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I.D # 7642

Calculas And Analytical

Geomentry.

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QNo1 (a) Estimate $\int \theta^4 \sqrt{1-\theta^2}\, d\theta$

Solution:- $\int \theta^4 \sqrt{1-Q^2}\, d\theta$

$\Rightarrow \int (\theta^4 - Q^2)\, d\theta$

$\Rightarrow \int Q^4\, d\theta - \int Q^2\, d\theta$

$\Rightarrow$ Now solving

$\Rightarrow \int \theta^4\, d\theta$

Apply power rule.

$\int \theta^n d\theta = \frac{\theta^{n+1}}{n+1}$ with $n = 4$

$\int \theta^4 d\theta \boxed{= \frac{\theta^5}{5}}$

Now solving

$\Rightarrow \int \theta^2 d\theta$

Apply power rule $n = 2$

$\int \theta^2 d\theta = \frac{\theta^3}{3}$

Plugin solove integrals

$\Rightarrow \int Q^4\, d\theta - \int \theta^2\, d\theta$

$\boxed{\Rightarrow \frac{\theta^5}{5} - \frac{\theta^3}{3} + C}$

Q No 1 Part (b) (2)

Estimate $\int_0^1 x^3 (1+x^4)^3 dx$ Using Substitution method.

Solution:- $\int x^3 (1+x^4)^3$

Substitute $u = x^4 + 1$ $\frac{du}{dx} = 4x^3$

$dx = \frac{1}{4x^3} du$

$= \frac{1}{4} \int u^3 du$

Now Solving

$\int u^3 du$

Apply power rule:

$\int u^n du = \frac{u^{n+1}}{n+1}$ with $n = 3$

$= \frac{u^4}{4}$

Plug in Solved Integrals:

$\frac{1}{4} \int u^3 du$

$\frac{u^4}{16}$

undo Substitution

(3)

$$u = x^4 + 1$$

$$= \frac{(x^4+1)^4}{16}$$

$$\int x^3 (x^4+1)^3 \, dx$$

$$\boxed{= \frac{(x^4+1)^4}{16} + C}$$

$$\int_0^1 f(x)\, dx = \boxed{\frac{15}{16}} = \boxed{0.9375}$$

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Q2 Part (a) ④

Illustrate the center and radius of the sphere $x^2+y^2+z^2+3x-4z+1$.

Sol:-> 3

$$x^2+y^2+z^2+3x-4z+1 =$$

$$\Rightarrow (x^2+3x)+y^2+z^2-4z+1 = 0$$

$$\left(x^2+3x+\left(\tfrac{3}{2}\right)^2\right)+(y-0)^2+\left(z^2-4z+\left(\tfrac{-4}{2}\right)^2\right)$$
$$= -1+\left(\tfrac{3}{2}\right)^2+\left(\tfrac{-4}{2}\right)^2$$

$$\Rightarrow \left(x+\tfrac{3}{2}\right)^2+(y)^2+(z-2)^2 = \frac{21}{4}$$

So (x_0, y_0, z_0) = Centre.

$$= \left(-\tfrac{3}{2}, 0, 2\right)$$

and Radius $a = \sqrt{\frac{21}{4}}$

QNo 2 Part (b) 5

The region between the Curve $y=\sqrt{x}$, $0 \le x \le 4$ and the x-axis is revolved about the x-axis to generate a solid. Apply the Integration. Find the volume of Solide.

Solution:- given that

$$y=\sqrt{x} \qquad 0 \le x \le 4$$

$$\Rightarrow a \le x < b$$

As $V = \int_a^b \pi y^2 dx$

$$V = \int_0^4 \pi (\sqrt{x})^2 dx$$

$$V = \pi \int_0^4 x \, dx$$

$$V = \pi \cdot \frac{x^2}{2} \Big|_0^4$$

$$V = \frac{\pi}{2}\left[(4)^2 - 0\right]$$

$$\boxed{V = 8\pi}$$

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Q No3 If $A = 2i - 4j + \sqrt{5}k$ and $B = -2i + 4j - \sqrt{5}k$ then illustrate the vectore project A^{B}

Solution:-

$$A = 2i - 4j + \sqrt{5}k \qquad B = -2i + 4j - \sqrt{5}k$$

$$B \cdot A = (-2i + 4j - \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$B \cdot A = -4 - 16 - 5 = \boxed{-25}$$

$$(A \cdot A) = (2i - 4j + \sqrt{5}k) \cdot (2i - 4j + \sqrt{5}k)$$

$$(A \cdot A) = 4 + 16 + 5 = \boxed{25}$$

So

$$\mathrm{Proj}_A{}^{B} = \left(\frac{B \cdot A}{A \cdot A}\right)(A)$$

$$= \left(\frac{-25}{25}\right)(2i - 4j + \sqrt{5}k)$$

$$\mathrm{Proj}_A{}^{B} = -\frac{50}{25}i + \frac{100}{25}j - \frac{56}{25}k$$

Q No 4 Find the area of the region b/w the graph and the x-axis.
where $y = -x^2 + 5x - 4$, $[0, 2]$

Solution

$y = -x^2 + 5x - 4$

~~$x^2 + 5x$~~

$a = 1, \ b = 5, \ c = 0$

The Discriminant is given by

$\Delta = b^2 - 4 \cdot a \cdot c$

$= (5)^2 - (4 \cdot 1 \cdot (-4))$

$= 25 + 16 = 41$

The solution are found using the formula

$x = \dfrac{-b \pm \sqrt{\Delta}}{2 \cdot a}$

$x = \dfrac{(-5) \pm \sqrt{41}}{2 \cdot 1} = \dfrac{-5 \pm \sqrt{41}}{2}$

$\boxed{x = \dfrac{-5 + \sqrt{41}}{2}}$

$\boxed{x = \dfrac{-5 - \sqrt{41}}{2}}$

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QNo5

Part (a)

Estimate the ~~following~~ Angle between $A = i - 2j - 2k$ and $B = 6i + 3j + 2k$

Solution:-

$$A = i - 2j - 2k$$

$$|A| = \sqrt{1+4+4} = \sqrt{9} = 3$$

$$B = 6i + 3j + 2k$$

$$|B| = \sqrt{36+9+4} = \sqrt{49} = 7$$

$$\theta = \cos^{-1}\left(\frac{A \cdot B}{|A||B|}\right)$$

$$\theta = \cos^{-1}\frac{(i-2j-2k)\cdot(6i+3j+2k)}{3 \times 7}$$

$$\theta = \cos^{-1}\left(\frac{(1)(6)+(-2)(3)+(-2)(2)}{21}\right)$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right)$$

Q No 5 Part (b)

Change into a spherical coordinat equation for the sphere $x^2+y^2+(z-1)^2=1$

Sol:

$$x^2+y^2+(z-1)^2=1$$

$$\Rightarrow (\rho\sin\phi\cos\theta)^2+(\rho\sin\phi\sin\theta)^2+(\rho\cos\phi-1)^2=1$$

$$\Rightarrow \rho^2\sin^2\phi\cos^2\theta+\rho^2\sin^2\phi\sin^2\theta+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$

$$\Rightarrow \rho^2\sin^2\phi(\cos^2\theta+\sin^2\theta)+\rho^2\cos^2\phi+1-2\rho\cos\phi=1$$

$$\Rightarrow \rho^2(\sin^2\phi+\cos^2\phi)-2\rho\cos\phi=0$$

$$\rho^2=2\rho\cos\phi$$

$$\boxed{\rho=2\cos\phi}$$