Final Term Exam Summer Semester 2020

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Subject: Fluid Mechanics II

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Q1 - Define drag with it's components. Write down the equations for friction Drag coefficient for both Laminar and turbulant boundary layer.

Drag: A body which is wholly immersed in a homogenous fluid may be subjected to two kind of forces arising from relative motion between body and fluid. These forces are termed as drag and lift depending on wether force is parallel or at rigid angle to the motion

Components of drag: Drag forces on submerged body can have two components.

1) **Pressure drag F_p** ⇒ It is equal to the integration of components in the direction of motion of all pressure forces exerted on surface of the body.

$$F_p = C_p \rho \frac{V^2}{2} A \qquad C_p \rightarrow \text{depends on shape}$$

2) **Friction drag** ⇒ It is equal to the integration of components of shear stress along the surface of the body in direction of motion.

$$F_f = C_f \underbrace{\rho \frac{V^2}{2}}_{\text{shear stress}} BL \qquad C_f \rightarrow \text{depends on viscosity}$$

→ figure a

Friction drag of boundary layer

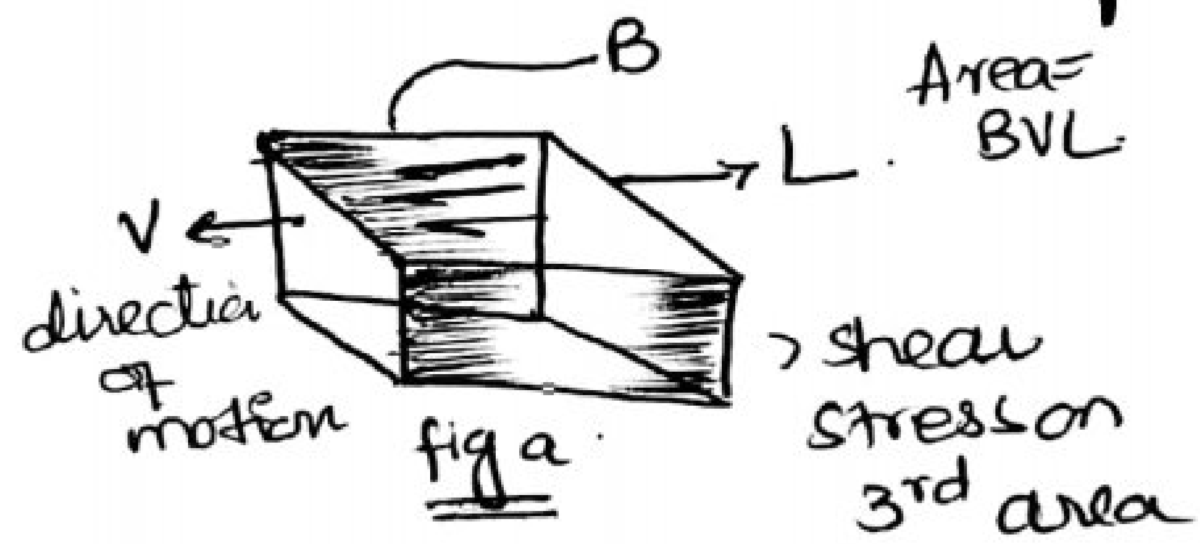


fig a

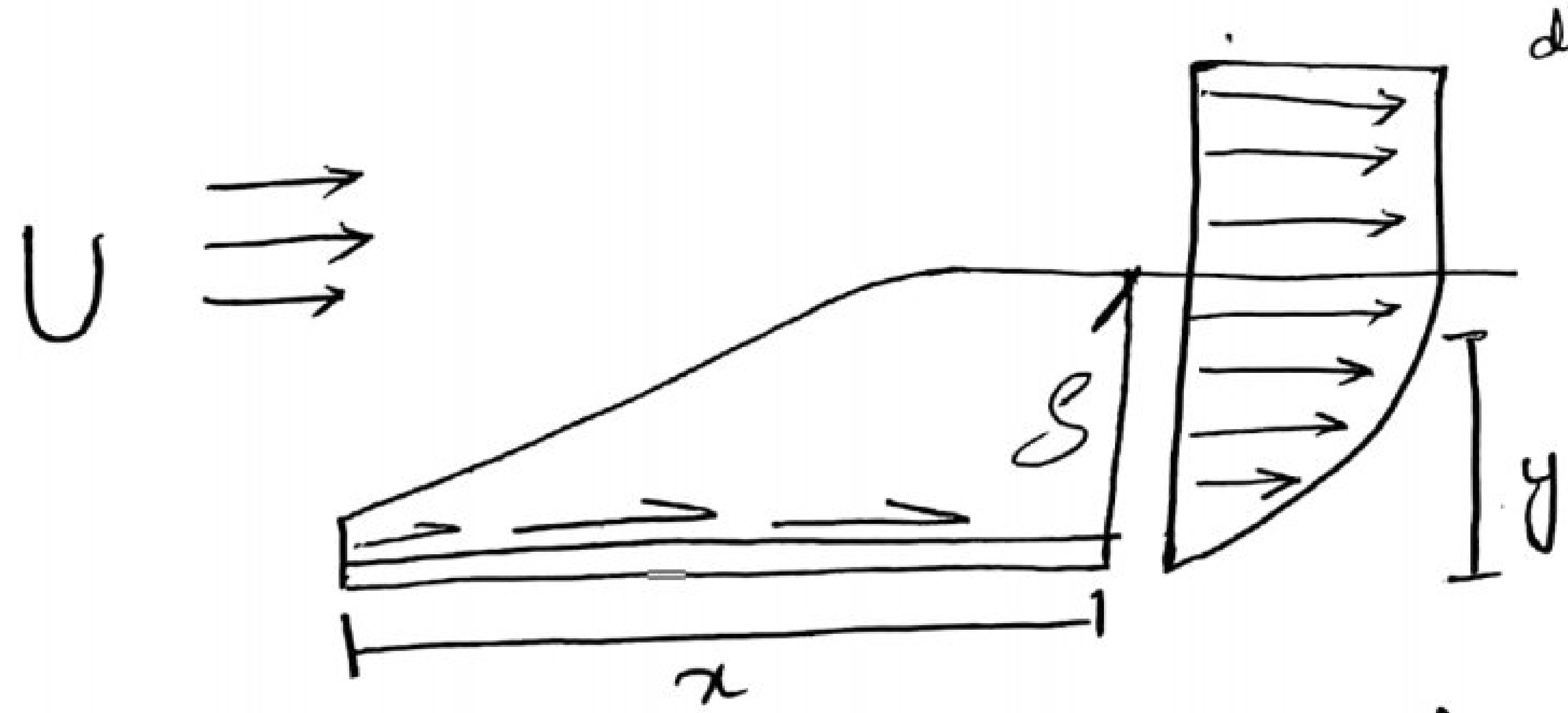
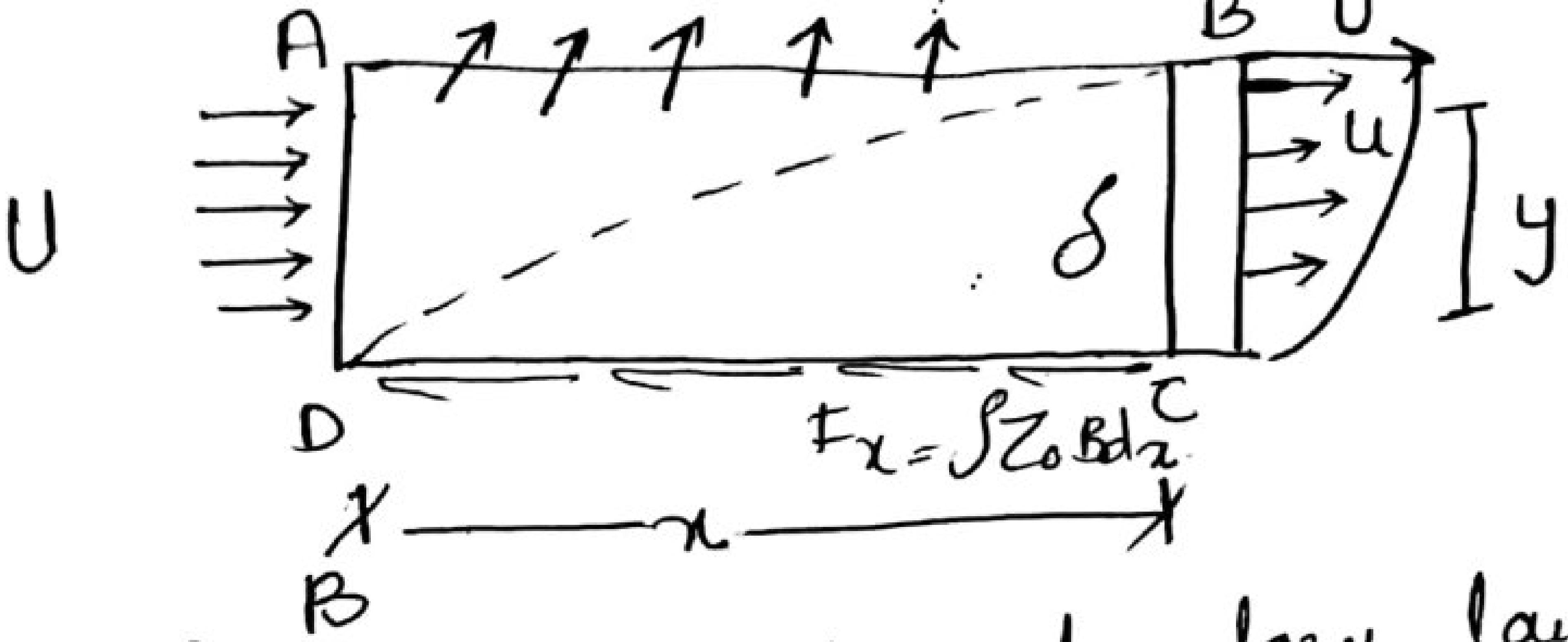


figure shows growth of boundary layer along one side of smooth plate in steady flow of incompressible fluid. Consider a control volume.



where δ is distance from boundary layer to plate. U is undisturbed velocity. Now $-F_x = -$ drag $=$ rate of momentum in x-direction (Leaving through BC + rate of momentum in x direction leaving through AB - rate of momentum in x direction entering through DA)

According to impulse momentum principle :-

$\Delta P = P_{out} - P_{in}$

$\Sigma f = \frac{d(P)}{d(t)} \Rightarrow \Sigma F = \frac{d(mv)}{dt} = \frac{\rho \times vol \times V}{dt} \Rightarrow F = \rho QV$

where $\frac{dm}{dt} = \rho Q$

$$\Sigma F_x = \rho Q_2 V_2 - \rho Q_1 V_1$$

$A \rightarrow \rho U(UB\delta)$

$C \rightarrow \rho B \int_0^\delta u^2 \cdot dy$

$B \rightarrow \rho (UB\delta - B\int_0^\delta u\, dy) U$

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right)$$

$$\therefore \eta = y/\delta$$

$$\frac{u}{U} = f(\eta)$$

putting values

$$F_x = \rho B \int_0^\delta u(U-u)\, dy$$

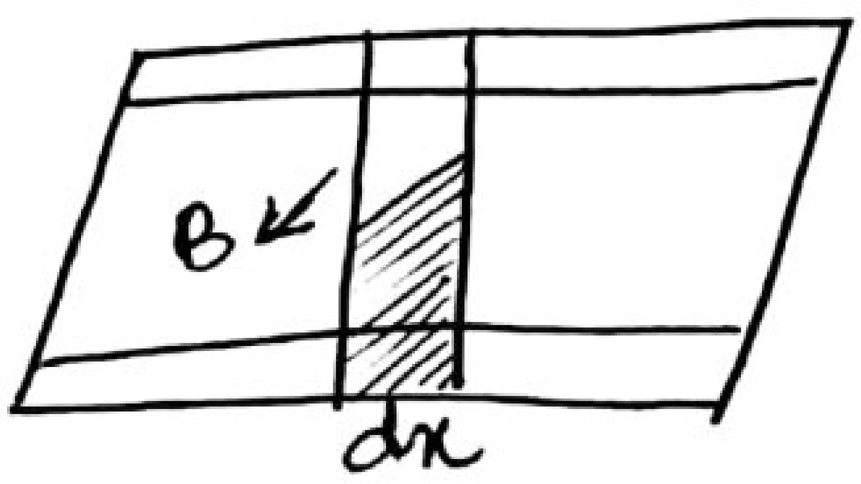
Solving this: $F_x = \rho B U^2 \delta \alpha$ where α is a function of boundary layer velocity distribution only.

Now to find local wall shear stress.

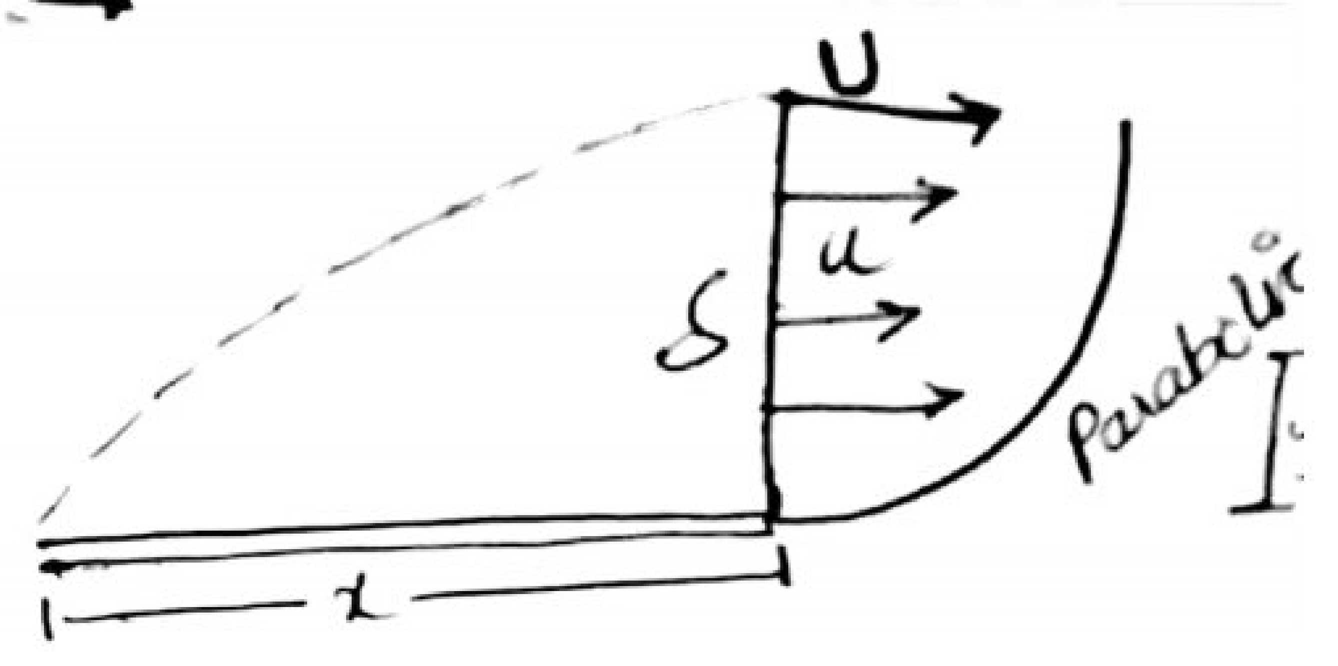
$$\tau = \frac{F_x}{Area} \Rightarrow \tau_0 = \frac{dF_x}{B \cdot dx}$$

$$F_x = \rho B U^2 \delta \alpha$$

$$\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$$ in general equation of shear stress.



Laminar boundary Layer:



$$\frac{u}{U} = F\left(\frac{y}{\delta}\right)$$

Assume $\eta = \frac{y}{\delta}$ or $y = \eta\delta$

Thus, $\frac{u}{U} = f(\alpha)$ or $u = Uf(\eta)$

In case of Laminar flow $\tau_0 = \mu\left(\frac{du}{dy}\right)$

$$= \frac{\mu}{\delta}\left(\frac{du}{d\eta}\right) = \frac{\mu U}{\delta}\left[\frac{df(\eta)}{d\eta}\right].$$

Solving the equation

$$\tau_0 = \frac{\mu U B}{\delta} \rightarrow (1)$$

As general equation is $\tau_0 = \rho u^2 \alpha \frac{d\delta}{dx}$

Equating both equations

$$\frac{\mu U B}{\delta} = \rho U^2 \alpha \frac{d\delta}{dx}$$

or

$$\delta \, d\delta = \frac{\mu B}{\rho U \alpha} dx$$

Integrating the equation

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} + C$$

Now at $x = 0$, $\delta = 0$ Thus $C = 0$

$$\frac{\delta^2}{2} = \frac{\mu B}{\rho U \alpha} x$$

or $\delta = \sqrt{\frac{2\mu B}{\rho U \alpha} x}$ or $\sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}}$

Multiplying and dividing by "x"

$$\delta = \sqrt{\frac{2B}{\alpha}} \cdot \sqrt{\frac{\mu x}{\rho U}} \cdot \frac{x}{\sqrt{x} \cdot \sqrt{x}}$$

Where $\alpha = 0.135$

$B = 1.63$

$R_x = \frac{\rho U x}{\mu}$

$$\delta = \frac{4.91}{\sqrt{R_x}} \cdot x \quad \text{or} \quad \frac{\delta}{x} = \frac{4.91}{\sqrt{R_x}}.$$

Now $\tau_0 = \frac{\mu U \beta}{\delta}$

Thus putting values

$$\boxed{\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}}$$

Where R_x is local reynolds number. It should be noted that R_x increases linearly in downstream direction

Now: $F_f = B \int_0^{L} \tau_0 \, dx \Rightarrow$

$$\tau_0 = 0.332 \frac{\mu U}{x} \sqrt{R_x}$$

$$R_x = \frac{x U \rho}{\mu}$$

Thus $F_f = 0.664 B \sqrt{\rho \mu L U^3}$.

where $F_f = C_f \rho \frac{V^2}{2} BL$

Equaling both

$$C_f = 1.328 \sqrt{\frac{\mu}{\rho L U}} = \frac{1.328}{\sqrt{R}}$$

where R is based on characteristic length of whole plate. The laminar boundary layer will remain laminar if $2x$ is of about 500000.

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Turbulant boundary layer:

Figure b shows that velocity distribution in turbulent boundary layers shows a much steeper gradient near wall and flatter throughout remaining layer.

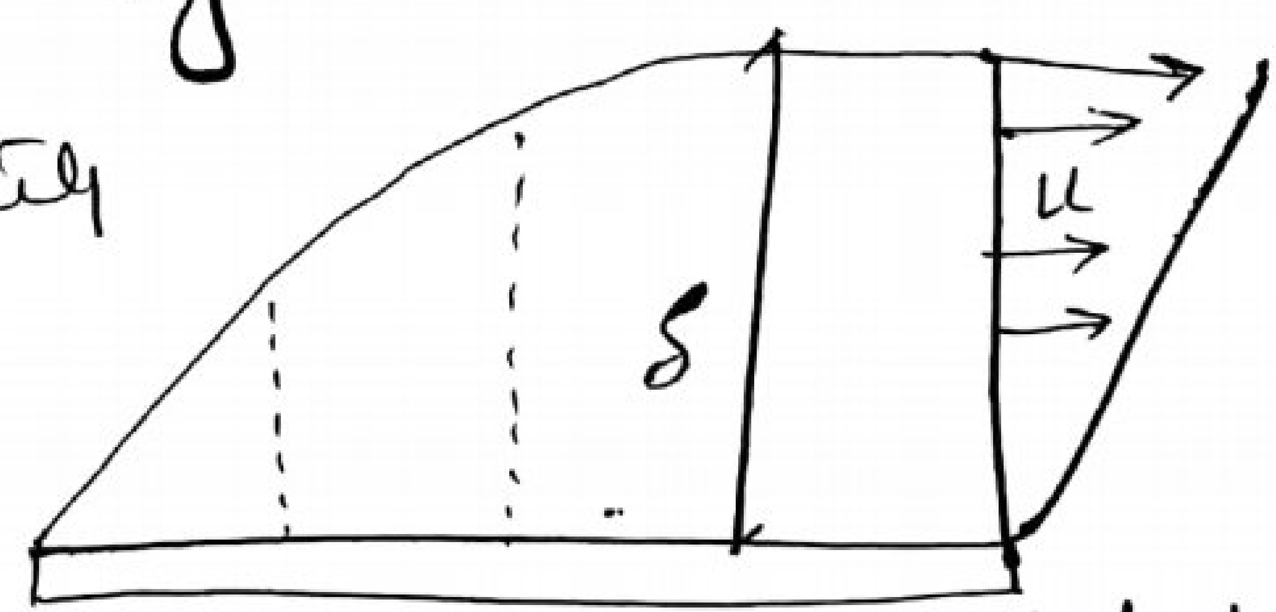

fig b

Resistance is less, so curve become straight.

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The shear stress is greater in turbulent than in laminar layer.

As we have $\tau_0 = f \rho \frac{v^2}{8}$ where v denotes average velocity of pipe

Now, we have obtained an approximate relation between V and U by using pipe factor equation of

$$\frac{V}{U_{max}} = \frac{1}{141.33\sqrt{f}}$$

Using friction factor of 0.028 from chart which is middle critical value

$$\boxed{U = 1.235V}$$

Now we have $\tau_0 = f \rho \frac{v^2}{8}$

As we know that $f = \frac{0.316}{R^{0.25}}$

Thus $\tau_0 = \frac{0.316}{\left(\frac{Dv}{\nu}\right)^{1/4}} \frac{\rho v^2}{8}$

Where $v = \frac{U}{1.235}$ Thus

$$\tau_0 = \frac{0.316}{\left(\frac{D}{\nu}\left(\frac{U}{1.235}\right)\right)^{1/4}} \cdot \frac{\rho}{8}\left(\frac{U}{1.235}\right)^2$$

& $D = 2\delta$

Thus $\tau_0 = \frac{0.023 \rho U^2}{\left(\frac{\delta U}{\nu}\right)^{1/4}}$

As we have $\tau_0 = \rho U^2 \alpha \frac{d\delta}{dx}$

Equating both and integrating for boundary condition of $x = 0$, $\delta = 0$

Thus $\delta = \left(\frac{0.0287}{\alpha}\right)^{4/5} \left(\frac{\nu}{Ux}\right)^{1/5} x$

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For $\alpha = 0.0972$.

$$\boxed{\frac{\delta}{x} = \frac{0.377}{(R\,x)^{1/5}}}$$

Putting values in equation

$$\tau_0 = 0.0587 \rho \frac{U^2}{2} \left(\frac{\nu}{Ux}\right)^{1/5}$$

Now

$$F_f = B \int_0^2 \tau_0 dx$$

$$F_f = 0.0735 \rho \frac{U^2}{2} \left(\frac{\nu}{UL}\right) BL$$

As $F_f = C_f \rho \frac{U^2}{2} BL$

Equating both

$$\boxed{C_f = \frac{0.0735}{R^{1/5}}}$$

R is less than 10^7 for: $5001000 < R < 10^7$

For $R > 10^7$

$$\boxed{C_f = \frac{0.455}{(\log \cdot R)^{2.58}}}$$

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Q1 : Part B :-

As specific energy $E = y + \frac{V^2}{2g}$

The flow Q per unit width b can be expressed as $q = \frac{Q}{b}$.

Now average velocity will be

$$v = \frac{Q}{A} = \frac{qb}{by} = \frac{q}{y}.$$

Thus, The point at which specific energy is at least among all is called **Critical point**

$$E = y + \frac{V^2}{2g} \Rightarrow y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right)$$

$$(E - y) = \frac{1}{2g}\left(\frac{q^2}{y^2}\right) \text{ or } (E-y)\,y^2 = \frac{q^2}{2g}$$

Thus plot of E vs y will be parabolic. For particular q, there will be two kind of possible values of y for a given E.

The equation is cubic with three roots with third root being negative. Point "C" represents dividing point between two regime of flow. Thus for given q, E, value of E is minimum & flow at that point is critical flow. Depth of flow is $\boxed{\text{critical depth } y_c}$ and velocity at that point is "$\boxed{\text{critical velocity } V_c}$"

Thus,

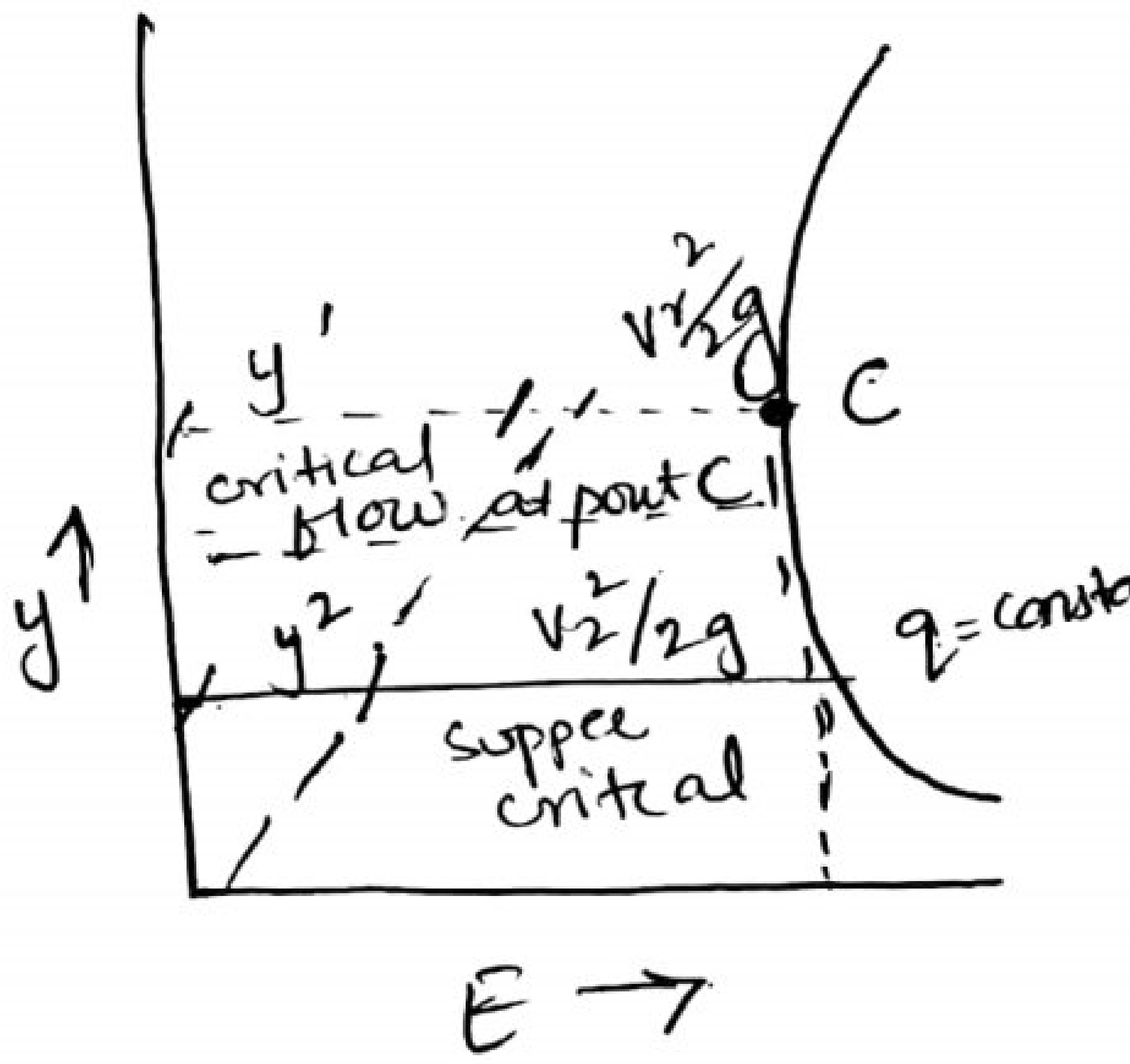
$$E = y + \frac{1}{2g}\left(\frac{q^2}{y^2}\right).$$

For minimum specific energy $\frac{dE}{dy} = 0$

Thus $\frac{dE}{dy} = 1 - \frac{2}{2g}\left(\frac{q^2}{y^3}\right) = 0$

$$\Rightarrow \frac{q^2}{gy^3} = 1 \Rightarrow q^2 = gy^3$$

$$\frac{q^2}{g} = y^3 \Rightarrow \boxed{\left(\frac{q^2}{g}\right)^{1/3} = y_{cr}}$$



Now, $q^2 = gy^3$

& $q = Vy \Rightarrow V^2 y^2 = gy^3$

or

$$V^2 = g y_{cr}$$

or $\boxed{V_{cr} = \sqrt{g y_{cr}}}$

Q2 Find the rectangular channel if water flows...
... Is flow super critical or sub-critical.

Given: water flows at rate, $Q = 3.5 \, m^3/s$.

Bed slope, $S_o = 0.0008$.

$n = 0.0219$.

Width of bed = Student ID = 7720 mm.

= 7.720m

Required:-

Depth of rectangular channel = ?

Critical depth, $y_c = ?$

Critical velocity = $V_c = ?$

flow is sub-critical or super critical = ?

Solution:

Manning Equation.

$$Q = \left(\frac{1}{n} R_n^{2/3} S_o^{1/2} \right) A \rightarrow 1$$

Area = $7.720 \times d$

Parameter = $d + 7.720 + d$

Hydraulic Radius $R_n = \frac{\text{Area}}{\text{Parameter}}$

$$= \frac{7.720 \times d}{2d + 7.720}$$

put values in equation (1)

$$Q = \left(\frac{1}{n} R_n^{2/3} S_o^{1/2} \right) A$$

$$3.5 = \frac{1}{0.0219} \times \left(\frac{7.720}{2d + 7.720} \right)^{2/3} \times (0.0008)^{1/2} \times 7.720d$$

$$\frac{3.5 \times 0.0219}{(0.0008)^{1/2}} = \left(\frac{7.720d}{2d + 7.720} \right)^{2/3} \times 7.720d.$$

$$\frac{3.5 \times 0.0219}{(0.0008)^{1/2}} = \left(\frac{7.720d}{2d+7.720}\right)^{2/3} \times 7.720d.$$

$$\left(\frac{3.5 \times 0.219}{\sqrt{0.0008}}\right)^{3/2} = \frac{7.720d}{2d+7.720} \times 7.720d.$$

$$\Rightarrow (2.710)^{3/2} = \frac{59.5984d^2}{(2d+7.720)}$$

$$\Rightarrow 4.462 \times (2d + 7.720) = 59.5984\, d^2.$$

$$\Rightarrow 8.924\,d + 34.4466 = 59.5984\,d^2$$

$$\Rightarrow 59.5984d^2 - 8.924d - 34.4466 = 0.$$

$$\Rightarrow \boxed{d = 0.8387}$$

So, the depth of the channel is 0.3860 m.

Now, As q = discharge per unit width.

$$q = \frac{Q}{b} = \frac{3.5}{7.720}$$

$$\boxed{q = 0.453}$$

→ Critical depth, y_{cr}.

$$y_{cr} = \left(\frac{q^2}{g}\right)^{1/3}.$$

$$= \left(\frac{(0.453)^2}{9.81}\right)^{1/3} = \left(\frac{0.205}{9.81}\right)^{1/3} = (0.0209)^{1/3}$$

$$= (0.0209)^{0.333}.$$

y_{cr} = <u>0.2758 m</u>.

⇒ Critical Velocity, V_{cr}.

Using equation

$$V_{cr} = \sqrt{g y_{cr}}$$

$$V_{cr} = \sqrt{(9.81)(0.2758)} = \sqrt{2.705598}$$

$$V_{cr} = \underline{\underline{1.6448 \text{ m/s}}}$$

→ $\boxed{d = 0.8387}$ $\boxed{y_{cr} = 0.2758 m}$ $\boxed{V_{cr} = 1.6448 \text{ m/s}}$

As $V = \frac{Q}{A} = \frac{3.5}{7.720 \times 0.8387} = \frac{3.5}{2.979}$

$$\boxed{V = 0.5406 \text{ m/s}}$$

<u>As</u> $y > y_{cr}$

and $V < V_{cr}$

So, the flow is Sub critical flow.

(10)

Q3:- Find the friction drag on one side of smooth plate -- - - - - - - ---- -- and kinematic viscocity is $0.93 \times 10^{-4}\ m^2/s$.

Given Data:- Width of Smooth plate, $B = 200mm$
$= 0.2m$

length of Smooth plate, $L = 800mm$
$= 0.8m$

oil with specific gravity, $S = 0.89$

Undisturbed velocity, $U = 5m/s$

kinematic viscocity, $\nu = 0.93 \times 10^{-4}\ m^2/s$

Required data: Friction drag on one side of a smooth plate

$F_f = ?$

Solution:- Check the flow. As $\nu = 0.93 \times 10^{-4}\ m^2/s$

$$R = \frac{LU}{\nu} = \frac{(0.8)(5)}{0.93 \times 10^{-4}}$$

$R = 43010.75 < 500,000$ Thus flow is Laminar.

Now,
$$C_f = \frac{1.328}{\sqrt{R}} = \frac{1.328}{\sqrt{43010.75}}$$

$$C_f = 6.403 \times 10^{-3}$$

$$C_f = 0.0064$$

$$F_f = C_f \rho \frac{V^2}{2} BL$$

$$= (0.0064)(\rho_{oil} \times \gamma_{water}) \times \frac{(5)^2}{2} \times (0.2)(0.8)$$

$$= \left((0.0064)(0.89 \times 1000) \times \frac{5^2}{2} \times (0.2)(0.8)\right).$$

$$= \boxed{F_f = 11.392\ N}$$