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(1)

Q# 1 Ans:

Given:

$y[n] - 4y(n-1) + 4y(n+2) =$

$x[n] - x[n-1]$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$

$y_h[n] = C_1 2^n + C_2 n 2^n$

The particular solution is

$y_p(n) = K(-1)^n u[n]$

substituting this solution into the difference equation we obtain

$K[-1]^n u[n] - 4K(-1)^{n-1} u(n-1)$

$+ 4K(-1)^{n-2} u(n-2) =$

$(-1)^n u(n) - (-1)^{n-1} u(n-1).$

(1)

(2)

for $n-2$, $K(1+4+4)-2$

$K = 2/9$

$y(n) = [C_1 2^n + C_2 2^n + 2/9(-1)^n 4^n$

from the initial conditions:

we obtain $y(0)=1$, $y(1)=2$

Than

$C_1 + \frac{2}{9} = 1$

$C_1 = 7/9$

$2C_1 + 2C_2 - 2/9 = 2$

$\boxed{C_2 = 1/3}$

(2)

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Q # 1 (b)

Given:

$y(n) - 0.7y(n-1) + 0.1y(n-2) =$
$2x(n) - x(n-2)$

Solution:

$\lambda^2 - 0.7\lambda + 0.1 = 0$

$\lambda = 1/2, 1/5$

$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$

with

$x(n) = \delta(n)$ we have

$y(0) = 2$

$y(1) - 0.7(0) = 0$

$y(1) = 1.4$

Hence, $c_1 + c_2 = 2$

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(5)

And:

$1/2\, c_1 + 1/5 = 1.4$

$1.4 = 7/5$

$c_1 + \frac{2}{5} c_2 = 14/5$

These equations yield

$c_1 = \frac{10}{3}$, $c_2 = -4/3$

$h(n) = [\frac{10}{3} (1/2)^n - \frac{4}{3} (1/5)^n u(n)$

The step response is:

$s(n) = \frac{10}{3} \sum_{k=0}^{n} [(\frac{1}{2})^{n-k} - \frac{4}{3}$

$\sum_{k=0}^{n} (\frac{1}{5})^{n-k}$

$s(n) = \frac{10}{3} (\frac{1}{2})^n \sum_{k=0}^{n} 2^k - \frac{4}{3} (\frac{1}{5})^n$

(5)

$\sum_{k=0} 5^k$

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$$s(n) = \frac{10}{3}\left(\frac{1}{2}\right)^n \left(2^{n+1} - 1\right).$$

$$u(n) - 1/3$$

$$(1/5)^n (5^{n-1} - 1)(u(n)$$

" ———— " ———— "

Q# 2 (a):-

Determine causal signal $x[n]$ having Z-transform:

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Sol:

Taking inverse and Z transform:

$$\frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

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$A = 4, \; B = -3, \; C = -1$

$x(n) = (4(2)^n - 3 - n)\, u(n)$.

Q# 2(b):

Perform the circular convolution of following:

$x_1[n] = \{2, 1, 2, 1\}$.

$x_2(n] = \{1, 2, 3, 4\}$.

Sol:

Each sequence consist of 4 non-zero points for the purpose of illustrating the operations involved in circular convolution it is desired to graph each sequence as points on a circle Thus the sequence $x_1(n)$ and $x_2(n)$ are graphed as illustrated.

Now $x_3(n)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as specified begining with $m=0$ we have $x_3(m)$

$$x_3(m) = \sum_{n=0}^{3} x_1(n) x_2[(-n)]_N$$

$x_2(-n)_4$ is simply the sequence $x_2(n)$ folded and graphed on a circle

The product sequence is obtained by multiplying $x_1(n)$ with $x_2(-n)$ point by point. Finally we sum the values in product sequence

$$x_3(0) = 14$$

for $m=1$ we have:

$$x_3(1) = \sum_{n=0}^{3} x_1(n) x_2(1-n)_4$$

it is easily verified that

$x_2(1-n)_4$ is simply the sequence $x_2(-n)_4$ rotated counter clockwise by one unit in time.

This rotated sequence multiplies $x_1(n)$ to yield the product sequence also finally we sum the value in product sequence to obtain $x_3(1)$ Thus:

$x_3(1) = 16$

$n=2$ we have

$$x_3(2) \sum_{n=0}^{3} x_1(n) x_2(2-n)_4$$

Now $x_2(2-n)_4$ is the folded sequence.

(9)

$x_1(1) = 1$

$x_1(2) = 2 \quad x_1(n) : \quad -x_1(0) = 2$

$x_1(3) = 1$

$x_2(1) = 2$

$x_2(2) = 3 \quad x_2(n) : \quad -x_2(0) = 1$

$x_2(3) = 4$

(A)

$x_2(3)$

$x_2(2) = 3 \quad x_2(-n) 4 \quad x_2(0) = 1$

$x_2(1) = 2$

Folded sequence

(9)

(10)

$$4$$
$$6 - x_1(n)(-n)_4 - 2$$
$$2$$

(B)

product sequence

$x_2(0) = 1$

$x_2(3) = 4 - x_2[(1) - n)]_4 - x_2(1) = 2$

$x_2(2) = 3$

Folded sequence rotated by one unit in time

(c)

$$1$$
$$8 - x(n) x_2(1-n)_4 - 4$$
$$3$$

product sequence

(10)

$x_2(1) = 2$

$x_2(0) = 1 - x_2(2-n)_4 - x_2(2) = 3$

$x_2(3) = 4$

Folded sequence rotated by 2 unit in time

D)

$$2 - \underset{4}{\overset{2}{x_1(n)\, x_2(2-n)}} - 6$$

Product sequence

$x(2) = 4$

$x_2(1) = 2 - x_2(3-n)_4 - x_2(3) = 4$

$x_2(0) = 1$

Folded sequence rotated by 3 unit in time

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3

$4 - \{x_1 (n) x^2 (3-n)_4\} - 8$

1

product sequence

12

Q# 3 (a)

$$H_2 = \frac{b_0}{(1 - pz^{-1})^{+2}}$$

Determine b_a & b_p such that $H(w)$ satisfies the condition

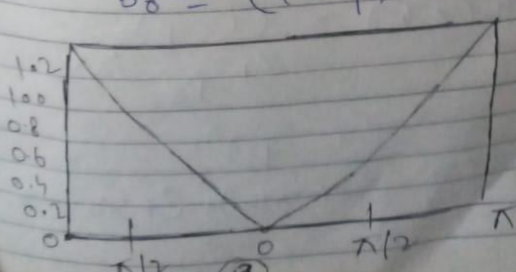
$$H_0 = \left| H \frac{\pi}{4} \right|^2 = 1/2$$

Sol:

At $w = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

$$b_0 = (1-p)^2$$



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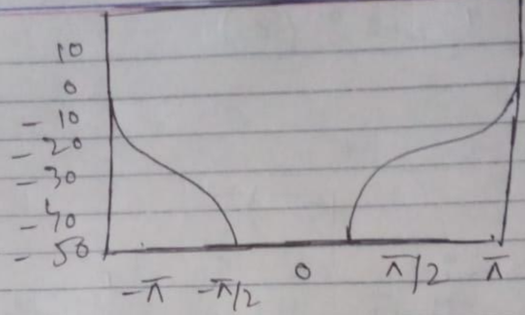
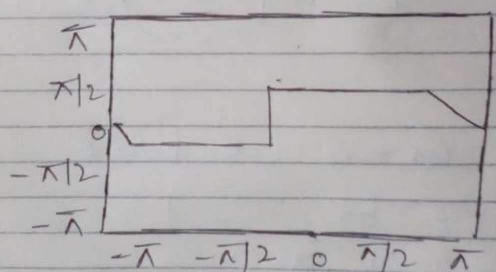


20log10 |H(w)|



$$w = \pi/4$$

$$H(\pi/4) = \frac{(1-p)^2}{(1-pe^{-i\pi/4})^2}$$

$$= \frac{(1-p)^2}{[1-p\cos(\pi/4)+ip\sin(\pi/4)^2}$$

(15)

15

$$= \frac{(1-p)^2}{[1-p\sqrt{2} + \dot{\imath}p/\sqrt{2})^2}$$

So:

$$= \frac{(1-p)^4}{[(1-p\sqrt{2})^2 + p^2/2]^2} = 1/2$$

equivalently:

$$\sqrt{2}(1-p)^2 = 1 + p^2 - \sqrt{2}p$$

The system function for desired filter

$$\boxed{H(z) = \frac{0.46}{(1-0.322z^{-1})^2}}$$

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Q # 3 (b). :-

Sol:

The filter must have poles at $p_{12} = re$

And zero at $z = 1$ and $z = -1$

(consequently the sum system function)

$$H(z) = \frac{G(z-1)(z+1)}{(z-ir)(z+ir)}$$

$$H(z) = \frac{G z^2 - 1}{z^2 + r^2}$$

The gain factor is determined evaluating the frequency response $H(\omega)$ of the filter at

$$\omega = \pi/2$$

$$H = (\pi/2) = G \frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(\omega)$ at

$$\omega = 4\pi/9$$

we have

$$\left| H(4\pi/9)^2 \right| = \frac{(1-r^2)^2}{4}$$

$$\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

Equivalently:

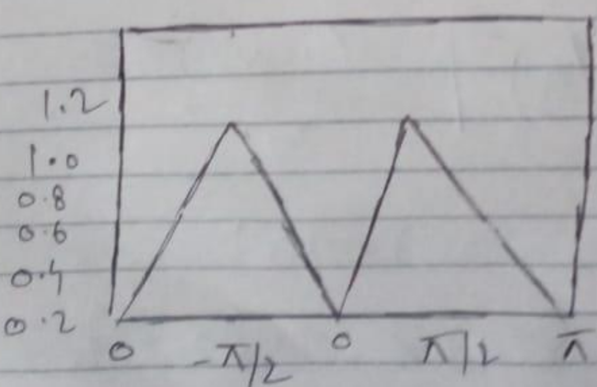
$1.94(1-r^2)^2 = 1 - 1.88r^2 + r^4$

The value of $r^2 = 0.7$

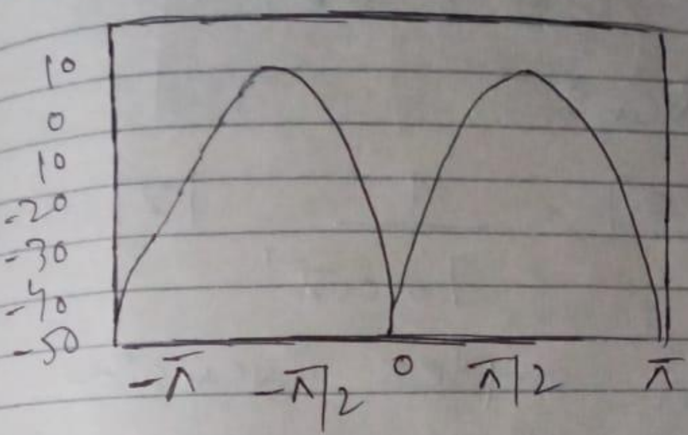
satisfies this equation therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1 - z^{-2}}{1 + 0.7z^{-2}}$$

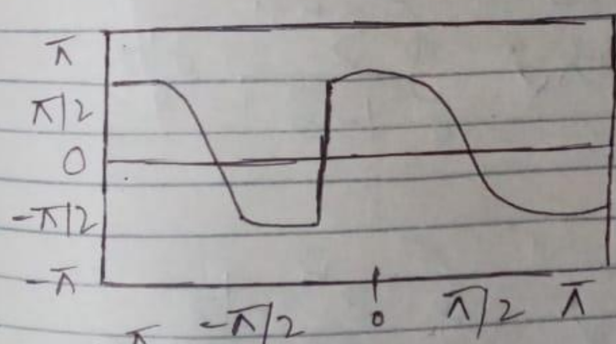
its frequency response is illustrated



$H\,|(\omega)|$



$20\log(H(w))$.



phase (radians).

megnitute and phase respone of a simple bandpass filter is

$$H(z) = 0.15[(1-z^{-2})/(1+0.7z^{-2})].$$

Q# 4 (b)

Given:

$$x(z) = \frac{1}{1-az^{-1}} \quad |z|>|a|$$

using complex inversion integral we have:

$$x(n) = \frac{1}{2\pi i} \oint_c \frac{z^{n-1}}{1-a^{z-1}} dz$$

$$= \frac{1}{2\pi i} \oint_c \frac{z^n \, dz}{z-a}$$

where c is circle at radius greater than (a) we shall evaluate this integral using (3,4,2) with

$f(z) = z^n$

we distinguish 2 cases.

1) if $n \geq 0$ $f(z) = z^n$

has an nth order

pole of $z=0$ which is also inside c Thus there are contribution from both poles for $n=1$ we have

$$x(-1) = \frac{1}{2\pi i} \oint \frac{1}{z^2(z-a)} dz$$

$$= \frac{d}{dz} ($$

$$= \frac{1}{z-a} \Big|_{z=0} + \frac{1}{z} \Big|_{z=a} = 0$$

if $n = -2$ we have:

$$x(-2) = \frac{1}{2\pi i} \oint \frac{1}{z^2(z-a)} dz$$

$$\frac{d}{dz}\left(\frac{1}{z-a}\right)\bigg|_{z=0} + \frac{1}{z^2}\bigg|_{z=a} = 0$$

By continuing in same way we can:

Show that $x(n) = 0$

for $n<0$ Thus

$$\boxed{x(n) = a^n u(n)}$$

Q# 4 (a)

$$x(n) = \begin{cases} 1 & 0 \le n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

Sol:

(2)

$$X(\omega) = \sum_{n=0}^{L-1} x(n) e^{-j\omega n}$$

$$X(\omega) = \sum_{n=0}^{L-1} e^{-j\omega n}$$

$$= \frac{1 - e^{j\omega L}}{1 - e^{-j\omega}}$$

$$= \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

The magnitude and phase of $X(\omega)$ are illustrated for $L=10$

The N point DFT of $x(n)$ is simply $X(\omega)$ evaluated at the set of N equally spaced frequency.

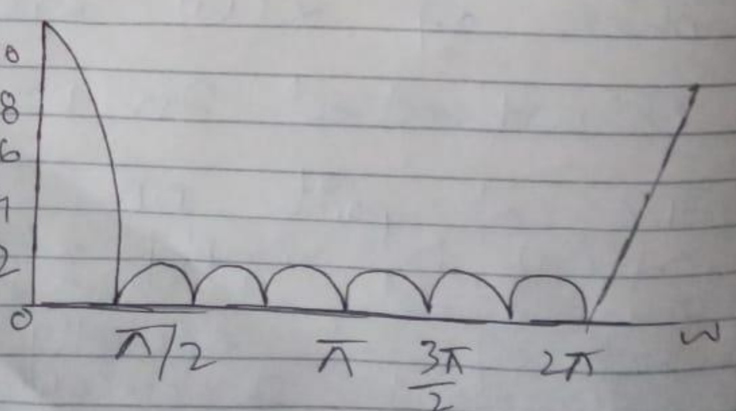
$\omega K = 2\pi K/n$

$K = 0, 1, 2 - - N-1$

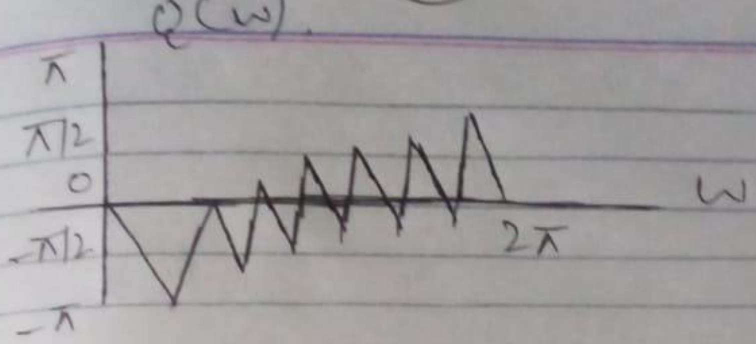
$$x(K) = \frac{1 - e^{-i2\pi KL/N}}{1 - e^{-i2\pi K/N}}$$

$K = 0, 1, - - N-1$

$$x(K) = \frac{\sin(\pi - KL/N)}{\sin(\pi K/N)} e^{-i\pi K(L-1)/N}$$



$|x(\omega)|$



if N is selected such that N = L

then discrete Fourier transform become:

$$X(K) = \begin{cases} L & K=0 \\ 0 & K=1,2,\ldots L-1 \end{cases}$$

Thus there is only one non-zero value

$X(w) = 0$

$2\pi K/L$

$K = 0.$

Q# 5 (a) Ans :-

i) **Minimum Sampling Rate:**

By sampling therem:

$F_1 = 100\,Hz \quad F_2 = 200\,Hz$

$F_s \geq F_{max}$

So:

F_2 is max (greater then F_1)

$F_s \geq 2 \times 100$

$\boxed{F_s = 200\,Hz}$

ii) As $F_s = 100\,Hz$

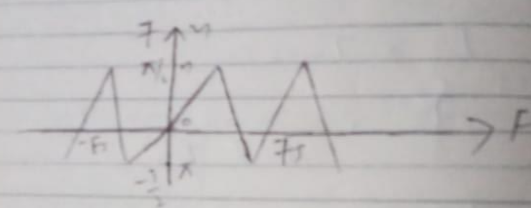
$F = \frac{100}{2} \Rightarrow \boxed{50\,Hz}$

This is the max frequency that can be represented by the sampled signal

As:

$X_a[n] = 3\cos 2\pi\left(\frac{50}{100}\right)n + 4\sin\left(\frac{100}{100}\right)n$

$= 3\cos\pi\left(\frac{5}{10}\right)n + 4\sin 2\pi n$



The effect of sampling rate on the newly generated discrete time signal is that That will be no dislay phenomenon mean There will not present unwanted component in Re reconstruct of signal The reconstruct original signal

iii) Folding frequency:

$\frac{F_1}{2} \Rightarrow 100/2$

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$f_1 = 50Hz$, $f_2 = 100Hz$

Both frequency are either equal or greater the folding frequency

Hence for ideal interpolation we can consider the original signal

$X_0(t) = 3\cos 100\pi + 4\sin 200\pi t$

Since only the frequency components at 100Hz are present on the sampled signal

$$\boxed{y_0(t) = 3\cos 100\pi t}$$

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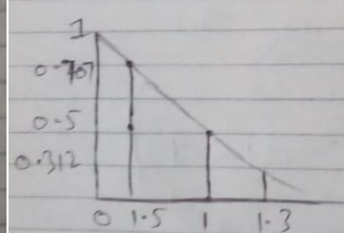
Q (8) 5 (b) Ans:

$$x(n) = \begin{cases} 0.5^n & , n > 0 \\ 0 & , n < 0 \end{cases}$$

$F_s = 2Hz$

$F_s = \frac{1}{T} \Rightarrow T = \frac{1}{F_s}$

$\Rightarrow \frac{1}{2} = 0.5sec$



X_n	0.5^n
0	1
0.5	0.707
1	0.5
1.5	0.3

(ii)

$L = 2^n$

$n = 3bit$

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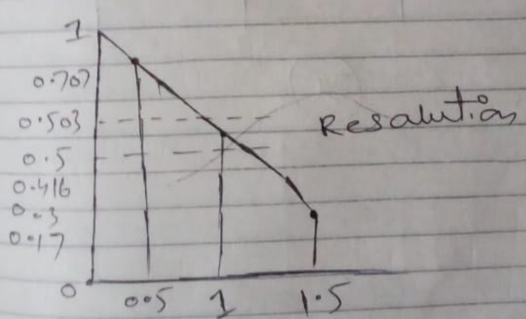
$L = 2^3 = 8$ Level

$$\text{Resolution} = \frac{X_{max} - X_{min}}{L}$$

$$= 1 - 0 / 8$$

$$\boxed{\text{Resolution} = 0.125}$$



iii)

Ds Iv signal	Direction	Reading	Error
1	1.0	1.0	0.0
0.85	0.8	0.9	-0.1
0.707	0.7	0.7	0.0
0.603	0.6	0.6	0.0
0.5	0.5	0.5	0.0
0.42	0.4	0.4	0.0
0.35	0.3	0.4	-0.1
0.17	0.1	0.1	-0.1