Name = Iqrar Hussain

ID = 7877

Section = A

Subject = Differential Equations

Submitted To = Shomaila Mazhar

Exam = Final Term

Date = 24/9/2020

QNo1 => Find the Fourier Series representation of

$$F(t) = 1+t, -\pi \le t \le \pi$$

Sol =) $1+t \quad -\pi \le t \le \pi$

Here we use the formula

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos t + \sum_{n=1}^{\infty} b_n \sin t \rightarrow eq\ (1)$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(t)\,dt$$

$$a_0 = \frac{1}{2\pi}\int_{\pi}^{\pi} (1+t)\,dt$$

$$a_0 = \frac{1}{2\pi}\left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$a_0 = \frac{1}{2\pi}\left(\pi-(-\pi)+\frac{\pi^2}{2}-\left(-\frac{\pi^2}{2}\right)\right)$$

$$a_0 = \frac{1}{2\pi}\left(\pi-(-\pi)+\frac{\pi}{2}-\left(-\frac{\pi^2}{2}\right)\right)$$

$$a_0 = \frac{1}{2\pi}\left(2\pi+\frac{2\pi^2}{2}\right)$$

$$\boxed{a_0 = \frac{1}{2\pi}(2\pi+\pi^2)}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)(\cos nt)\,dt$$

$$a_n = \frac{1}{\pi}\left(\left.(1+t)\frac{\sin nt}{n}\right|_{-\pi}^{\pi} - \int \frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)$$

$$a_n = \frac{1}{\pi}\left(\left.(1+t)\frac{\sin \cancel{nt}}{\cancel{n}} - \frac{\cos nt}{n^2}\right|_{-\pi}^{\pi}\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$a_n = \frac{-1}{n^2\pi}\left(-1-(-1)\right)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\,dt$$

$$b_n = \frac{1}{\pi}\left|(1+t)\int_{-\pi}\sin nt - \int\left(\sin nt\,\frac{d}{dx}(1+t)\,dt\right.\right.$$

$$b_n = \frac{1}{\pi}\left(\left.\frac{(1+t)(-\cos nt)}{n}\right|_{-\pi}^{\pi} - \int \frac{-\cos nt}{n}(1)\right)$$

$$bn = \frac{1}{\pi}\left(-\frac{(1+t)(\cos nt)}{n}\Big|_{-\pi}^{\pi} + \frac{\sin \cancel{n}x}{n^2}^{\,0}\Big|_{-\pi}^{\pi}\right)\Big)$$

$$bn = \frac{-1}{n\pi}\left((1+\pi)(\cos n\pi) - (1+(x))(\cos n(x))\right)$$

$$bn = \frac{-1}{n\pi}\left(\cos n\pi + \pi \cos n\pi - \cos nA + A \cos n A\right)$$

$$bn = \frac{-1}{n\cancel{\pi}}\left(2\cancel{\pi} \cos n\pi\right)$$

Here $\cos n\pi = \frac{(-1)^{n+1}}{n}$

$$bn = \frac{2}{n}(-1)^{n+1}$$

So equation become

$$f(x) = \frac{1}{2\pi}\left(2\pi + \pi^2\right) + 0 + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n} \sin t$$

Qno2) ⇒ Calculate the Characteristic equation the Eigen values of the System. Where A is given by

$$\begin{matrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{matrix}$$

Sol ⇒ Eigen values = ?

Step = 1

We have

$$(A - \lambda I)X = 0 \qquad A = \text{Given matrix}$$

Step = 2

We have, The Characteristic equation is given by

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step#3

$$\lambda^3 - \left|\begin{matrix}\text{Sum of} \\ \text{Digonal elem.}\end{matrix}\right| \lambda^2 + \left|\begin{matrix}\text{Sum of} \\ \text{Diagonal} \\ \text{minor}\end{matrix}\right| \lambda - |A| = 0 \longrightarrow (B)$$

Sum of Diagonal elements $= 1+1+2 = 4$

Sum of Digonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$= (-6) + (2) + (1)$

$= -6 + 2 + 1$

$= -3$

By putting values in eq (B)

$$\lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0 \quad \text{—— (C)}$$

$$A = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$$

$= 1(2-8) - 0 + 1(6-0)$

$= -6 + 6$

$= 0$

By putting values in eq ⊖

$\lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda^2 - 4\lambda - 3) = 0$

$\lambda = 0$

$\lambda^2 - 4\lambda - 3 = 0$

Using Quadratic Formula

$a = 1$
$b = -4$
$c = -3$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2a}$$

$$= \frac{4 \pm \sqrt{16+12}}{2} = \frac{4 \pm \sqrt{28}}{2}$$

$$\lambda = \frac{4+\sqrt{28}}{2}, \quad \lambda = \frac{4-\sqrt{28}}{2}$$

We have eigenvalues,

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$ required Solution

QNo3) Solve the following system of linear equations

$$5x + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2z = 1$$
$$x + y + z + m = 0$$

Sol ⇒

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_4\ R_2$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \quad -1/5 \times R_3$$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 8/5 & 1/5 \end{array}\right]$$ $5 \times R_3$ and $5 \times R_4$

$$\left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right]$$ $5R_3$ and $5R_4$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right]$$ $\frac{1}{5} \times R_1$

$$\begin{pmatrix} 1 & 0 & 4/5 & 2/5 \\ 0 & -1 & 6/5 & 1/5 \\ 0 & -5 & 6 & 4 \\ 0 & 0 & 7 & 8 \end{pmatrix}$$ $R_2 \times 5$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & \frac{2}{5} & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \qquad \underline{R_3 - R_4}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 8/7 & 1/7 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \qquad \begin{array}{l} \underline{R_3 \longleftrightarrow R_4} \\ \underline{\frac{1}{7} \times R_3} \\ \underline{\frac{1}{3} \times R_4} \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \qquad l_2 \times -5$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & \frac{3}{5} \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -\frac{4}{21} \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

(11)

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 26/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad \underline{5/4 \times R_1}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 & 0 & \frac{31}{21} \\ 0 & 0 & 1 & 0 & -\frac{11}{21} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{3}{4} \\ 0 & 1 & 0 & 0 & \frac{31}{21} \\ 0 & 0 & 1 & 0 & -\frac{11}{21} \\ 0 & 0 & 0 & 1 & \frac{1}{3} \end{array}\right]$$

$$(x, y, z, m) = \left(\frac{3}{4}, \frac{31}{21}, \frac{-11}{21}, \frac{1}{2}\right)$$

$$x = \frac{3}{4}$$

$$y = \frac{31}{21}$$

$$z = \frac{-11}{21}$$

$$m = \frac{1}{3}$$

QNo4) ⇒ Verify that $U(x,t) = \sin(x+2t)$ is a Solution of the one-dimensional wave equation

Sol ⇒ <u>Given data</u>

$$U(x,t) = \sin(x+2t)$$

Differentiat w.r.t x partialy

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\sin(x+2t)$$

$$\frac{\partial u}{\partial u} = \cos(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial u}{dx} = \cos(x+2t)(1+0)$$

$$\frac{\partial u}{dx} = \cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \sin(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\frac{\partial^2 u}{\partial x^2} = \sin(x+2t)(1+0)$$

$$\frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)$$

And

$$u(x,t) = \sin(x+2t)$$

Differentiate w.r.t 't'

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial t}\sin(x+2t)$$

$$\frac{\partial u}{\partial t} = \cos(x+2t)(0+2)$$

$$\frac{\partial u}{\partial t} = 2\cos(x+2t)$$

$$\frac{\partial^2 u}{\partial t^2} = (2) - \sin(x+2t)(0+2)$$

$$\frac{\partial^2 u}{\partial t^2} = -4\sin(x+2t)$$

As we know that one dimensional wave equation

is = $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

(15)

$$-4\sin(x+2t) = c^2\left[-\sin(x+2t)\right]$$

$$-4\sin(x+2t) = -c^2\sin(x+2t)$$

$$-4\sin(x+2t) + c^2\sin(x+2t) = 0$$

For the orbitary constant $c = \pm 2$

$$-4\sin(x+2t) + (\pm 2)^2\sin(x+2t) = 0$$

$$-4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for the orbitrary constant

$$c = 2$$