct) + \cos(2x+2ct)$

(ii) $W = \tan(2x+ct)$

(i) $W = \sin(x+ct) + \cos(2x+2ct)$

Given $\frac{\partial^2 W}{\partial t^2} = c^2 \frac{\partial^2 W}{\partial x^2} \rightarrow \textcircled{1}$

Now $\frac{\partial W}{\partial t} = \frac{\partial}{\partial t} [\sin(x+ct) + \cos(2x+2ct)]$

$$= \frac{\partial}{\partial t} (\sin(x+ct)) + \frac{\partial}{\partial t} (\cos(2x+2ct))$$

$$\frac{\partial W}{\partial t} = c \cos(x+ct) - 2c \sin(2x+2ct)$$

Now $\frac{\partial^2 W}{\partial t^2} = \frac{\partial}{\partial t} [c \cos(x+ct) - 2c \sin(2x+2ct)]$

$$\frac{\partial^2 W}{\partial t^2} = -c^2 \sin(x+ct) - 4c^2 \cos(2x+2ct)$$

Now $\frac{\partial W}{\partial x} = \frac{\partial}{\partial x} [\sin(x+ct) + \cos(2x+2ct)]$

$$\frac{\partial W}{\partial x} = \cos(x+ct) - 2 \sin(2x+2ct)$$

$$\frac{\partial^2 W}{\partial x^2} = \frac{\partial}{\partial x} [\cos(x+ct) - 2 \sin(2x+2ct)]$$

$$\frac{\partial^2 W}{\partial x^2} = -\sin(x+ct) - 4 \cos(2x+2ct)$$

(i) \Rightarrow

$$-c^2 \sin(x+ct) - 4c^2 \cos(2x+2ct) = c^2 [-\sin(x+ct) - 4\cos(2x+2ct)]$$

$$-c^2 \sin(x+ct) - 4c^2 \cos(2x+2ct) = -c^2 \sin(x+ct) - 4c^2 \cos(2x+2ct)$$

$$0=0 \text{ (Satisfied).}$$
(ii) $W = \tan(2x+ct)$

$$\text{Now } \frac{\partial W}{\partial t} = c \sec^2(2x+ct)$$

$$\text{P } \frac{\partial^2 W}{\partial t^2} = \frac{\partial}{\partial t} (c \sec^2(2x+ct))$$

$$= c \cdot 2 \sec(2x+ct) \tan(2x+ct)$$

$$\text{Now } \frac{\partial W}{\partial x} = 2 \sec^2(2x+ct)$$

$$\frac{\partial^2 W}{\partial x^2} = 4 \sec^2(2x+ct) \tan(2x+ct)$$

$$0 = 4c^2 \sec^2(2x+ct) \tan(2x+ct) = 4c^2 \sec^2(2x+ct) \tan(2x+ct)$$

$$0=0 \text{ (Satisfied).}$$

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Q.No2:- Expand the following function in a Fourier Series.

$$F(x) = x, -\pi < x \leq 0 \\ = 2x, 0 \leq x < \pi.$$

Given function is.

$$F(x) = \begin{cases} x; & -\pi < x \leq 0. \\ 2x; & 0 \leq x < \pi. \end{cases}$$

We have to find the Fourier Co-efficients.
 a_0, a_n fn.

$$\text{Now } a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) dx = \frac{1}{\pi} \int_{-\pi}^0 x dx + \frac{1}{\pi} \int_0^{\pi} 2x dx$$

$$= \frac{1}{\pi} \left[\frac{x^2}{2} \right]_{-\pi}^0 + \frac{2}{\pi} \left[\frac{x^2}{2} \right]_0^{\pi}$$

$$= \frac{1}{\pi} \left[0 - \frac{\pi^2}{2} \right] + \frac{2}{\pi} \left[\frac{\pi^2}{2} - 0 \right]$$

$$a_0 = \frac{-\pi + \pi}{2} = \frac{\pi}{2} \rightarrow \textcircled{1}$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} F(x) \cos nx dx.$$

$$= \frac{1}{\pi} \int_{-\pi}^0 x \cos nx dx + \frac{1}{\pi} \int_0^{\pi} 2x \cos nx dx.$$

$$= \frac{1}{\bar{\lambda}} \left[x \left(\frac{\sin nx}{n} \right) - \left(\frac{-\cos nx}{n^2} \right) \right]_0^{\bar{\lambda}} + \frac{2}{\bar{\lambda}} \left[x \left(\frac{\sin nx}{n} \right) - \left(\frac{-\cos nx}{n^2} \right) \right]_0^{\bar{\lambda}}$$

$$a_n = \frac{1}{\bar{\lambda}} \left[\frac{\cos(0)}{n^2} - \frac{\cos n\bar{\lambda}}{n^2} \right] + \frac{2}{\bar{\lambda}} \left[\frac{\cos n\bar{\lambda}}{n^2} - \frac{\cos(0)}{n^2} \right]$$

$$= \frac{1}{\bar{\lambda}} \left[\frac{1 - (-1)^n + 2(-1)^n - 2}{n^2} \right]$$

So =

$$a_n = \begin{cases} \frac{-2}{\bar{\lambda} n^2} & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}$$

$$b_n = \frac{1}{\bar{\lambda}} \int_{-\bar{\lambda}}^{\bar{\lambda}} f(x) \sin nx \, dx = \frac{1}{\bar{\lambda}} \int_{-\bar{\lambda}}^0 x \sin nx \, dx + \frac{2}{\bar{\lambda}} \int_0^{\bar{\lambda}} x \sin nx \, dx$$

$$= \frac{1}{\bar{\lambda}} \left[x \left(\frac{-\cos nx}{n} \right) - \left(\frac{-\sin nx}{n^2} \right) \right]_0^0 + \frac{2}{\bar{\lambda}} \left[x \left(\frac{-\cos nx}{n} \right) - \left(\frac{-\sin nx}{n^2} \right) \right]_0^{\bar{\lambda}}$$

$$\textcircled{3}. b_n = \frac{1}{\bar{\lambda}} \left[\frac{-\bar{\lambda} \cos n\bar{\lambda}}{n} \right] + \frac{2}{\bar{\lambda}} \left[\frac{-\bar{\lambda} \cos n\bar{\lambda}}{n} \right] = \frac{-3 \cos n\bar{\lambda}}{n} = \frac{-3(-1)^{n+1}}{n}$$

So the required Fourier Series is.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

$$= \frac{\pi}{4} - \frac{2}{x} \sum_{n=1}^{\infty} \frac{\cos(2x-1)^{n+1}}{(2x-1)^2} + 3 \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \sin nx}{n}$$

Q3:- Solve the Initial Value Problem.

$$y'' - 4y' + 13y = 8 \sin 3x, \quad y(0) = 1 \text{ and } y'(0) = 2.$$

Given, $y'' - 4y' + 13y = 8 \sin 3x$.

We have to find $y = y_c + y_p$.

For y_c the characteristic (auxiliary eqn) is:

$$m^2 - 4m + 13 = 0.$$

$$m = \frac{4 \pm \sqrt{16 - 52}}{2} \Rightarrow m = \frac{4 \pm 6i}{2}$$

$$m = 2 \pm 3i \quad \alpha = 2 \text{ and } \beta = 3.$$

$$\text{So } y_c = e^{2x} \{ C_1 \cos 3x + C_2 \sin 3x \}$$

For y_p let

$$y_p = \frac{1}{m^2 - 4m + 13} 8e^{3ix}$$

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$$= 8 \int mg \frac{e^{3sx}}{(3s)^2 - 4(3s) + 13}.$$

$$= 8 \int mg \frac{e^{3ix}}{-9 - 12s + 13}.$$

$$= 8 \int mg \frac{e^{3ix}}{4 - 12s}.$$

$$y_p = 2 \int mg \frac{e^{3ix} \times (1 + 3s)}{(1 - 3i)(1 + 3s)}.$$

$$y_p = 2 \int mg \frac{(1 + 3s)(e^{3sx})}{(1)^2 - (3s)^2}$$

$$y_p = \frac{2}{10} \left(\int mg (1 + 3s)(\cos 3x + s \sin 3x) \right)$$

$$y_p = \frac{2}{10} [\sin 3x + 3 \cos 3x]$$

So the General Solution is.

$$y = y_c + y_p.$$

$$y = C_1 e^{2x} \cos 3x + C_2 e^{2x} \sin 3x + \frac{2}{10} (\sin 3x + 3 \cos 3x)$$

Now use the Initial condition. $y(0) = 1$.

$$y(0) = C_1 e^{10} \cos(0) + C_2 e^{10} \sin(0) + \frac{2}{10} (\sin(0) + 3 \cos(0)).$$

$$1 = C_1(1) + 0 + 0 + \frac{2}{10} (3(1)).$$

$$1 = C_1 + \frac{6}{10} = C_1 = 1 - \frac{6}{10} = \frac{4}{10} = \frac{2}{5}$$

again use the another initial condition.

$$y'(0) = 2.$$

$$\begin{aligned} \text{So, } y' &= C_1 2e^{2x} (\cos 3x + C_1 e^{2x} (-3\sin 3x) \\ &+ C_2 2e^{2x} \sin 3x + (2e^{2x} (3\cos 3x) \\ &+ \frac{2}{10} (\cos 3x - 3\sin 3x)) \end{aligned}$$

$$\begin{aligned} y'(0) &= C_1 2e^{(0)} \cos(0) + C_1 e^{(0)} (-3\sin(0)) \\ &+ C_2 2e^{(0)} \sin(0) + C_2 e^{(0)} (3\cos(0)) \\ &+ \frac{2}{10} (\cos(0) - 3\sin(0)) \end{aligned}$$

$$2 = 2C_1 + 0 + 0 + C_2 \cdot 3(1) + \frac{2}{10} (1 - 3(0))$$

$$2 = 2C_1 + 3C_2 + \frac{2}{10}$$

$$2 = 2(\frac{2}{5}) + 3(2 + \frac{2}{10})$$

$$\text{Use } C_1 = \frac{2}{5}$$

$$\frac{1}{3} (2 - \frac{4}{5} - \frac{2}{10}) = C_2 \Rightarrow C_2 = \frac{1}{3} (\frac{22}{10} - \frac{2}{10}) = \frac{1}{3}$$

So The General Solution is

$$y = \frac{2}{5} e^{2x} \cos 3x + \frac{1}{3} e^{2x} \sin 3x + \frac{2}{10} [\sin 3x + 3 \cos 3x]$$

b The required Solution.

Q No 4. $(D^2 - DD')z = \cos x \cos 2y$.

$(D^2 - DD')z = \cos x \cos 2y$.
The auxiliary equation is.

$$m^2 - m = 0 \Rightarrow m = 0, m = 1.$$

Hence the complementary function is given by.

$$Z_c = f_1(y) + f_2(y+x).$$

For the Particular Integral, we have.

$$Z_p = \frac{1}{D^2 - DD'} \cos x \cos 2y.$$

$$= \frac{1}{2} = \frac{1}{D^2 - DD'} [\cos(x-2y) + \cos(x+2y)].$$

$$= \frac{1}{2} \left[\frac{1}{D^2 - DD'} \cos(x+2y) + \frac{1}{D^2 - DD'} \cos(x-2y) \right]$$

$$= \frac{1}{2} \cos(x+2y) - \frac{1}{6} \cos(x-2y).$$

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Hence the complete solution is given by.

$$Z = f_1(y) + f_2(y+x) + \frac{1}{2} \cos(x+2y) + \frac{1}{6} \cos(x-2y).$$

Ans.