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Paper: Calculus and Analytical

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Question 1 part A

$$\frac{3x^3 - 5x^2 + 5}{(x^2+1)}$$

Solution:

By Using Quotient Rule.

$$y = \frac{3x^2 - 5x^2 + 5}{(x^2+1)}$$

$$y = \frac{(x^2+1)(9x^2-10x) - (3x^2-5x^2+5)(2x)}{(x^2+1)^2}$$

$$y = \frac{9x^4 - 10x^3 + 9x^2 - 10x - 6x^4 + 10x^3 - 10x}{(x^2+1)2}$$

$$y = \frac{9x^4 - 6x^4 - 10x^3 + 10x^3 + 9x^2 - 10x}{(x^2+1)^2}$$

$$y = \boxed{\frac{3x^4 + 9x^2 - 10x}{(x^2+1)^2}}$$ Ans

Question 1 part 2

Differentiate $\frac{(x^2+1)^2}{x^2-1}$ with respect to x

Solution = $\frac{(x^2+1)^2}{x^2-1}$

$= \frac{(x^2+1)(x^2-1)}{(x^2-1)}$

$= \frac{(x^2+1)\cancel{(x^2-1)}}{\cancel{(x^2-1)}}$

$= \frac{d}{dx}(x^2+1) = 2\frac{d}{dx}x^{2-1} \cdot \frac{d}{dx}(1)$

$= 2x \cdot 0$

$= 2x$ Ans

Question No: 2
part b

$\Rightarrow$ Find $\frac{dy}{dx}$ if $y = \sqrt{\frac{1-x}{1+x}}$ Using chain rule.

Solution $y = \sqrt{\frac{1-x}{1+x}}$

$$= \sqrt{\frac{(1-x)}{(1+x)} \times \frac{(1-x)}{(1-x)}}$$

$$y = \frac{1-x}{\sqrt{1-x^2}} \quad — \ 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-x^2}\,\frac{d}{dx}(1-x) - (1-x)\frac{d}{dx}\left(\sqrt{1-x^2}\right)}{\left(\sqrt{1-x^2}\right)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1-x^2}\,\frac{d}{dx}(1-x) - (1-x)\frac{d}{dx}\left(\sqrt{1-x^2}\right)}{\left(\sqrt{1-x^2}\right)^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt{1-x^2} + (1-x) \cdot \frac{x}{\sqrt{1-x^2}}}{1-x^2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt{1-x^2} + (1-x) \cdot \frac{x}{\sqrt{1-x^2}}}{1-x^2}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} = \frac{-1+x^2+x-x^2}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} = \frac{-1+x}{\sqrt{1-x^2}}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} = -\left(\frac{1-x}{\sqrt{1-x^2}}\right)$$

From (1)

$$(1-x^2)\frac{dy}{dx} = -y$$

$$(1-x^2)\frac{dy}{dx} + y = 0$$

Answer

Question No 2
part (A)

$$y = [1+2\sqrt{x})^3 x^{2/3}$$

using chain rule

$$y = (x+2\sqrt{x})^3 \cdot x^{2/3}$$

$$y = (x+2\sqrt{x} \cdot x)^2$$

$$y = (x+2x^{3/2})^2$$

let $y = m^2$, where

$$m = x + 2x^{3/2}$$

$$\frac{dm}{dx} = 1+3x^{1/2}$$

$$\boxed{dm = (1+3\sqrt{x}\, dx} \text{ — (1)}$$

$$\frac{dy}{dx} = \frac{dy}{dx} \times \frac{du}{dx} \text{ — (A)}$$ Chain Rule.

$$\frac{dy}{dx} = (2u)(1+3\sqrt{x}\,dx)$$

$$\boxed{\frac{dy}{dx} = 2(x+2x^{3/2})^2 (1+3\sqrt{x})}$$

Q3 part (a)

Find the Integration of $\int \frac{1}{\sqrt{x^3}} dx$

Solution : $\int \frac{1}{\sqrt{x^3}} dx$

$= \int x^{-3/2} dx$

$= \frac{x^{-3/2+1}}{-3/2+1} = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} = \frac{-2}{\sqrt{x}} + C$

Ans

Q3 part B

Find the Integration of $\int \frac{1}{(6x+7)^6} dx$

Solution $\int \frac{1}{(6x+7)^6} dx$ —— (A)

$= \int (6x+7)^{-6} dx$ = let $6x+7 = y$ —— (1)

put (1) in (A)

$= \int \frac{1}{(y)^6} dy$

$= \int y^{-6} dy$

$= \frac{y^{-5}}{-5} + C = \frac{-1}{5y^5} + C$

put the value of y =

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$$= \frac{-1}{5}\left(\frac{1}{(6x+7)^5}\right) + C \quad \text{Ans}$$

Finish paper.