Department of Electrical Engineering
Sessional Assignment
Date: 01/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:		**Total Marks:**	20

Student Details

Name:	**M.kamran**	**Student ID:**	**13752**

Q1.	(a)	Determine the response $y(n)$, $n \ge 0$, of the system described by the second order difference equation $$y(n)-3y(n-1)-4y(n-2) = x(n) + 2x(n-1)$$ To the input $x(n) = 4^n u(n)$.	Marks 6
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) = 0.6y(n-1)-0.8y(n-2) + x(n)$$	
Q2.	(a)	Determine the causal signal $x(n)$ having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	Marks 6
	(b)	Determine the partial fraction expansion of the following proper function $$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$$	
Q. 3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $\left\|H(\frac{\pi}{4})\right\|^2 = \frac{1}{2}$.	Marks 4

	(b)	Design a two-pole bandpass filter that has the center of its passband at $\omega = \pi/2$, zero in its frequency response characteristics at $\omega = 0$ and $\omega = \pi$ and its magnitude response in $\frac{1}{\sqrt{2}}$ at $\omega = 4\pi/9$.	
Q 4	(c)	A finite duration sequence of Length L is given as $$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & \text{otherwise} \end{cases}$$ Determine the N- point DFT of this sequence for $N \ge L$	Marks 4
	(d)	Compute the DFT of the four-point sequence $$x(n) = (0 \;\; 1 \;\; 2 \;\; 3)$$	

Name: M. Kamran (1) ID# 13752

Q#1 1 (a):

Determine the responce $y(n)$. $n \geq 0$ of the system described by the second order differencial equation.

$$y(n) - 3y(n-1) - 4y(n-2) = x(n) + 2x(n-1) \quad (1)$$

when the input sequence is

$$x(n) = 4^n u(n)$$

Solution:

We have allready determined the solution to the homogenous difference equation for this system

$$y_h(n) = C_1(-1)^n + C_2(4)^n \quad — \quad (2)$$

The particular solution is assumed to be an exponential sequence of the same from as $x(n)$, Normally we could assume a solution of the form.

$$y_p(n) = K(4)^n u(n) \quad —$$

However we observe that $y_p(n)$ is allready contained in the homogenous solution. so that this particullar solution is redundat. Instead we select the particular solution to be linearly independent of the terms contained in the homogenous solution In fact we treat this situation in the same manner as we have already treated multiple roots in the characteristic equation. thus we assume that

$$Y_p(n) = Kn(4)^n u(n) \quad -(3)$$

upon substitution we obtained

$$Kn(4)^n u(n) - 3K(n-1)(4)^{n-1} u(n-1) - 4K(n-2)(4)^{n-2} u(n-2)$$

$$= (4)^n u(n) + 2(4)^{n-1} u(n-1)$$

To Determine K we evalute this equation for any $n \geq 2$. where a none of the unit step terms vanish. to simply the arithmatic we select $n=2$ from which we obtaind $K = 6/5$ therfore

$$Y_p(n) = \frac{6}{5} n(4)^n u(n) \quad (4)$$

the total soultion to the difference equation is obtained by adding

$$Y(n) = C_1(-1)^n + C_2(4)^n + \frac{6}{5} n(4)^n \quad n \geq 0 \quad (5)$$

where the constant C_1 and C_2 are determind such that the initial condition are satasfy. to accomplished this we return to (1) thus which we obtained.

$$Y(0) = 3y(-1) + 4y(-2) + 1$$

$$Y(1) = 3y(0) + 4Y(-1) + 6$$

$$= 13y(-1) + 12y(-2) + 9$$

on the other hand (5) evaluated at $n=0$ and $n=1$ yeilds

$$y(0) = C_1 + C_2$$

$$y(1) = -C_1 + 4C_2 + 24/5$$

We can now equate these two sets of relations to obtaind C_1 and C_2. In so doing we have the responce due to initial condition $Y(-1)$ and $Y(-2) = 0$ then we have (the zero-input response) and the zero state or force state or forced responces.

Since we have already soleved for the zero input responce we can simply the computation above by setting $v(-1) = v(-2) = 0$, then we have.

$$C_1 + C_2 = 1$$

$$-c_1 + 4c_2 + \frac{24}{5} = 9$$

Hence $c_1 = -1/25$ and $c_2 = \frac{26}{25}$, so $y(n)$ is

$$\boxed{y_{zs}(n) = -\frac{1}{25}(-1)^n + \frac{26}{25}(4)^n + \frac{6n(4)^n}{5} \quad n \geq 0}$$

Q#1 (b)

Determine the impulse responce and unit step responce of the system described by difference equation.

$$y(n) = 0.6y(n-1) - 0.8y(n-2) + x(n)$$

Soultion:

The characteristic equation is

$$\pi^2 - 0.6\pi + 0.08 = 0$$

$$\pi = 0.2, 0.4 \quad \text{Hence}$$

$$y(n) = c_1 \frac{1^n}{5} + c_2 \frac{2^n}{5}$$

with $x(n) = \delta(n)$ the Initial condition are:

$$y(0) = 1$$

$$y(1) - 0.6y(0) = 0 \Rightarrow y(1) = 0.6$$

Hence $C_1 + C_2 = 1$ and

$$\tfrac{1}{5} C_1 + \tfrac{2}{5} = 0.6 \Rightarrow C_1 = -1,\ C_2 = 3$$

Therefore $h(n) = \left[-(1/5)^n + 2(2/5)^n\right] u(n)$

the Step responce is

$$s(n) = \sum_{k=0}^{n} h(n-k),\ n \geq 0$$

$$= \sum_{k=0}^{n} \left[2(2/5)^{n-k} - (1/5)^{n-k} \right]$$

$$= \boxed{\left\{ \frac{1}{0.12}\left[\left((2/5)^{n+1} - 1\right) - \frac{1}{0.16}\left[(1/5)^{n+1} - 1\right]\right\} u(n)\right.}$$

Q#2 (a):

Determine the causal signal $x(n)$ having the z-transform:

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Hint: take inverse z-transform using partial fraction method.

Solution:

$$X(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

$$= \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4, \; B = -3, \; C = -1$

Hence $\boxed{x(n) = \left[4(2)^n - 3 - n\right]u(n)}$

Q#2(b):

Determine the partial fraction expression of the following funtion:

$$X(z) = \frac{1}{1-1.5z^{-1}+0.5z^{-2}}$$

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Solution:

First we eliminate the negative powers by multiplying both numenator and denomenator by z^2

$$X(z) = \frac{z^2}{z^2-1.5z+0.5}$$

the poles of $X(z)$ are $p_1 = 1$ and $p_2 = 0.5$

$$\frac{X(z)}{z} = \frac{z}{(z-1)(z-0.5)} = \frac{A_1}{z-1} + \frac{A_2}{z-0.5}$$

multiplying equation with denomenitor

$$z = (z-0.5)A_1 + (z-1)A_2$$

Now set $z = P_1 = 1$

$$1 = (1-0.5)A_2 \text{ and } 0.5 = (0.5-1)A_2$$

$A_2 = 1$ result of expression is

$$\frac{X(z)}{z} = \frac{2}{z-1} - \frac{1}{z-0.5}$$

So we have pole position.

$$\frac{(z-A_K)X(z)}{z} = \frac{(z-P_K)A_1}{z-P_1} + \cdots\cdots A_K + \cdots\cdots \frac{(z-P_K)A_n}{z-P_N}$$

Consequatly with $z = P_K$ yields the Kth coeffient as

$$\boxed{A_K = \left.\frac{(z-P_K)X(z)}{z}\right|_{z=P_K} \quad K=1,2\ldots N}$$

Q #3 (a)

A two pole low pass filter has the system responce

$$H(z) = \frac{b_0}{(1 - pz^{-1})^2}$$

Determine the values of b_0 and p such that the frequency responce $H(w)$ satisfy the condition $H(0) = 1$ and

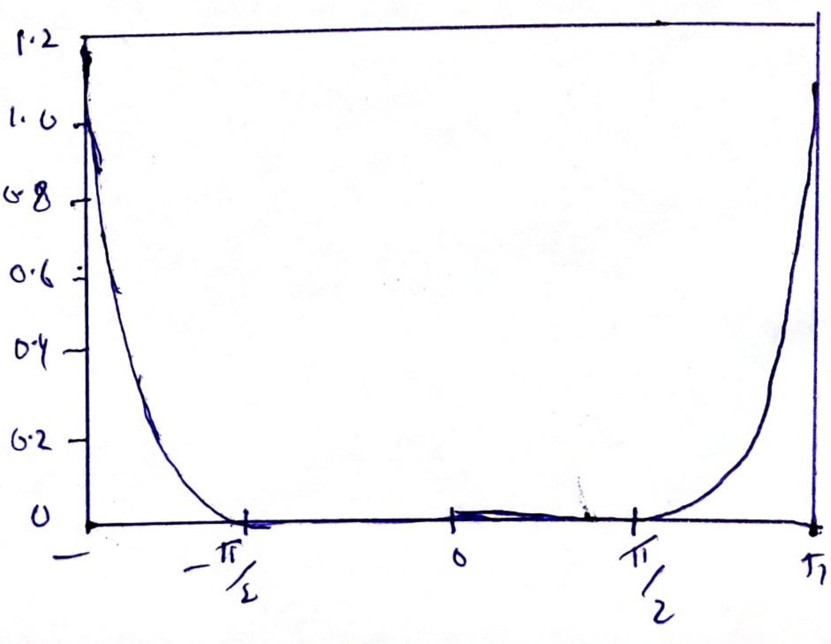
$$|H(\tfrac{\pi}{4})|^2 = \frac{1}{2}$$

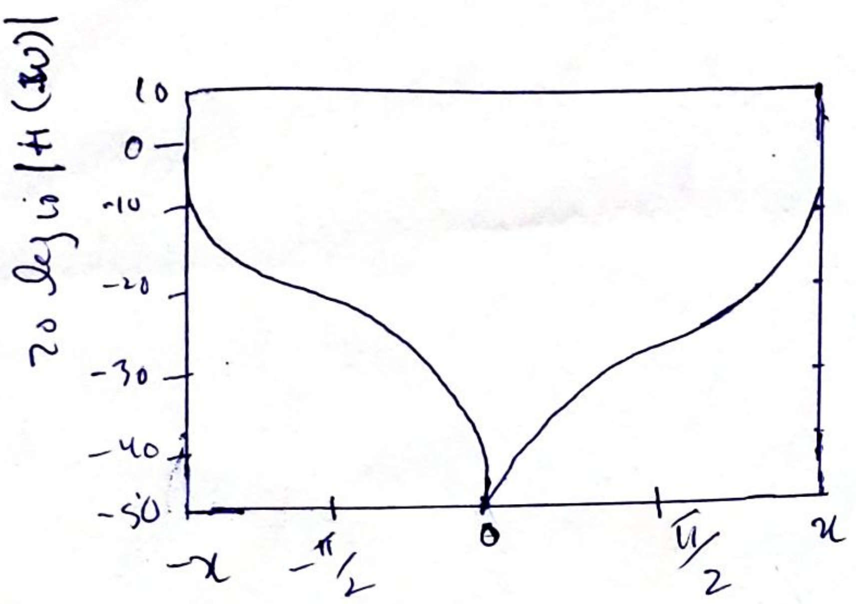
Solution:

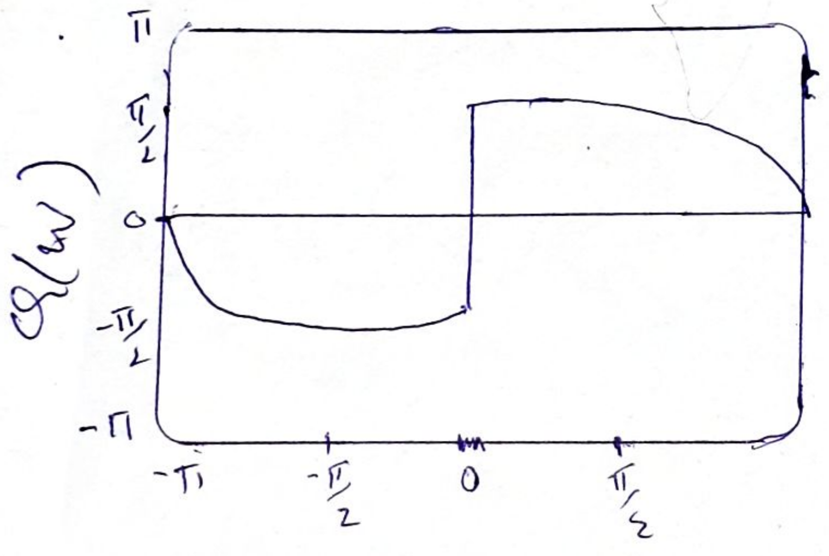
at $w_0 = 0$ we have

$$H(0) = \frac{b_0}{(1 - p)^2} = 1$$

hence $b_0 = (1 - p)^2$







at $\omega = \pi/4$

$$H(\pi/4) = \frac{(1-p)^2}{(1-pe^{-j\pi/4})^2}$$

$$= \frac{(1-p)^2}{1-p\cos(\pi/4)+jp\sin(\pi/4))^2}$$

$$= \frac{(1-p)^2}{(1-p/\sqrt{2}+jp/\sqrt{2})^2}$$

hence

$$\frac{(1-p)^2}{[(1-p/\sqrt{2})^2+p^2/2]^2}$$

$$= 1/2$$

or equivalently.

$$\sqrt{2}(1-p)^2 = 1+p^2-\sqrt{2}p$$

the value of $p = 0.32$ satisfying this equation.

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}$$

Q#3 (b)

Design a two-pole band pass filter that has the center of its passband at $w = \pi/2$ zero in its frequency responce characteristic at $w = \pi$ and its magnitude responce is $1/\sqrt{2}$ at $w = 4\pi/9$.

solution:

clearly the filter must have

at

$$P_{1,2} = re^{\pm j\pi/2}$$

and zeros at $z = 1$ and $z = -1$

consiquently the system function is

$$H(z) = \frac{G\cdot(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= \frac{G\, z^2 - 1}{z^2 + r^2}$$

the value of r is determine by evaluating $H(w)$ at $w = 4\pi/9$. Thus we have

$$\left| H\left(\frac{4\pi}{9}\right) \right|^2 = \frac{(1-r^2)^2}{4} \frac{2 - 2\cos(8\pi/9)}{1 + r^4 + 2r^2\cos(8\pi/9)}$$

$$= \frac{1}{2}$$

or consiquently

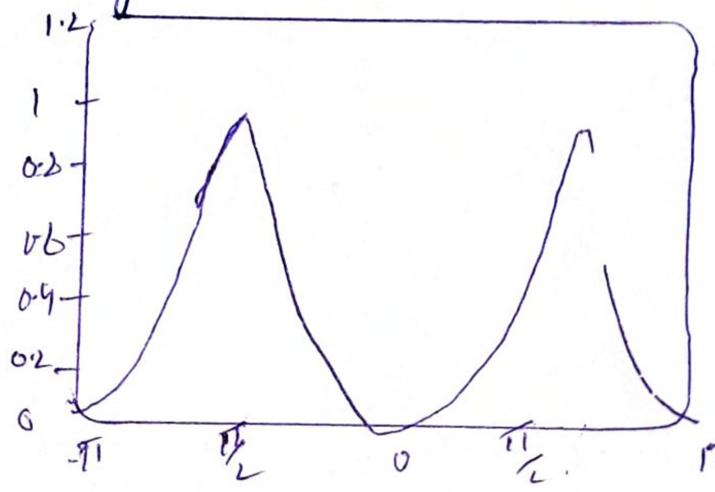
$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

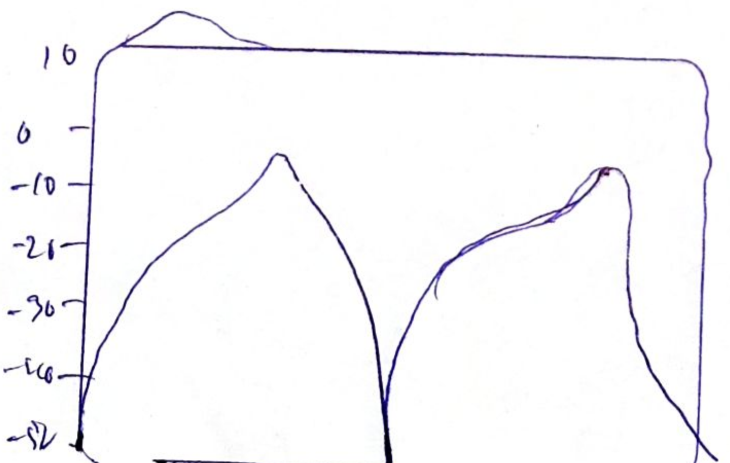
the value of $r^2 = 0.7$ satisfy the equation.

so the system function for the desired filter is:

$$H(z) = 0.15\frac{1-z^{-2}}{1+0.7z^{-2}}$$

so frequency respons illastrated in fig





Q#4(a) a finite duration sequence of length L is given as.

$$x(n) = \begin{cases} 1, & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases}$$

Determine the N-point of DFT of this sequence for $N \geq L$

Solution:

the fourier tranform of all this Sequence is.

$$X(w) = \sum_{n=0}^{L-1} x(n) e^{-jwn}$$

$$= \sum_{n=0}^{L-1} e^{-jwn} = \frac{1 - e^{-jwL}}{1 - e^{-jwL}}$$

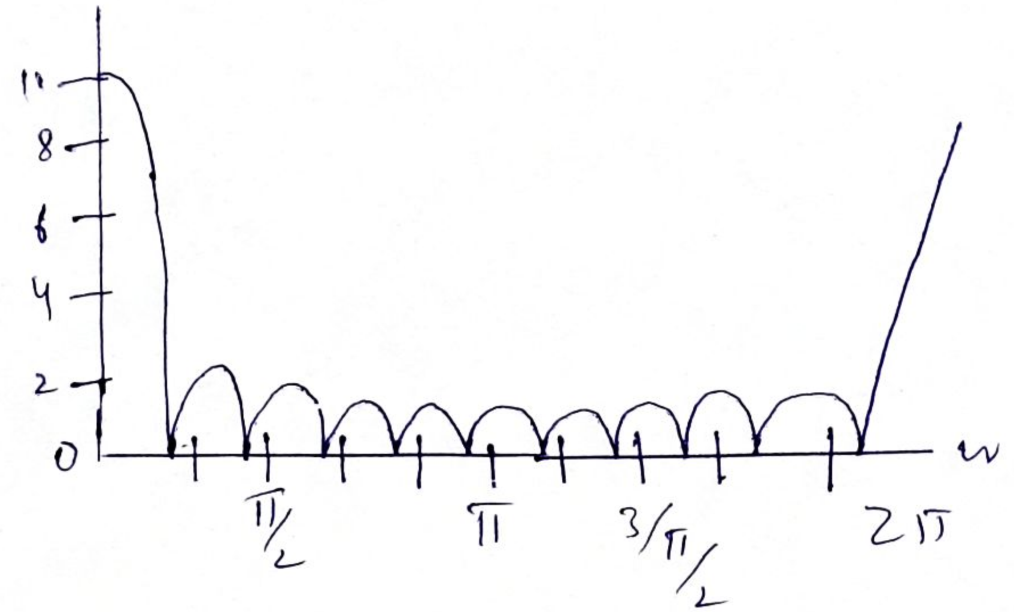
$$= \frac{\sin(wL/2)}{\sin(w/2)} e^{-jwL - n/2}$$

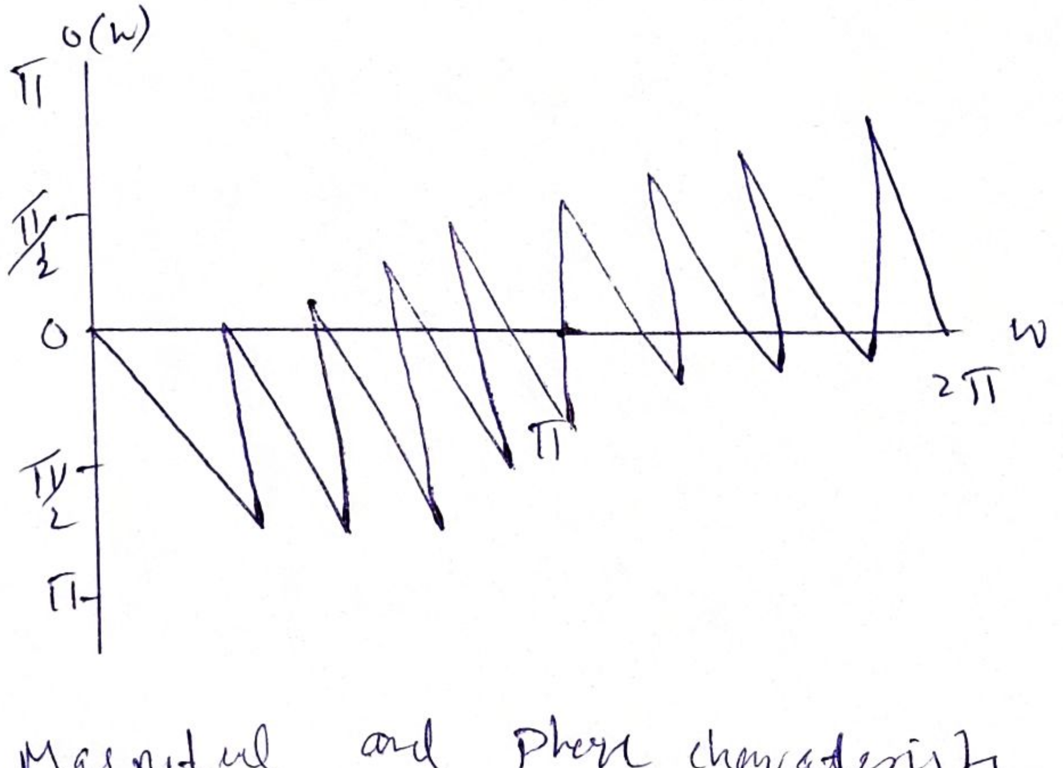
the magnitude of $x(w)$ and phase illustrated in fig for $L=10$ the N point DFT of $x(n)$ is simply $x(w)$ evaluated at the set of N equally spaced frequency $w_L = 2\pi K/N$, $K = 0, 1 \ldots N-1$

Hence

$$X(K) = \frac{1 - e^{-j 2\pi K L / N}}{1 - e^{-j 2\pi K/N}} \quad K = 0, 1 \ldots N-1$$

$$= \frac{\sin(\pi K L/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$





Magnitude and phase characteristics of fourier transform for signal x.

Q # 4 (b)

compute the DFT of four point sequence.

$$x(n) = (0, 1, 2, 3)$$

Solution:

the first is to Determine the matrix W_4. By exploiting the periodically property of W_4.

$$W_N^{K+N/2} = W_N^K$$

the matrix W_4 may be expressed as.

$$W_4 = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^4 & W_4^6 \\ W_4^0 & W_4^3 & W_4^6 & W_4^0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^1 & W_4^2 & W_4^3 \\ 1 & W_4^2 & W_4^0 & W_4^2 \\ 1 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix}$$

$$= \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{pmatrix}$$

then

$$X_4 = W_4 x_4 = \begin{pmatrix} 6 \\ -2+2j \\ -2 \\ -2-2j \end{pmatrix}$$

The IDFT of X_4 may be determine by conjugating the element in W_4 to obtaind W_4