

NAME: HAMMAD RAJ

ID: 16654

SUBJ: LINEAR ALGEBRA.

Q4) Let $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ inverse $ad - bc$

$$M^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

enteries and d for Matrix. M among d
in term of put resuce in front of

$$(b) \begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix} \quad 9 - 8 = 1$$

$$(c) \begin{bmatrix} 2 & 3 \\ 3 & 3 \end{bmatrix} \quad 6 - 9 = -3$$

$$(d) A = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 4 & 1 & 3 \end{vmatrix} = 3$$

$$1 \cdot 3 \cdot 5 + 1 \cdot 3 \cdot 4 + 1 \cdot 2 \cdot 1 - 1 \cdot 3 \cdot 4$$

$$1 \cdot 3$$

$$15 + 8 + 2 - 12 - 10 - 2$$

$$(2)$$

Q3) A.

(a) The set V of all Matrices of the form $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix}$ where $a, b \in \mathbb{R}$ over \mathbb{R} with standard

addition & scalar Multiplication

Note that V is not closed under

tion for $a, b, c, d \in \mathbb{R}$ we have $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} \in V$
 $\begin{pmatrix} 1 & a \\ b & 1 \end{pmatrix} + \begin{pmatrix} 1 & c \\ d & 1 \end{pmatrix} = \begin{pmatrix} 2 & a+c \\ b+d & 2 \end{pmatrix} \notin V.$

(b) Polynomials of degree n does not form a vector space bcz they don't form a set closed under

for instance

$$x^n - x^n = 0$$

which is not of degree n

so don't get confused with the set of polynomials of degree or equal than $n+1$

$n+1$ we often work with this space

polynomial of degree n is a set which is not closed under addition.

e.g if $n=3$ then x^3+x^3 and $-x^3$ are both 3rd degree polynomial but their sum is not

$$x^3+x^3-x^3 = x^3$$

which is not a 3rd degree polynomial.

$$(Q2) \quad T(U+V) = T(U) + T(V)$$

$$T(cU) = cT(U)$$

Example: Determine whether

$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by

$$T([x, y, z]) = [x+y, x-y, z]$$

is a linear transformation

1. Let $U = [x_1, y_1, z_1]$ & $V = [x_2, y_2, z_2]$

then we want to prove $T(U+V) = T(U) + T(V)$

$$T(U+V) = T([x_1, y_1, z_1] + [x_2, y_2, z_2])$$

$$= T([x_1 + x_2, y_1 + y_2, z_1 + z_2])$$

$$= [x_1 + x_2, y_1 + y_2, z_1 + z_2]$$

$$\text{Therefore } T(U+V) = T(U) + T(V)$$

we want to prove $T(cU)$

$$= cT(U)$$

$$T(cU) = T(c[x_1, y_1, z_1])$$

$$= T([cx_1, cy_1, cz_1])$$

$$= [cx_1, cy_1, cz_1]$$

and

$$cT(U) = cT([x_1, y_1, z_1])$$

$$= c[x_1, y_1, z_1]$$

$$= [c(x_1), c(y_1), c(z_1)]$$

$$= [cx_1, cy_1, cz_1]$$

$$\text{So } T(cU) = cT(U).$$

Q1. Consider the following vectors in

$$\mathbb{R}^3 \quad v_1 = (1, -1, 0) \quad v_2 = (3, 2, -1)$$

$$v_3 = (3, 5, -2) \quad \text{(a) verify that the}$$

general vector $U = (x, y, z)$ can
be written as a linear
combination of v_1, v_2 and v_3