Abdul Aziz 13741

Differential Equations

Q1: $4y'' - 20y' + 20y = 0$

sol:

A second order homogeneous O.D.E has form of $ay'' + by' + cy = 0$

Now putting $y = e^{rt}$

$\Rightarrow \quad 4(e^{rt})'' - 20(e^{rt})' + 25(e^{rt}) = 0$

$\Rightarrow \quad 4 \cdot e^{rt}(4y^2 - 20y + 25) = 0$

Using quadratic formula

$a = 4, \quad b = -20, \quad c = 25$

$$y = \frac{-(20) \pm \sqrt{(-20)^2 - 4(4)(25)}}{2(4)}$$

$$y = \frac{20 \pm \sqrt{400 - 400}}{8}$$

$$y = \frac{20 \pm \sqrt{0}}{8}$$

$$y = \frac{\cancel{20}^{5}}{\cancel{8}_{2}} \Rightarrow y = 5/8$$

$\Rightarrow \quad y = e^{rt} \Rightarrow 4r^2 - 20r + 25 = 0$

$= \; ^5/_2$

$\therefore \; y = C_1 y_1 + C_2 y_2 = C_1 e^{5/2 t} + C_2 t e^{5/2 t}$

$\Rightarrow \; y' = \frac{5}{2} C_1 e^{5/2 t} + C_2 \left(e^{5/2 t} + \frac{5}{2} t e^{5/2 t} \right)$

Q2 (b) (b)

$x^2 y'' + 3xy' + y = 0$

sol:- put

$y = x^r$

$\Rightarrow \; x^2 (x^r)'' + 3x (x^r)' + x = 0$

$\Rightarrow \; x^r (x^2 + 12r + 1) = 0$

Using quadratic formula

$a = 1 \quad b = 2 \, , \; c = 1$

$$x = \frac{-(+2) \pm \sqrt{(2)^2 - 4(1)(1)}}{2(1)}$$

$$\Rightarrow \; x = \frac{-2 \pm \sqrt{4 - 4}}{2}$$

$$x = \frac{-2}{2} \Rightarrow \boxed{x = -1}$$

$$\Rightarrow \quad y = c_1 x^r + c_2 \ln(x) x^r$$

$$\Rightarrow \quad c_1 x^{-1} + c_2 \ln(x) x^r$$

$$\Rightarrow \quad c_1 x^{-1} + c_2 \ln(x) x^{-1}$$

$$\Rightarrow \quad y = \frac{c_1}{x} + \frac{c_2 \ln(x)}{x} \quad \text{Ans.}$$

Part (a)

$$y'' + 2y' + y = 0 \; ; \; y(0) = 4, \; y'(0) = -6$$

$$\Rightarrow \frac{d^2}{dx^2} y(x) + 2 \frac{d}{dx} 8y(x) + y(x) = 0$$

$$\int \frac{d^2}{dx^2} y(x) + 2 \int \frac{d}{dx} \cancel{8}(x) + \int y(x) = 0$$

$$y'(x) + 2y(x) + \frac{y^2(x)}{2} = 0$$

$$-6 + 2(4) + \frac{y^2}{2} = 0$$

$$-6+8+\frac{y^2}{2}=0$$

$$2+\frac{y^2}{2}=0$$

$$\frac{y^2}{2}=-2$$

$$\sqrt{y^2}=\sqrt{-4}$$

$$\boxed{y=2i}$$

Q:3 Sol:-

For Homo- eq

$$y''+y'-6y=0$$

Auxilary equation

$$\lambda^2+\lambda+6y=0$$
$$\lambda^2+\lambda-6=0$$
$$\lambda^2+3\lambda-2\lambda-6=0$$
$$(\lambda+3)-2(\lambda+3)=0$$

$$\lambda+3=0,\ \lambda-2=0$$

$$\lambda=-3\ ,\ \lambda=-2$$

Roots are real and distant

$y = c_1 e^{-3x} + c_2 e^{2x}$

Choice for y

$y = K_1 x^3 + K_2 x^2 + K_1 x + K_0$

$y_p' = 3K_3 x^2 + 2K_2 x + K_0$

$y_p'' = 6K_3 x - 2K_2$

Put in 2

$6K_3 x - 2K_2 - 3K_3 x^2 + 2K_2 x + K_1 - 6K_3 x^3 - 6K_2 x^2 - 6K_1 x - 6K_0$
$6x^3 - 3x^2 + 12x$

$-6K_3 = 6$	$-6K_2 + 3K_1 = -3$	$6K_3 x + 2K_2 + K_1 = 12x$
$\boxed{K_3 = -1}$	$-6K_2 + 3(-1) = -3$	$6(-1) + 2(0) + K_1 = 12$
	$-6K_2 - 3 = -3$	
	$K_2 = 0$	$K_1 = -2$

$-2K_1 + K_1 + K_0 = 0$

$-2(0) - 2 + 14 = 0$

$\boxed{K_0 = 2}$

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Q.4 sol: $y'' - 4y' + 4y = x^2 e^{2x}$

for equation

$y'' - 4y' + 4y = 0$

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$

$\lambda(\lambda - 2) - 2(\lambda - 2) = 0$

$(\lambda - 2)(\lambda - 2) = 0$

$\lambda = 2, \lambda = 2$

Roots are real & equal

$y = (c_1 + c_2 x) e^{2x}$

$y_1 = e^{3x}, \quad y_1 = x e^{2x}$

$y_1' = 2e^{2x}, \quad y_1' = e^{2x} + 2x e^{2x}$

Q.5 $\quad e^{\sqrt{3}x}$, $xe^{\sqrt{3}x}$

$$y_1 = e^{\sqrt{3}x} , \quad y_2 = xe^{\sqrt{3}x}$$

$$y = c_1 e^{\sqrt{3}x} + c_2 x e^{\sqrt{3}x}$$

$$\lambda_1 = \sqrt{3} , \quad \lambda_2 = \sqrt{3}$$

$$\lambda - \sqrt{3} = 0 , \quad \lambda = \sqrt{3} = 0$$

$$(\lambda \sqrt{3})(\lambda - \sqrt{3}) = 0$$

$$\lambda^2 \sqrt{3}\lambda - \sqrt{3} + 3 = 0$$

$$\lambda^2 - 2\sqrt{3}\lambda + 3 = 0$$

$$a = -2\sqrt{3}\lambda ; \quad b = 3$$

Have ODE $\quad y'' + ay' + b = 0$

$$y'' + ay' + by = 0$$

$$y'' - 2\sqrt{3}\lambda y' + 3y = 0$$

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