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Section = B

Semester = 7th

Subject = Differential

Question : 1

$$x + 3y + 5z + 2t = 2$$

$$-y + 3z + 4t = 0$$

$$2x + y + 9z + 6t = -3$$

$$3x + 2y + 4z + 8t = -1$$

Solution:-

using Gauss jordan method.

$$x + 3y + 5z + 2t = 2$$

$$-y + 3z + x = 0$$

$$2x + y + 9z + 6t = -3$$

$$3x + 2y + 4z + 8t = -1$$

writing system in matrix form

$$2t + x + 3y + 5z = 2$$

$$x = y + 3z = 0$$

$$6t + 2x + y + 9z = -3$$

$$8t + 3x + 2y + 4z = -1$$

$$= \left[\begin{array}{cccc|c} 2 & 1 & 3 & 5 & 2 \\ 0 & 1 & -1 & 3 & 0 \\ 6 & 2 & 1 & 9 & -3 \\ 8 & 3 & 2 & 4 & -1 \end{array} \right]$$

$$= \left[\begin{array}{cccc|c} 2 & 0 & 4 & 2 & 2 \\ 0 & 1 & -1 & 3 & 0 \\ 6 & 2 & 1 & 9 & -3 \\ 8 & 3 & 2 & 4 & -1 \end{array} \right] \begin{array}{l} \text{Multi row 2 by } -1 \text{ and add to} \\ \text{row 1} \end{array}$$

$$= \left[\begin{array}{cccc|c} 2 & 0 & 4 & 2 & 2 \\ 0 & 1 & -1 & 3 & 0 \\ 6 & 0 & 3 & 3 & -3 \\ 8 & 3 & 2 & 4 & -1 \end{array} \right] \begin{array}{l} \text{Multi row 2 by } -2 \text{ and add} \\ \text{it to row 3} \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -1 & 3 & 0 \\ 6 & 0 & 3 & 3 & -3 \\ 8 & 0 & 5 & -5 & -1 \end{array} \right] \begin{array}{l} \text{Divide the row 1 by 2} \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -1 & 3 & 0 \\ 2 & 0 & 1 & 1 & -1 \\ 8 & 0 & 5 & -5 & -1 \end{array} \right] \begin{array}{l} \text{Divide the row 3 by 3} \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & -3 & -1 & -3 \\ 8 & 0 & 5 & -5 & -1 \end{array} \right] \begin{array}{l} \text{Multi row 1 by 2 and add it} \\ \text{to row 3} \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & -3 & -1 & -3 \\ 0 & 0 & -11 & -13 & -9 \end{array} \right] \begin{array}{l} \text{Multi row 1 by } -8 \text{ and add} \\ \text{it to row 4} \end{array}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & -11 & -13 & 9 \end{array} \right] \text{ Multi the row 3 by } -1$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 1 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 11 & 13 & 9 \end{array} \right] \text{ Multi the row 4 by } -1$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -2 \\ 0 & 1 & -1 & 3 & 0 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 11 & 13 & 9 \end{array} \right] \text{ Multi row 3 by } -1 \text{ and add it to row 1}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -2 \\ 0 & 1 & -10 & 0 & -9 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 11 & 13 & 9 \end{array} \right] \text{ Multi row 3 by } -3 \text{ and add it to row 2}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -2 \\ 0 & 1 & -10 & 0 & -9 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & -28 & 0 & -30 \end{array} \right] \text{ Multi row 3 by } -13 \text{ and add it to row 4}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & -1 & 0 & -2 \\ 0 & 1 & -10 & 0 & -9 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 1 & 0 & \frac{15}{14} \end{array} \right] \text{ Divide row 4 by } -28$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{13}{14} \\ 0 & 1 & -10 & 0 & -9 \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 1 & 0 & \frac{15}{14} \end{array} \right] \text{ Add row 4 to row 1}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{13}{14} \\ 0 & 1 & 0 & 0 & \frac{12}{7} \\ 0 & 0 & 3 & 1 & 3 \\ 0 & 0 & 1 & 0 & \frac{15}{14} \end{array} \right] \text{ Multi row 4 by 10 and add it to row 2}$$

$$= \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{13}{14} \\ 0 & 1 & 0 & 0 & \frac{12}{7} \\ 0 & 0 & 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 1 & 0 & \frac{15}{14} \end{array} \right] \text{ Multi row 4 by -3 and add it to row 3}$$

Convert the augmented matrix into a system of linear equations.

$$t = -\frac{13}{14}$$

$$x = \frac{12}{7}$$

$$z = +\frac{3}{4}$$

$$y = \frac{15}{14}$$

The possible solution of system is the ordered 4-tuple ~~tuple~~

$$(t, x, y, z) = \left(-\frac{13}{14}, \frac{12}{7}, \frac{15}{14}, +\frac{3}{4} \right)$$

check if the given ordered 4-tuple a solution of system of equations

$$= \begin{cases} \frac{12}{7} + 3 \times \frac{15}{14} + 5 \times \left(-\frac{3}{4} \right) + 2 \times \left(-\frac{13}{14} \right) = 2 \\ -\frac{15}{34} + 3 \times \left(-\frac{3}{4} \right) + \frac{12}{7} = 0 \\ 2 \times \frac{12}{7} + \frac{15}{14} + 9 \times \left(-\frac{3}{4} \right) + 6 \times \left(-\frac{13}{14} \right) = -3 \\ 3 \times \frac{12}{7} + 2 \times \frac{15}{14} + 4 \times \left(-\frac{3}{4} \right) + 8 \times \left(-\frac{13}{14} \right) = -1 \end{cases}$$

Simplify the equalities

$$2 = 2$$

$$0 = 0$$

$$-3 = -3$$

$$-1 = -1$$

Since all the equalities are true so ordered 4 tuple is the solution of system.

$$(t, x, y, z) = \left(-\frac{13}{14}, \frac{12}{7}, \frac{15}{14}, -\frac{3}{14} \right) \text{ Ans.}$$
