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Solution 1:2:1, 2:1:1, 2:0:2

| P | E |
|---|---|
| A | E |

| P | P |
|---|---|
| A | E |

| P | A |
|---|---|
| A | A |

Let x, y and z be the cost/kg pakistan, Egyptain, American cotton respectively then according to the given condition

$$\frac{1}{4}x + \frac{2}{4}y + \frac{1}{4}z = 40$$
$$\frac{2}{4}x + \frac{1}{4}y + \frac{1}{4}z = 80$$
$$\frac{2}{4}x + \frac{2}{4}y + \frac{2}{4}z$$

$\Rightarrow A$

$$1x + 2y + 1z = 160$$
$$2x + 1y + z = 200$$
$$1x + 1z = 120$$

$\Rightarrow B$

In matrix form, we can write it as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$$

Let $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $\underline{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $\underline{B} = \begin{bmatrix} 160 \\ 200 \\ 120 \end{bmatrix}$

$$A\underline{X} = \underline{B}$$

$$\Rightarrow A_1 = \begin{bmatrix} \underline{160} & 2 & 1 \\ \underline{200} & 1 & 1 \\ \underline{120} & 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{bmatrix},$$

First $|A| = \begin{vmatrix} 1 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$ Exp by $R_1$

$= 1\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$

$= 1(1-1\times 0) - 2(2\times 1 - 1\times 1) + 1(2\times 0 - 1\times 1)$

$= 1(1-1-0) - 2(2-1) + 1(0-1)$

$= 1(0) - 2(1) + 1(0)$

$= 0 - 2 + 0$

$= -2$

Now

$|A| = \begin{vmatrix} 100 & 2 & 1 \\ 200 & 1 & 1 \\ 120 & 0 & 1 \end{vmatrix}$ Exp by $R_1$

$= 100\begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} + 1\begin{vmatrix} 200 & 1 \\ 120 & 0 \end{vmatrix}$

$100(1\times 1 - 0)(0-1) - 2(200-120) + 1(0-120)$

$|A| = 100(1) - 2(80) + 1(-120)$

$= 100 - 160 - 120 \Rightarrow |A| = -120$

Similarly

$$|A_2| = \begin{vmatrix} 1 & 160 & 1 \\ 2 & 200 & 1 \\ 1 & 120 & 1 \end{vmatrix}$$ Exp by $R_1$

$$1\begin{vmatrix} 200 & 1 \\ 120 & 1 \end{vmatrix} - 160\begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix}$$

$$|A_2| = 1(200-120) - 160(2-1) + 1(240-200)$$

$$|A_2| = 1(80) - 160(1) + 1(40)$$

$$|A_2| = 80 - 160 + 40$$

$$|A_2| = 120 - 160$$

$$|A_2| = -40$$

then

$$|A_3| = \begin{vmatrix} 1 & 2 & 160 \\ 2 & 1 & 200 \\ 1 & 0 & 120 \end{vmatrix}$$ Exp by $R_1$

$$= 1\begin{vmatrix} 1 & 200 \\ 0 & 120 \end{vmatrix} - 2\begin{vmatrix} 2 & 200 \\ 1 & 120 \end{vmatrix} + 160\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(120 - 0) - 2(240 - 200) + 160(0 - 1)$$

$$= 1(120) - 2(40) + 160(-1)$$

$$= 120 - 80 - 160$$

$$|A_3| = 120 - 240$$

$$|A_3| = -120$$

Now according to Cramer's Rule's

$$x = \frac{|A_1|}{|A|} = \frac{-120}{-2} = 60$$

$$y = \frac{|A_2|}{|A|} = \frac{+40}{+2} = \frac{40}{2} = 20$$

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$$z = \frac{|A_3|}{|A|} = \frac{-120}{-2} = \frac{120}{2} = 60$$

$$(x, y, z) = (60, 20, 60)$$

Pakistan = 60

Egyption = 20

American (USA) = 60

The End