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Subject: Electromagnetic Field (EMF)
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Assignment:

Solve problem 4.1, 4.2, 4.3, 4.5, and 4.7 of course book.

4.9) The value of E at $P(p=2, \phi=40^\circ, z=3)$ is given as $E = 1002p - 2002p + 300z$ V/m. Determine the incremental work require to move a 20nC charge a distance of 6um:

a) In the direction of a_p :
the incremental work is given

by $dw = -qE dl$, where in this case $dl = dp a_p = 6 \times 10^{-6}$

$$dw = -(20 \times 10^{-9} \text{ C}) (100 \text{ V/m}) (6 \times 10^{-6} \text{ m}) = -12 \times 10^{-9} \text{ J} = -12 \text{ nJ}$$

b) In the direction of a_ϕ : In this case $dl = 2d\phi a_\phi = 6 \times 10^{-6} a_\phi$ and so

$$dw = -(20 \times 10^{-9}) (-200) (6 \times 10^{-6}) = 2.4 \times 10^{-8} \text{ J} = 24 \text{ nJ}$$

c) In the direction of a_z : Here,

$$dl = dz a_z = 6 \times 10^{-6} a_z \quad \text{So } dw = -(20 \times 10^{-9}) (300) (6 \times 10^{-6}) = -3.8 \times 10^{-8} \text{ J} = 36 \text{ nJ}$$

d) In the direction of E : Here $d\mathbf{l} = 6 \times 10^{-6} d\mathbf{r}$

$$\text{where } 2E = 1002\hat{p} = \frac{200^2\hat{p} + 3000a_z}{[100^2 + 200^2 + 300^2]^{1/2}} = 0.267a_p - 0.5852a_x + 0.802a_z$$

Thus

$$dW = -(20 \times 10^{-6}) (1002\hat{p} - 200a_p - 300a_z) \cdot [0.267a_p - 0.802a_z] (6 \times 10^{-6}) = -44.9$$

(e) In the direction of $G = 22x - 32y + 4az$: In this case $d\mathbf{l} = 6 \times 10^{-6} 2G$, where

$$2G = \frac{2ax - 32y + 4az}{[2^2 + 3^2 + 4^2]^{1/2}} = 0.3712 - 0.557a_y + 0.743a_z$$

So now,

$$dW = -(20 \times 10^{-6}) [100a_p + 2002\hat{p} + 300a_z] \cdot [0.3712a_x - 0.557a_y + 0.743a_z] (6 \times 10^{-6}) = -(20 \times 10^{-6}) [37.1(a_p \cdot a_x) - 55.7(a_p \cdot a_y) - 74.8(2\hat{p} \cdot 2x) + 11.4(a_p \cdot a_y) + 222.9] (6 \times 10^{-6})$$

(4.2) Let $E = 400ax - 300ay - 560az$

In the neighborhood of point $P(6, 2, 3)$ find the incremental work done in moving a 4-C charge a distance of 1mm in the direction specified by:

(a) $ax^2 + 2y + 2z$; we write

$$dw = -qE \, dl = -4(400ax - 300ay + 500az) \\ \frac{(ax + ay + az)}{\sqrt{3}} (10^{-3}) \\ = \frac{(4 \times 10^{-3})}{\sqrt{3}} (400 - 300 + 500) = -1.39 \text{ J}$$

(b) $-2ax + 3ay - az$: The computation is similar to that of part a. but we change the direction:

$$dw = -qE \cdot dl = -4(400ax - 300ay - 500az) \\ = \frac{(-2ax + 3ay - az)}{\sqrt{14}} (10^{-3}) \\ = - \frac{(4 \times 10^{-3})}{\sqrt{14}} (-800 - 900 - 500) = 2.35 \text{ J}$$

4.3

IF $E = 1202p \text{ V/m}$. Find the incremental amount of work done is moving a 50um charge a distance of 2mm from:

2) $P(1, 2, 3)$ toward $Q(2, 1, 4)$ The vector along this direction will be $Q - P = (1, -1, 1)$ from $dw = -qE \, dl = (50 \times 10^{-6}) \left[\frac{1200p(ax - ay - az)}{\sqrt{3}} \right]$

$$= (50 \times 10^{-6}) (120) [(ap-2x) \cdot (ap-ay)] \frac{1}{\sqrt{3}} (2 \times 10^{-3})$$

At P, $Q = \tan^{-1}(2/1) = 63.4^\circ$. Thus $(ap \cdot ax) =$

$$\cos(63.4) = 0.447 \quad \& \quad (ap \cdot ay) = \sin(63.4) =$$

0.894 substituting these we obtained

$$dw = 3.1 \text{ uJ}$$

4.5

Compute the value of $\int_A^P G \cdot dl$ for $G = 2yax$ with $A(1, -1, 2)$ & $P(2, 1, 2)$ using the path

(a) straight line segment $A(1, -1, 2)$ to $(2, 1, 2)$

$$\int_A^P G \cdot dl = \int_A^P 2y dx$$

The change in x occurs when moving b/w A and P during which $y=1$ Thus

$$\int_A^P G \cdot dl = \int_A^P 2y dx = \int_1^2 2(1) dx = 2$$

(b) Straight line segment A (1, -1, 2) to C (2, -1, 2) to P (2, 1, 2) in this case the change in x occurs

$$\int_A^P G \cdot dl = \int_A^C 2y dx = \int_1^2 2(-1) dx = -2$$

4.7

Repeat problem 4.6 for $G = 3xy^2 \mathbf{i} + 2zay$ Now think abt dlf in that the path does matter.

(a) straight line: $y = x - 1, z = 1$ we obtain

$$\int G \cdot dl = \int_2^4 3xy^2 dx + \int_1^3 3x(x-1)^2 dx + \int_1^3 2(1) dy = 90$$

(b) Parabolic $6y = x^2 + 2, z = 1$ we obtain

$$\int G \cdot dl = \int_2^4 3xy^2 dx + \int_1^3 2ndy = \int_2^4 \frac{1}{12} x(x^2+2)^3 dx + \int_1^3 2(1) dy = 80 + 2 = 82$$

