Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: ______ **Total Marks:** 50

Student Details

Name: ______ **Student ID:** ______

Q1.	(a)	Determine the response y(n), $n \geq 0$, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks** 7 **CLO** 2
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks** 7 **CLO** 2
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks** 6 **CLO** 2
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > \lvert a\rvert$$	**Marks** 6 **CLO** 2
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and $\left\lvert H(\frac{\pi}{4})\right\rvert^2 = \frac{1}{2}$.	**Marks** 6 **CLO** 3

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	**Marks** **6** **CLO** **3**
Q 4	(a)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$ Determine the N- point DFT of this sequence for $N \geq L$	**Marks** **6** **CLO** **2**
	(b)	Perform the circular convolution of the following two sequences. Solve the problem step by step $$x_1(n) = \{\underset{\uparrow}{2}, 1, 2, 1\}$$ $$x_2(n) = \{\underset{\uparrow}{1}, 2, 3, 4\}$$	**Marks** **6** **CLO** **2**

Name : Fayaz Ullah
ID : 13854
subject : DSP
Final Paper

Question 1

(a):

Consider the difference eq

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$

The homogenous eq is

$$y(n) - 4y(n-1) + 4y(n-2) = 0$$

The characteristic eq is,

$$\lambda - 4\lambda^{-1} + 4\lambda^{-2} = 0$$

$$\lambda^2 - 4\lambda + 4 = 0$$

Determine the root of the equation

$\lambda^{-2} - 2\lambda - 2\lambda + 4 = 0$

$\lambda(\lambda - 2) - 2(\lambda - 2) = 0$

$(\lambda - 2)(\lambda - 2)$

$\lambda = 2, 2$

Hence,

$yh(n) = C_1 2^n + C_2 n 2^n$

The particular solution is,

$yp(n) = k\{-0^n u(n)]$

By substituting this solution into difference eq, we obtain

$k(-1)^n u(n) - 4k(-1)^{n-1} u(n-1) + 4(k(-1)^{n-2} u$

$(-1) u(n) - (-1)^{n-1} u(u-1)$

For $n = 2$

$K(1+4+4) = 2$

$K(9) = 2$

$K = \frac{2}{9}$

So,

$$y(n) = \left[c_1 2^n + C_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

From ~~the~~ initial condition we obtain $y(0) = 1$, $y(1) = 2$

~~then~~,

$$c_1 + \frac{2}{9} = 1$$

$$\boxed{c_1 = \frac{7}{9}}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$\boxed{c_2 = \frac{1}{3}}$$

Question No 1

(b):

Consider the difference equation

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$

To obtain the homogenous eq

$x(n) = 0$

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 0$$

Determine the solution to the homogenous $y_h(n) = \lambda^n$

Substitute the solution obtained in the homogenous equation

$$\lambda^n - 0.7\lambda^{n-1} + 0.1\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 0.7\lambda + 0.1) = 0$$

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

The roots are

$$(\lambda - 0.5)(\lambda - 0.2) = 0$$

$$\lambda = \frac{1}{2}, \frac{1}{5}$$

General form of solution ⑤

$$y_h(n) = C_1(\lambda_1)^n + C_2(\lambda_2)^n$$

$$y(n) = C_1(0.2)^n + C_2(0.5)^n \quad \text{—— ①}$$

$\lambda = \frac{1}{2}$, $\lambda = \frac{1}{5}$ then

$$y_h(n) = C_1 \frac{1}{2}^n + C_2 \frac{1}{5}^n$$

with $x(n) = \delta(n)$, we have

$$y(0) = 2$$

$$y(1) - 0.7y(0) = 0$$

$$y(1) = 1.4$$

Here,

$$C_1 + C_2 = 2$$

$$\frac{1}{2}C_1 + \frac{1}{5} = 1.4 = 7/5$$

$$C_1 + \frac{2}{5}C_2 = \frac{14}{5}$$

These equation yield

$$C_1 = \frac{10}{3}, \quad C_2 = -4/3$$

$$h(n) = \left[\frac{10}{3}\left(\frac{1}{2}\right)^n - \frac{4}{3}\left(\frac{1}{5}\right)^n\right] u[n]$$

Now step responce is

$$s(n) = \sum_{k=10}^{n} h(n-k)$$

$$= \frac{10}{3}\sum_{k=0}^{n}\left(\frac{1}{2}\right)^{n-k} - \frac{4}{3}\sum_{k=0}^{n}\left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3}\left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2^k - \frac{4}{3}\left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5^k$$

$$\boxed{s(n) = \frac{10}{3}\left(\frac{1}{2}\right)^n (2^{n+1}-1)u(n) - \frac{1}{3}\left(\frac{1}{3}\right)^n (5^{n+1}-1)u[n]}$$

Question 2

(a): Causal signal

$$u(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Hint: Take inverse z-transform

Sol:

z-transform

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

the expression is written as.

$$x(z) = \frac{1}{(1-\frac{2}{z})(1-\frac{1}{z})^2}$$

$$= \frac{1}{\left(\frac{z-2}{z}\right)\left(\frac{z-1}{z}\right)^2}$$

$$= \frac{1}{\frac{(z-2)(z-1)^2}{z^3}}$$

$$= \frac{z^3}{(z-2)(z-1)^2} \quad \text{———} \quad (1)$$

$x(z)$ has a simple pole at $p_1 = 2$ & double $p_2 = p_3 = 1$.

The partial fraction is,

$$X(z) = \frac{z^3}{(z-2)(z-1)^2} = \frac{A_1}{z-2} + \frac{A_2}{z-1} + \frac{A_3}{(z-1)^2}$$

to find A_1, A_2, & A_3, we will proceed as in the case of distinct pole to find A_1. we will multiply both side by $(z-2)$ and result $z=-2$

$$(z-2)X(z) = A_1 + \frac{z-2}{z-1}A_2 + \frac{z-2}{(z-1)^2}A_3$$

$$A_1 = \left.\frac{(z-2)x(z)}{z}\right|_{z=2}$$

$$\boxed{A_1 = 4}$$

$$A_2 = A_1 + \frac{z-2}{z-1}$$

$$\boxed{A_2 = -3}$$

$$A_3 = A_1 + \frac{z-2}{z-1}A_2$$

$$\boxed{A_3 = -1}$$

Hence,

$$\boxed{x(n) = [4(2)^n - 3 - n]\,u(n)}$$

Question No 2

(b): we have,

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}} dz$$

$$= \frac{1}{2\pi j} \oint_c \frac{z^n dz}{z-a}$$

where C is a circle at radius greater than |a|. We shall evaluate this integral with $f(z) = z^n$

we dinguish two cases

1): If $n \geq 0$, $f(z)$ has only zeros and hence no poles inside C. The only pole inside C is $z=a$, Hence

$$x(n) = f(z_0) = a^n \quad n \geq 0$$

2): If $n<0$, $f(z) = z^n$ has an nth-order pole at $z=0$, which is also inside C. Thus there are contributions from both poles.

For $n=-1$, we have

$$x(-1) = \frac{1}{2\pi j}\oint_c \frac{1}{z(z-a)}\,dz$$

$$= \frac{1}{z-a}\bigg|_{z=0} + \frac{1}{z}\bigg|_{z=a} = 0$$

If $n=-2$, we have

$$x(-2) = \frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)}\,dz$$

$$= \frac{d}{dz}\left(\frac{1}{z-a}\right)\bigg|_{z=0} + \frac{1}{z}\bigg|_{z=a} = 0$$

By continuing in the same way we can show that $x(n)=0$ for $n<0$.

Thus $x(n) = a^n u(n)$

Question No 3

(a):

Sol: At $\omega = 0$, we have

$$H(0) = \frac{b^0}{(1-P)^2} = 1$$

hence,

$$b_0 = (1-P)^2$$

At $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4) + jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2} + jP\sqrt{2})^2}$$

Hence,

$$\frac{(1-P)^4}{\left[(1-P/\sqrt{2})^2 - P^2/\sqrt{2}\right]^2} = \frac{1}{2}$$

or equivalently,

$$\sqrt{2}(1-P)^2 = 1 + P^2 - \sqrt{2}P$$

the value of $P = 0.32$ satisfies this eq consequently, the system function desired filter is,

$$\boxed{H(z) = \frac{0.46}{(1-0.32z^{-1})^2}}$$

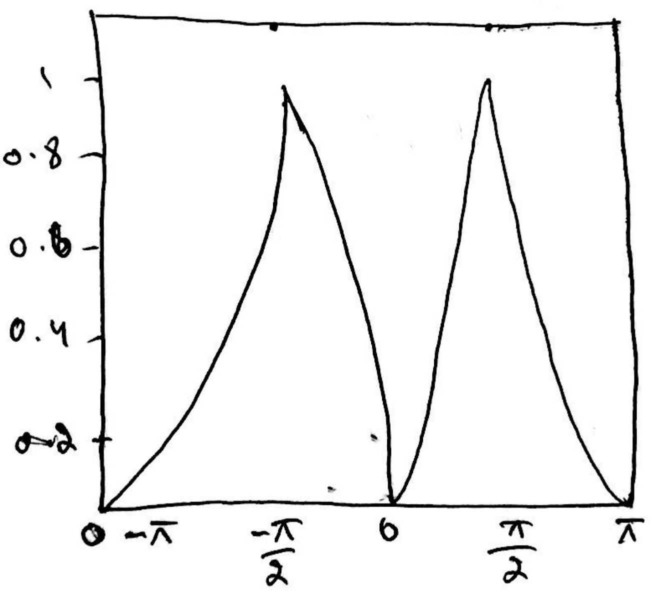
(b):

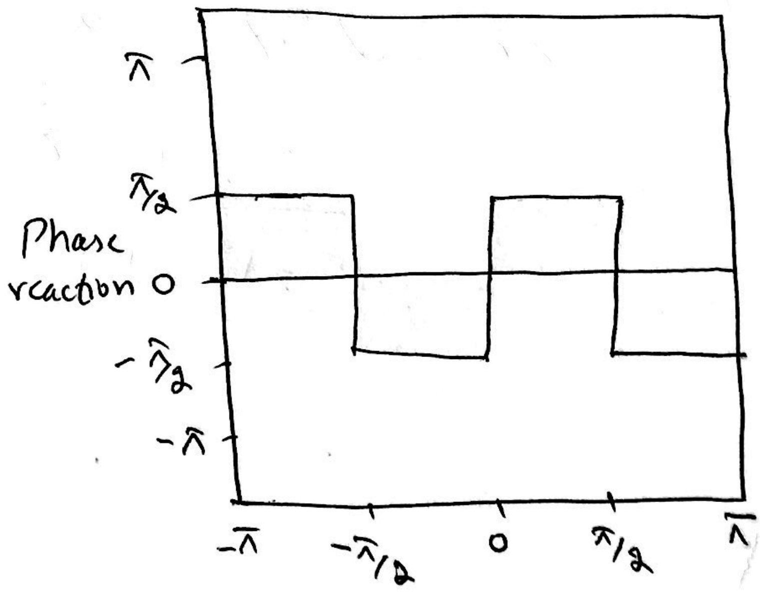
Sol: the filter must have poles at, $P_{1,2} = re^{\pm j\pi/2}$

and zeros at $z=1$, & $z=-1$,

$$H(z) = \frac{G(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\,\frac{z^2-1}{z^2+r^2}$$





The frequency response $H(\omega)$ of the filter at $\omega = \pi/2$,

$$H(\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determine by evaluating $H(\omega)$ at $\omega = 4\pi/9$,

$$H\left(\frac{4\pi}{9}\right)^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(r^2\pi/9)}{1+r^4+2r^2\cos(r\pi/9)} = \frac{1}{2}$$

or equivalently

$$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$$

the value of $r^2 = 0.7$, satisfies the eq, therefore, the system function for the desired filter is,

$$\boxed{H(z) = \frac{0.5 + z^{-2}}{1+0.7z^{-2}}}$$

Question 4

(a):

the fourier transform is,

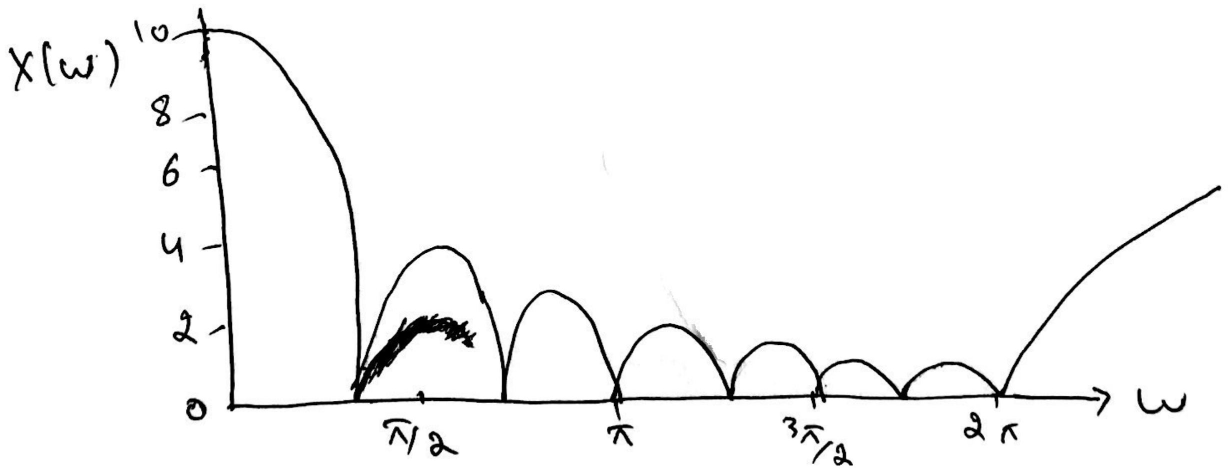
$$X(\omega) = \sum_{n=0}^{i-1} x(n) e^{-j\omega n}$$

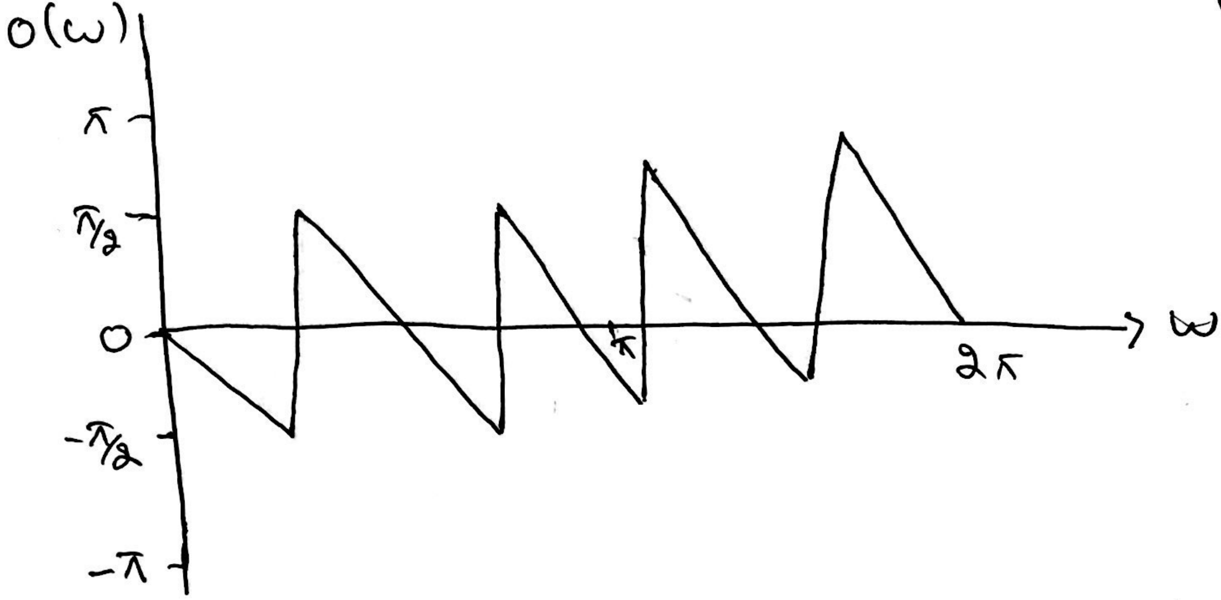
$$= \sum_{n=0}^{i-1} e^{-j\omega n} = \frac{1-e^{-j\omega n}}{1-e^{-j\omega}} = \frac{\sin(\omega/2)}{\sin(\omega/2)} e^{-j\omega(n-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustration for $i=10$. The N-point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally speed frequencies $\omega_k = 2\pi K/N$, $K = 0, 1 \cdots N-1$,

$$X(k) = \frac{1-e^{-j2\pi k_2/N}}{1-e^{-j2\pi k/N}}, \quad K = 0, 1, \cdots, N-1$$

$$= \frac{\sin(\pi k_2/N)}{\sin(\pi k/N)} e^{-j\pi k(L-1)/N}$$





If N is selected such that $N = L$, then the DFT become,

$$X(k) = \begin{cases} L, & k = 0 \\ 0, & k = 1, 2, \ldots\ldots -2, -1 \end{cases}$$

Thus, there is only one non-zero value in DFT. This is apparent from observation of $X(\omega)$ since $X(\omega) = 0$, at the frequencies $\omega_k = 2\pi k/L$, $k \neq 0$. The reader should verify that $x(n)$ can be recovered from $X(k)$ by performing an L-point DFT.

Question No 4

(b): $x_1(n) = \{2, 1, 2, 1\}$

$x_2(n) = \{1, 2, 3, 4\}$

Each sequence consists of four nonzero points. For circular convolution we have to graph each sequence as points on a circle. The sequence ~~is~~ will be graped in counterclock.

we will begin with $m=0$,

$$x_3(0) = \sum_{n=1}^{3} x_1(n)\, x_2((-n))N$$

$x_2((-n))_4$ is the sequence $x_2(n)$.

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$.

$x_3(0) = 14$

For $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

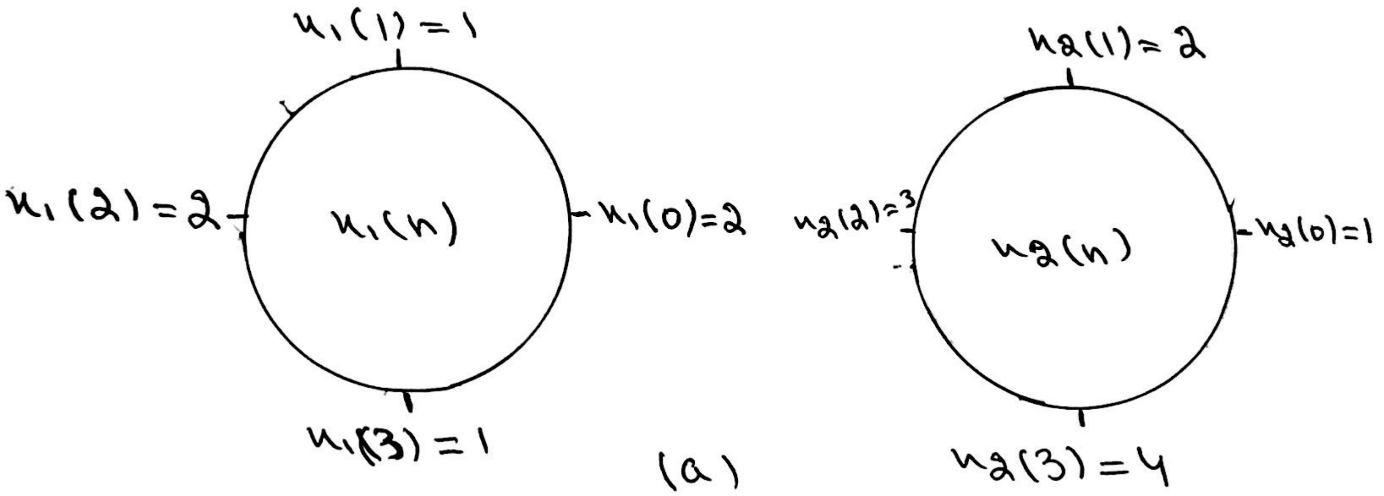
It is easily verified that $x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$ rotated counterclockwise by one unit.

$$x_3(1) = 16$$

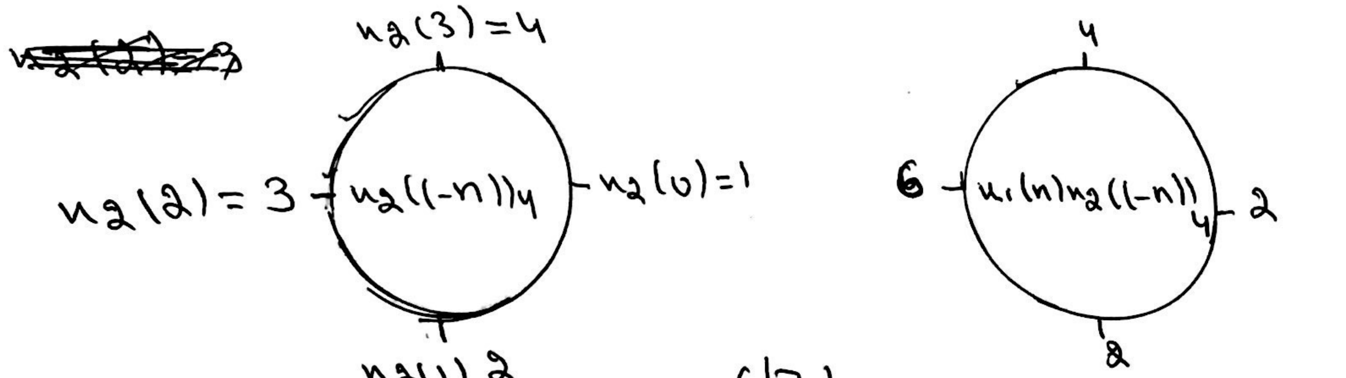
For $m=2$ we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\,x_2((2-n))_4$$

Now, $x_2((2-n))$ is the folded sequence rotated two units of time is counterclockwise direction.



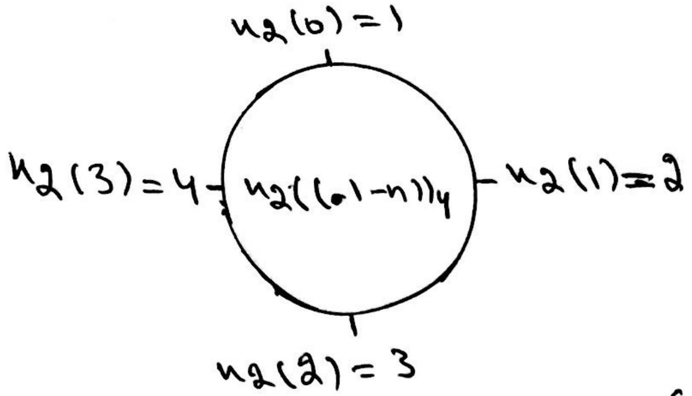
(a)



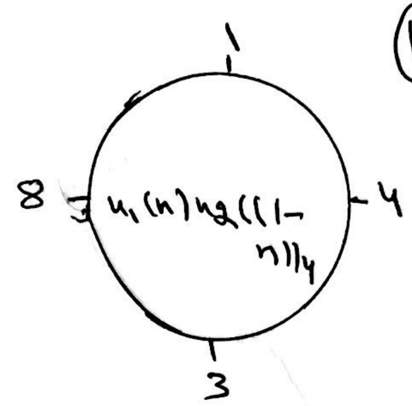
(b)

Folded sequence

Product sequence

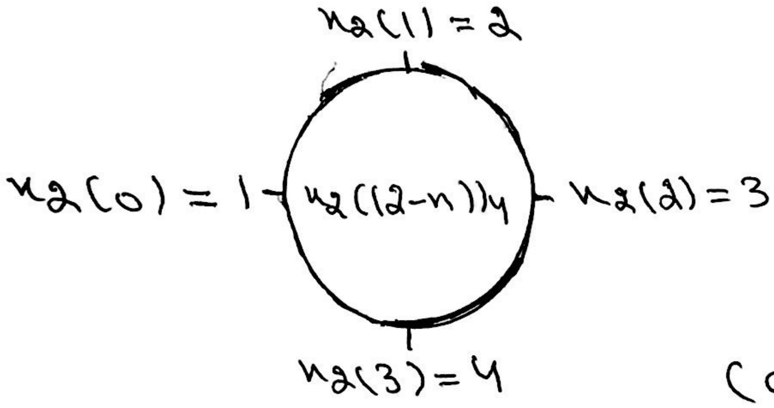


(c)

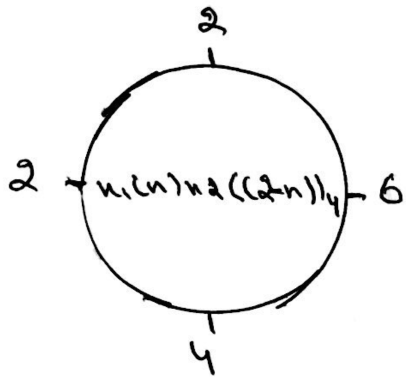


Folded sequence rotated by one unit in time

Product sequence

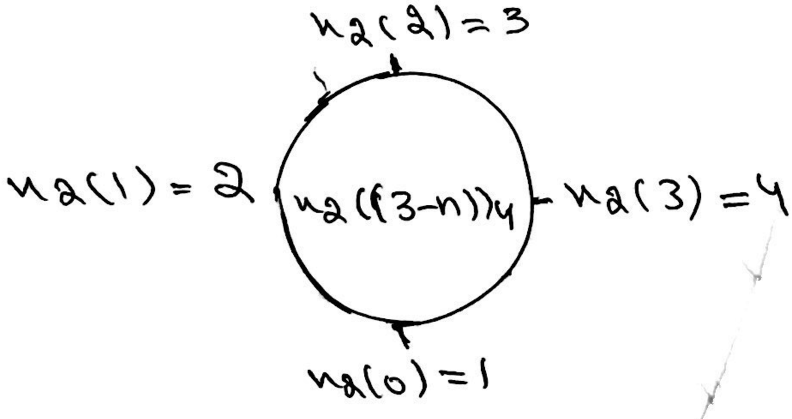


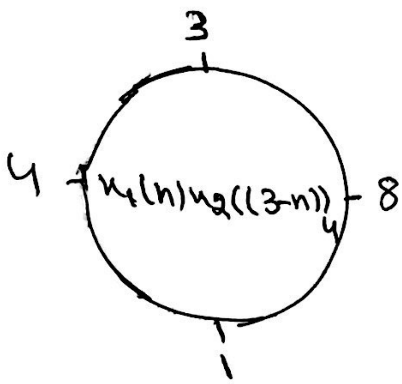
(d)



Folded sequence rotated by two units in time

Product sequence





Folded sequence rotated by three units in time

Product sequence