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SECTION: A

CIVIL ENGINEERING DEPARTMENT

APPLIED CALCULUS

FINAL TERM EXAM

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QUESTION : 1

$$P = (4,1,3)$$
$$Q = (1,2,4)$$

Cordinate of $P = (4,1,3)$

$$OP = 4j + 1j + 3k$$

Or $OQ = \vec{OQ} - \vec{OP}$

$$= (i + 2j + 4k) - (4j + 1j + 3k)$$

$$= 3j + 1j + 1k \text{ ——— } ①$$

Distance b/w P and Q $= |PQ|$

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{11} \text{ ——— } ②$$

Let M be the point which divided PQ in ratio 1:3 then by ratio position vector of M $= \vec{OM}$

$$= \frac{3(4j + 1j + 3k) + (1)(i \cancel{+} 2j + k)}{1 + 3}$$

$$= \frac{12j + 3j + 9k + i + 2j + 4k}{4}$$

$$= 13j + 5j + 13k \text{ ——— } ③$$

QUESTION NO 2

$$\int \frac{4x^3 + 10x + 4}{2x^2 + x} \, dx$$

$$\Rightarrow \quad 2\int \frac{2x^3 + 5x + 2}{x(2x+1)} \, dx$$

$$2\int \left(\frac{11x + 4}{2x(2x+1)} + \frac{2x-1}{2} \right) dx$$

$$2\left(\frac{1}{2} \int \frac{11x+4}{x(2x+1)} \, dx + \int x \, dx - \frac{1}{2} \int 1 \, dx \right.$$

$\Rightarrow$ Solving $\displaystyle\int \frac{11x + 4}{x(2x+1)} \, dx$

$$\Rightarrow \quad \int \left(\frac{3}{2x+1} + \frac{4}{x} \right) dx$$

$$\Rightarrow \quad 3\int \frac{1}{2x+1} \, dx + 4\int \frac{1}{x} \, dx$$

Plugging in solved integrals.

$$\frac{1}{2} \int \frac{11x + 4}{x(2x+1)} \, dx + \int x \, dx - \frac{1}{2} \int 1 \, dx$$

$$\Rightarrow \frac{3\ln(2x+1)}{4} + 2\ln(x) + \frac{x^2}{2} - \frac{x}{2}$$

$$2\int \frac{2x^3+5x+2}{x(2x+1)}\,dx$$

$$\Rightarrow \frac{3\ln(2x+1)}{2} + 4\ln(x) + x^2 - x$$

$$\Rightarrow \frac{3\ln(2x+1) + 8\ln x + 2x^2 - 2x}{2} + c.$$

QUESTION NO 3 (a)

$$\int_0^2 x^2 e^x \, dx$$

$\Rightarrow$ Integration by parts $\therefore f = x^2$, $g' = ex$

$$\int fg' = fg - \int f'g$$

$$\Rightarrow x^2 e^x - \int 2xe^x \, dx$$

Solving $\int 2xe^x \, dx$

$$\Rightarrow 2 \int xe^x dx$$

Solving $\int xe^x dx$

$$\Rightarrow xe^x - \int e^x dx$$

Solving $\int e^x dx$

$$\Rightarrow e^x$$

Plugging all in original integral

$$\Rightarrow x^2 ex - 2xe^x + 2e^x + C$$

$$\int_0^2 fx \, dx = 2e^2 - 2$$

$$= 12.778$$

(3b)

$$\int_0^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$$

Let $u = \sqrt{x} = \frac{du}{dx} = \frac{1}{2\sqrt{x}} \Rightarrow dx = 2\sqrt{x}\, du$

$\Rightarrow 2\sqrt{x}\, du = dx \Rightarrow du = \frac{dx}{2\sqrt{x}}$

$\Rightarrow 2\int \sin(u)\, du$

Solving $\int \sin(u)\, du = -\cos(u)$

$\Rightarrow -2\cos(u)$

$\Rightarrow -2\cos(\sqrt{x}) + c$

$$\int_1^2 fx(dx) = 2\cos(1) - 2\cos\sqrt{2}$$

$$= -2\left(\cos\sqrt{2} - \cos(1)\right)$$

$$= 0.7687$$

QUESTION NO 4

First compute U_x and U_{xx}.

$$U_x = \frac{-1}{2}\left(x^2+y^2+z^2\right)^{-3/2}\left(\frac{2}{2x}\left(x^2+y^2+z^2\right)\right)$$

$$= \frac{x}{\left(x^2+y^2+z^2\right)^{3/2}}$$

$$U_{xz} = \frac{\left(x^2+y^2+z^2\right)^{3/2} - 3x^2\sqrt{x^2+y^2+Z^2}}{\left(x^2+y^2+z^2\right)^3}$$

$$= \frac{2x^2-y^2-z^2}{\left(x^2+y^2+z^2\right)^{\frac{5}{2}}}$$

By symettry,

$$U_{yy} = \frac{2y^2+x^2-z^2}{\left(x^2+y^2+z^2\right)^{\frac{5}{2}}}$$

$$U_{zz} = \frac{2x^2-y^2-x^2}{\left(x^2+y^2+z^2\right)^{\frac{5}{2}}}$$

Therefore

$$U_{xx}+U_{yy}+U_{zz} = \frac{\left(2x^2-y^2-z^2\right)+\left(2y^2-x^2-z^2\right) + 2z^2-y^2-x^2}{\left(x^2+y^2+Z^2\right)^{5/2}}$$

$$U_{xx}+U_{yy}+U_{zz} = 0$$