Department of Electrical Engineering
Final Exam Assignment
Date: 27/06/2020

Course Details

Course Title:	Digital Signal Processing	**Module:**	6th
Instructor:	Engr Phir Mehar Ali Shah	**Total Marks:**	50

Student Details

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Q1.	(a)	Determine the response y(n), n ≥ 0, of the system described by the second order difference equation $$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1)$$ To the input $x(n) = (-1)^n u(n)$. And the initial conditions are y (-1) = y (-2) = 0.	**Marks 7** **CLO 2**
	(b)	Determine the impulse response and unit step response of the systems described by the difference equation. $$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2)$$	**Marks 7** **CLO 2**
Q2.	(a)	Determine the causal signal x(n) having the z-transform $$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$ (Hint: Take inverse z-transform using partial fraction method)	**Marks 6** **CLO 2**
	(b)	Evaluate the inverse z- transform using the complex inversion integral $$X(z) = \frac{1}{1-az^{-1}} \qquad \lvert z\rvert > \lvert a\rvert$$	**Marks 6** **CLO 2**
Q.3	(a)	A two- pole low pass filter has the system response $$H(z) = \frac{b_o}{(1-pz^{-1})^2}$$ Determine the values of b_o and p such that the frequency response H(ω) satisfies the condition H(0) = 1 and $\left\lvert H(\frac{\pi}{4})\right\rvert^2 = \frac{1}{2}$.	**Marks 6** **CLO 3**

<table>
<tr><td rowspan="2"></td><td rowspan="2">(b)</td><td rowspan="2">Design a two-pole bandpass filter that has the center of its passband at ω = π/2, zero in its frequency response characteristics at ω = 0 and ω = π and its magnitude response in $\frac{1}{\sqrt{2}}$ at ω = 4π/9.</td><td>Marks
6</td></tr>
<tr><td>CLO
3</td></tr>
<tr><td rowspan="4">Q 4</td><td rowspan="2">(a)</td><td rowspan="2">A finite duration sequence of Length L is given as

$$x(n) = \begin{cases} 1, & 0 \le n \le L-1 \\ 0, & otherwise \end{cases}$$
Determine the N- point DFT of this sequence for N ≥ L</td><td>Marks
6</td></tr>
<tr><td>CLO
2</td></tr>
<tr><td rowspan="2">(b)</td><td rowspan="2">Perform the circular convolution of the following two sequences. Solve the problem step by step

$$x_1(n) = \left\{ \underset{\uparrow}{2}, 1, 2, 1 \right\}$$
$$x_2(n) = \{ \underset{\uparrow}{1}, 2, 3, 4\}$$</td><td>Marks
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<tr><td>CLO
2</td></tr>
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Q1 (a)

$$y(n) - 4y(n-1) + 4y(n-2) = x(n) - x(n-1).$$

soln:

$$y_h(n) = \lambda^n$$

$$\lambda^n - 4\lambda^{n-1} + 4\lambda^{n-2} = 0$$

$$\lambda^{n-2}(\lambda^2 - 4\lambda + 4) = 0$$

~~The~~ $\lambda^2 - 4\lambda + 4 = 0$

The Roots are $\lambda = 2, 2$ Hence.

$$y_h(n) = C_1\lambda_1^n + C_2\lambda_2^n$$

$$= C_1(2)^n + C_2(2)^n$$

$$y_h(n) = C_1 2^n + C_2 n 2^n$$

The particular solution is.

$$y_p(n) = K(-1)^n u(n).$$

substituting this solution into the difference equation. we obtain.

$$K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$$

For $n = 2$, $K(1+4+4) = 2$.

$$\boxed{K = 2/9}$$

The total solution is.

$$y(n) = \left[c_1 2^n + c_2 n 2^n + \frac{2}{9}(-1)^n\right] u(n)$$

From the initial conditions we obtain.

$y(0) = 1$ and $y(1) = 2$. Therefore.

$$c_1 + \frac{2}{9} = 1$$

$$\Rightarrow \boxed{c_1 = \frac{7}{9}}$$

$$2c_1 + 2c_2 - \frac{2}{9} = 2$$

$$\boxed{c_2 = 1/3}$$

$$y_h(n) = \left[\frac{7}{9} y(-1) + \frac{2}{9} y(-2)\right](-1)^n.$$

$$+ \left[1/3\, y(-1) + \frac{2}{9} y(-2)\right](-1)^n \quad n \geq 0.$$

so

$$c_1 = 7/9 \quad \text{and} \quad c_2 = 1/3.$$

$$y_h(n) = (-1)^{n+1} + (1)^{n+2} \quad n \geq 0.$$

Q1 (b) Determine the impulse response of the systems described by the difference equation.

$$y(n) - 0.7y(n-1) + 0.1y(n-2) = 2x(n) - x(n-2).$$

Soln: The characteristic equation is.

$$y_h(n) = \lambda^n$$

$$\lambda^n - 0.7\lambda^{n-1} + 0.1\lambda^{n-2} = 0$$

$$\lambda^2 - 0.7\lambda + 0.1 = 0$$

$\lambda = \frac{1}{2}$ and $\frac{1}{5}$ Hence.

$$y_h(n) = c_1 \lambda_1^n + c_2 \lambda_2^n$$

$$= c_1 (\tfrac{1}{2})^n + c_2 (\tfrac{1}{5})^n$$

$$y_h(n) = c_1 \frac{1}{2}^n + c_2 \frac{1}{5}^n$$

with $x(n) = \delta(n)$ we have.

$$y(0) = 2$$

$$y(1) - 0.7\, y(0) = 0$$

$$\boxed{y(1) = 1.4}$$

Hence $c_1 + c_2 = 2$

$$\frac{1}{2} c_1 + \frac{1}{5} = 1.4 = 7/5$$

$$c_1 + \frac{2}{5} c_2 = \frac{14}{5}$$

These equation yileds.

$$c_1 = \frac{10}{3}, \quad c_2 = -\frac{4}{3}$$

$$h(n) = \left[\frac{10}{3} (\tfrac{1}{2})^n - \frac{4}{3} (\tfrac{1}{5})^n \right] u(n).$$

The step response is.

$$s(n) = \sum_{k=0}^{n} h(n-k)$$

$$= \frac{10}{3} \sum_{k=0}^{n} \left(\frac{1}{2}\right)^{n-k} - \frac{4}{3} \sum_{k=0}^{n} \left(\frac{1}{5}\right)^{n-k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} \sum_{k=0}^{n} 2^{k} - \frac{4}{3} \left(\frac{1}{5}\right)^{n} \sum_{k=0}^{n} 5^{k}$$

$$= \frac{10}{3} \left(\frac{1}{2}\right)^{n} (2^{n+1} - 1) u(n) - \frac{1}{3} \left(\frac{1}{5}\right)^{n} (5^{n+1} - 1) u(n).$$

22 (a) Determine the casual signal $x(n)$ having Z-Transform.

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

Soln

$$x(z) = \frac{1}{(1-2z^{-1})(1-z^{-1})^2}$$

By partial fraction method.

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$$\frac{1}{(1-2z^{-1})(1-z^{-1})^2} = \frac{A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1})}{(1-2z^{-1})(1-z^{-1})^2}$$

$$1 = A(1-z^{-1})^2 + B(1-2z^{-1})(1-z^{-1}) + Cz^{-1}(1-2z^{-1}) \quad \text{——— } (1)$$

put $z = 1$

$$1 = A(1-0)^2 + B(1-2)(1-1) + C(1)(1-2)$$

$$1 = 0 + 0 - C$$

$$1 = -C$$

$$\boxed{C = -1}$$

put $z = 2$ in eq ①

$$1 = A\left(1-\frac{1}{2}\right)^2 + B\left(1-\frac{2}{2}\right)\left(1-\frac{1}{2}\right) + C\left(\frac{1}{2}\right)\left(1-\frac{2}{2}\right)$$

$$1 = A\left(\frac{1}{2}\right)^4 + B(1-1)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(1-1)$$

$$1 = \frac{A}{4} + B(0)\left(\frac{1}{2}\right) + C\left(\frac{1}{2}\right)(0)$$

So,

$$1 = \frac{A}{4} + 0 + 0$$

$$\boxed{A = 4}$$

put $z=3$ in eq ①

$$1 = A\left(1-\frac{1}{3}\right)^2 + B\left(1-\frac{2}{3}\right)\left(1-\frac{1}{3}\right) + C\left(\frac{1}{3}\right)\left(1-\frac{2}{3}\right)$$

$$1 = A\left(\frac{4}{9}\right) + B\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) + C(1/3)(1/3)$$

$$1 = \frac{4A}{9} + \frac{2}{9}B \cdot \frac{1}{9}C$$

$$1 + \frac{1}{4} - \frac{16}{9} = \frac{2}{9}B$$

$$-\frac{6}{94} \times \frac{9}{2} = B$$

$$\frac{12}{36} = B$$

$$\boxed{B = 3}$$

Hence $x(n) = [\,4(2)^n - 3 - n\,]\,u(n)$.

22 (b) Evaluate inverse Z-Transform using the complex inversion signal.

$$X(Z) = \frac{1}{1-aZ^{-1}} \quad |Z| > |a|.$$

Soln: we have.

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{Z^{n-1}}{1-aZ^{-1}} dZ = \frac{1}{2\pi j} \oint_c \frac{Z^n dZ}{Z-a}$$

where C is a circle at radius greater than $|a|$. we shall avaluate this integral using with $f(Z) = Z^n$. we distinguish two cases.

1. If $n \geq 0$, $f(Z)$ has only zeros and hence no poles inside C. The only pole inside C is $Z = a$.

$$x(n) = f(Z_0) = a^n \quad n \geq 0.$$

2. If $n < 0$, $f(Z) = Z^n$ has an nth-order poles at $Z=0$ which is also inside C. Thus there are contributions from both poles.. For $n = -1$ we have.

$$x(-1) = \frac{1}{2\pi j}\oint_C \frac{1}{z(z-a)}\,dz = \frac{1}{z-a}\Big|_{z=0} + \frac{1}{z}\Big|_{z=a} = 0$$

If $n = -2$ we have.

$$x(-2) = \frac{1}{2\pi j}\oint_C \frac{1}{z^2(z-a)}\,dz = \frac{d}{dz}\left(\frac{1}{z-a}\right)$$

$$\Big|_{z=0} + \frac{1}{z^2}\Big|_{z=a} = 0$$

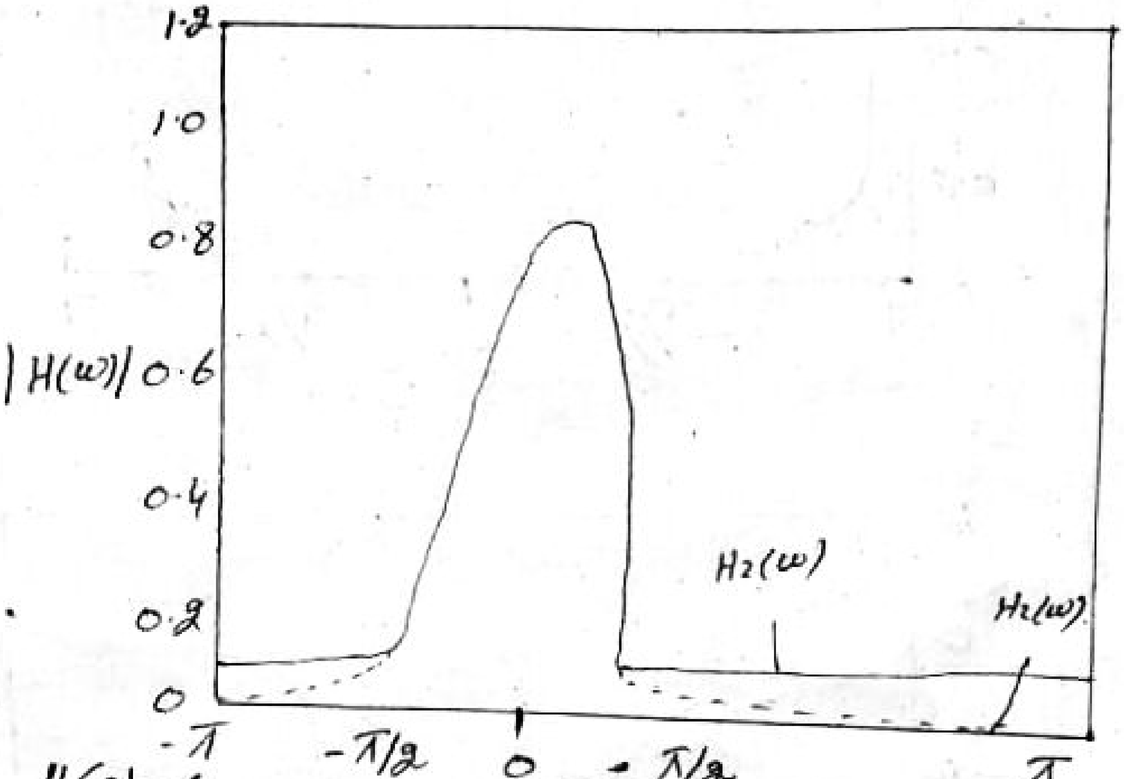
By continuing in the same way we ~~@~~ can show that $x(n) = 0$ for $n<0$. Thus.

$$x(n) = a^n u(n).$$

Q3
(a)

$$H(z) = \frac{b_0}{(1-pz^{-1})^2}$$

Determine the values of b_0 p such that the frequency response $H(\omega)$ satisfies the condition $H(0) = 1$ and $|H(\frac{\pi}{4})|_2 = \frac{1}{2}$



Sol/n

$H(0) = 1$ and $|H(\frac{\pi}{4})|^2 = 1/2$.

At $\omega = 0$ we have

$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$

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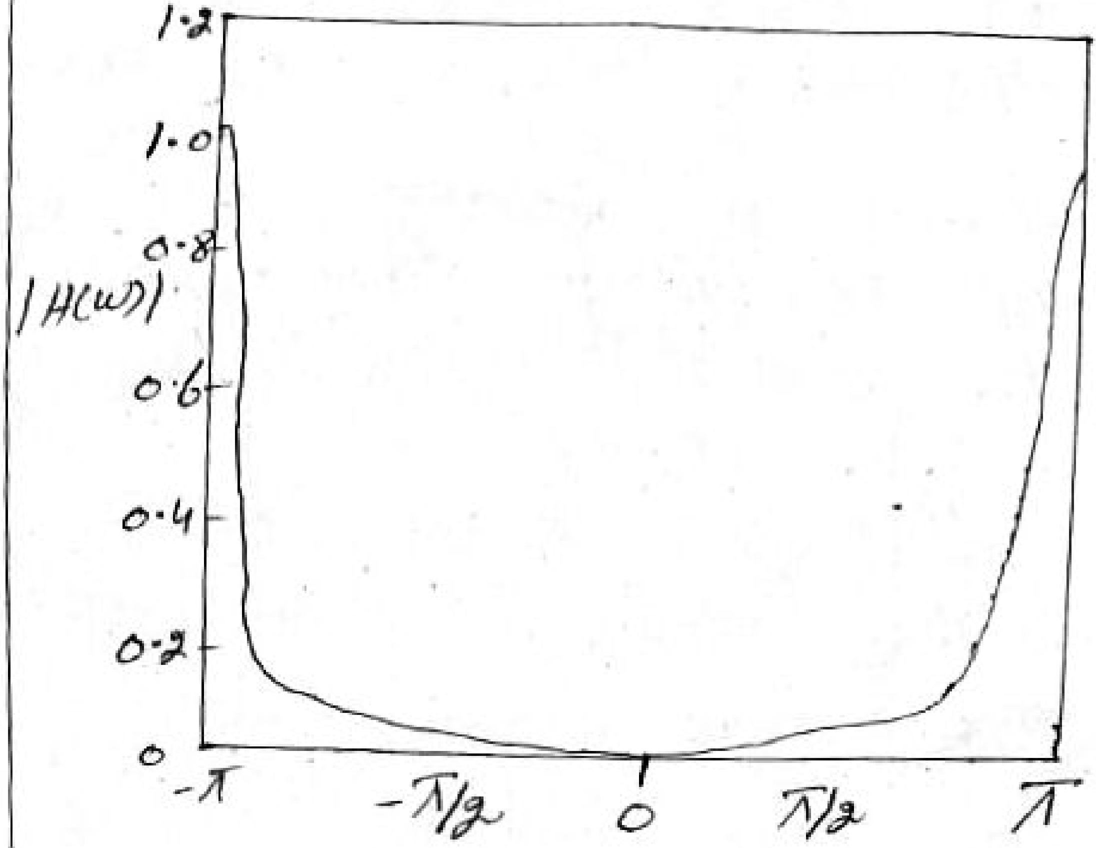
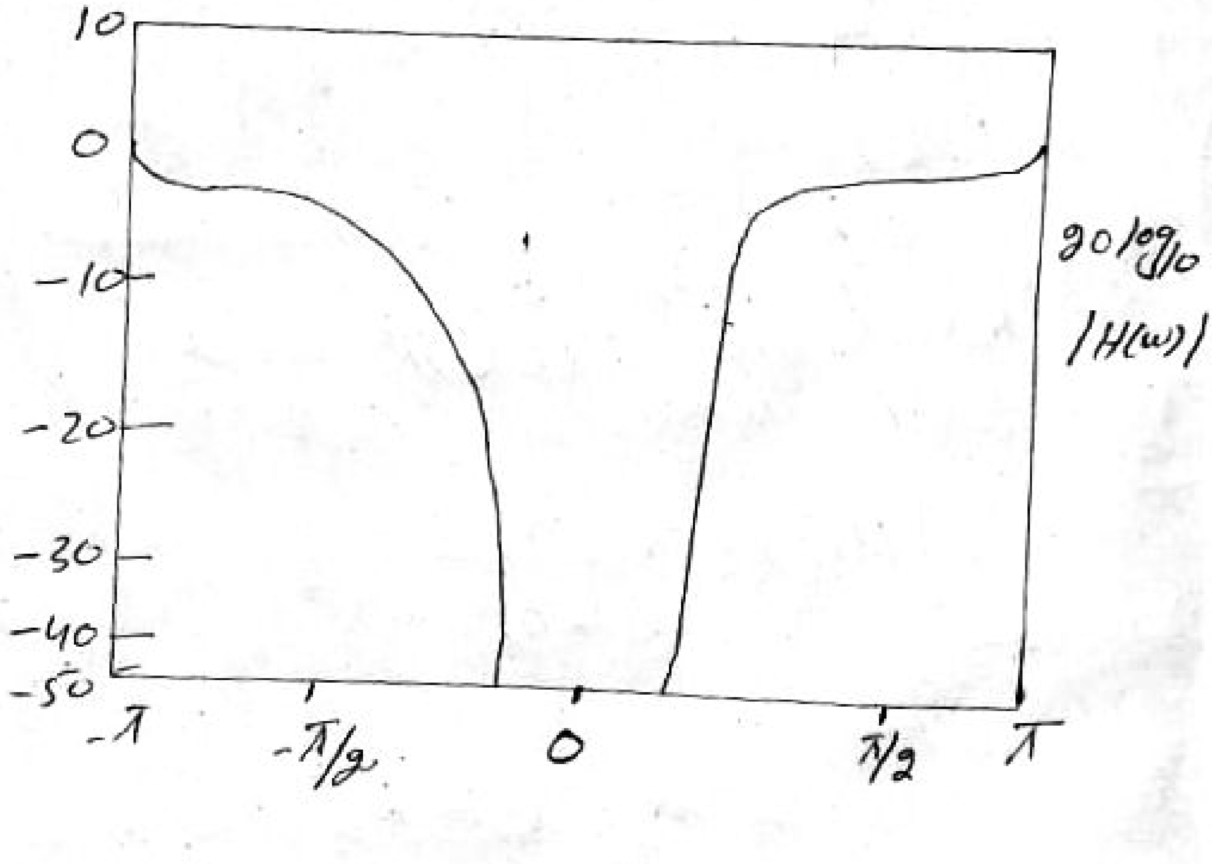




At $w = \pi/4$

$$H(\pi/4) = \frac{(1-P)^2}{(1-Pe^{-j\pi/4})^2}$$

$$= \frac{(1-P)^2}{(1-P\cos(\pi/4) + jP\sin(\pi/4))^2}$$

$$= \frac{(1-P)^2}{(1-P/\sqrt{2} + jP/\sqrt{2})^2}$$

Hence

$$\frac{(1-P)^4}{\left[(1-P/\sqrt{2})^2 + P^2/2\right]^2} = 1/2$$

equivalently

$$= \sqrt{2}\,(1-P)^2 = 1 + P^2 - \sqrt{2}\,P.$$

The value of $P = 0.32$ satisfies this equation.

Consequently the system function for the desired filter is

$$H(z) = \frac{0.46}{(1-0.32z^{-1})^2}.$$

The same principles can applied for the design of bandpass filters. Basically, the bandpass filters should contain one or more pairs of complex-conjugate poles near the unit circle in the vicinity of the frequency band that constitutes the passband of the filter.

Q3 (b) Design a two-pole bandpass filter that has the center at $w = \pi/2$ and frequency response at $w=0$ and $w=\pi$ and response is $1/\sqrt{2}$ at $w = 4\pi/9$.

Sol: clearly, the filter must have poles at.

$$P_{1.2} = re^{\pm j\pi/2}.$$

and zeros at $z=1$ and $z=-1$.

consequently the system function is.

$$H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$= G\frac{z^2-1}{z^2+r^2}$$

The gain factor is determined by evaluating the frequency response $H(w)$ of the filter at $w=\pi/2$. Thus we have.

$$H\left(\frac{\pi}{2}\right) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(w)$ at $w = 4\pi/9$.

Thus we have.

$$|H(\frac{4\pi}{9})|^2 = \frac{(1-r^2)^2}{4} \frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

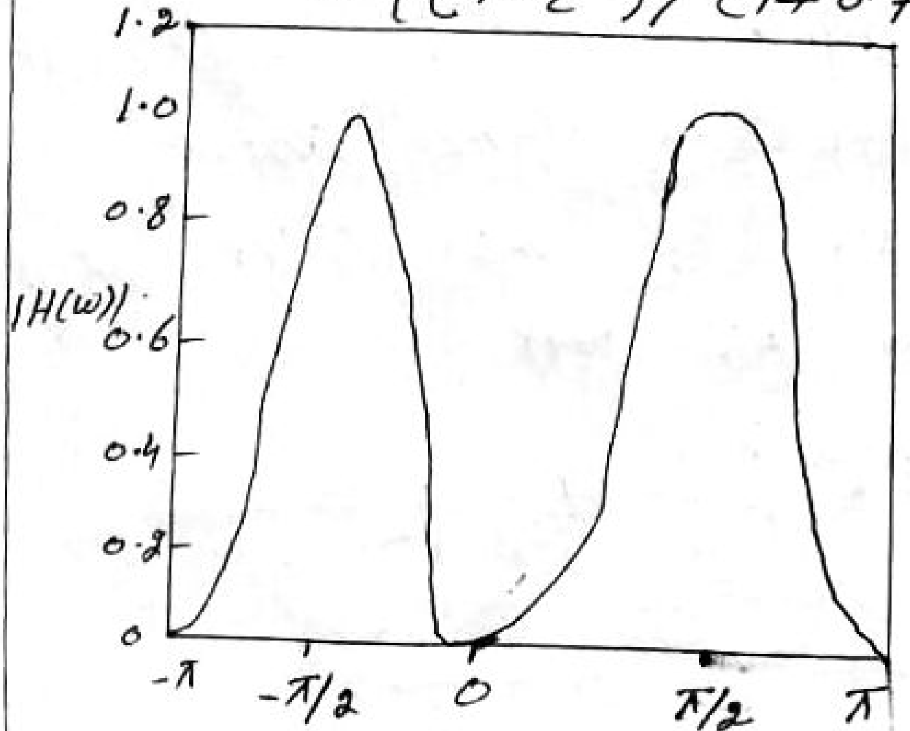
or equivalently.

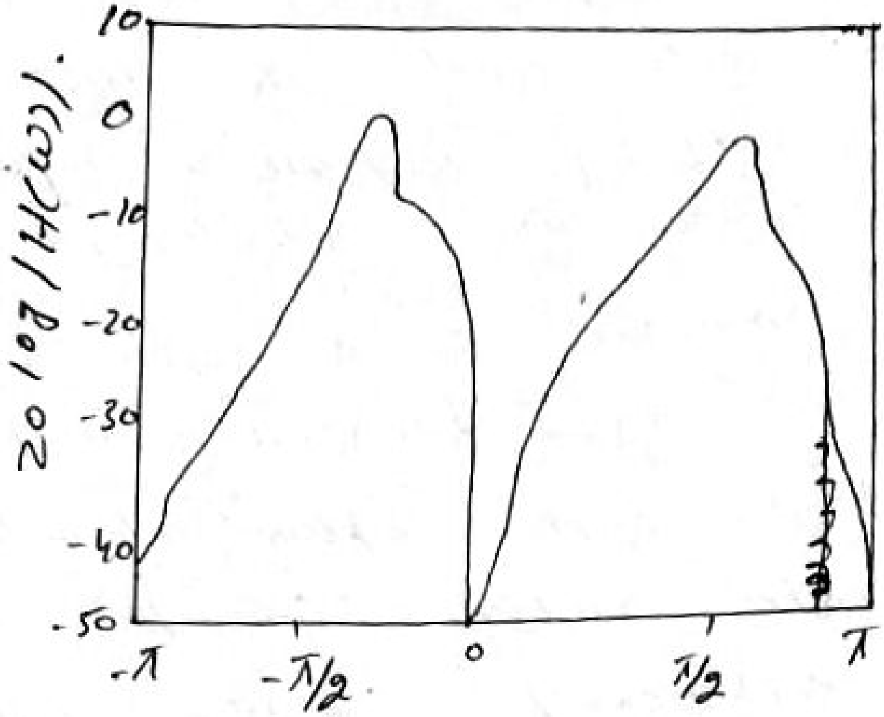
$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

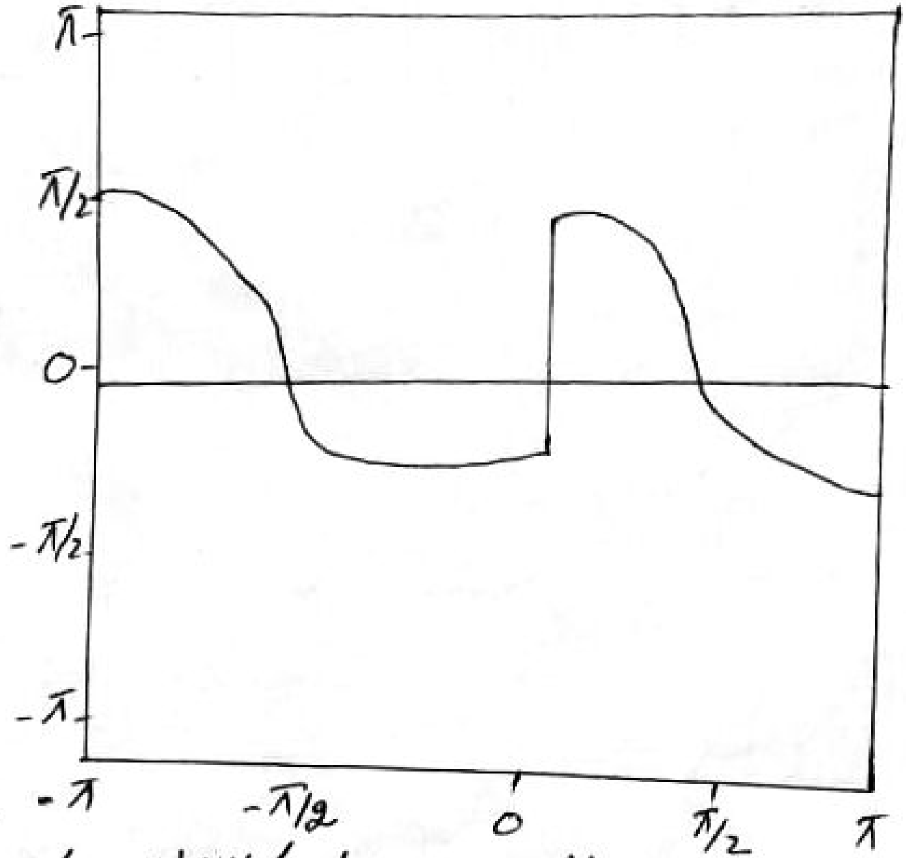
The value of $r^2 = 0.7$ satisfies this equation. Therefore the system function for the desired filter is

$$H(z) = 0.15 \frac{1-z^{-2}}{1+0.7z^{-2}}$$

$$H(z) = 0.15\left[(1-z^{-2})/(1+0.7z^{-2})\right].$$







It should be emphasized that the main purpose of the foregoing methodology for designing simple digital filters by pole-zero placement is to provide insight into the effect that poles and zeros have on the frequency response characteristics of systems. The methodology is not intended a a good method for designing filters with well-specified passband and stopband characteristics. systematic methods for the design of sophisticated digital filters for practical applications.

Q4 (a) A finite duration sequence of length L is given as

$$x(n) = \begin{cases} 1 & 0 < n \le L-1 \\ 0 & \text{otherwise} \end{cases}$$

N-point DFT of this sequence is for $N \ge L$.

Sol/n: The fourier Transform of this sequence is.

$$X(w) = \sum_{n=0}^{L-1} x(n)\, \bar{e}^{\,jwn}$$

$$= \sum_{n=0}^{L-1} e^{-jwn} = \frac{1-e^{-jwL}}{1-\bar{e}^{\,jw}}$$

$$= \frac{\sin(wL/2)}{\sin(w/2)} \cdot e^{-jw(L-1)/2}$$

The magnitude and phase of $X(w)$ are illustrated for $L=10$. The N-point DFT of $x(n)$ is simply $X(w)$ evaluated at the set of N equally spaced frequencies.

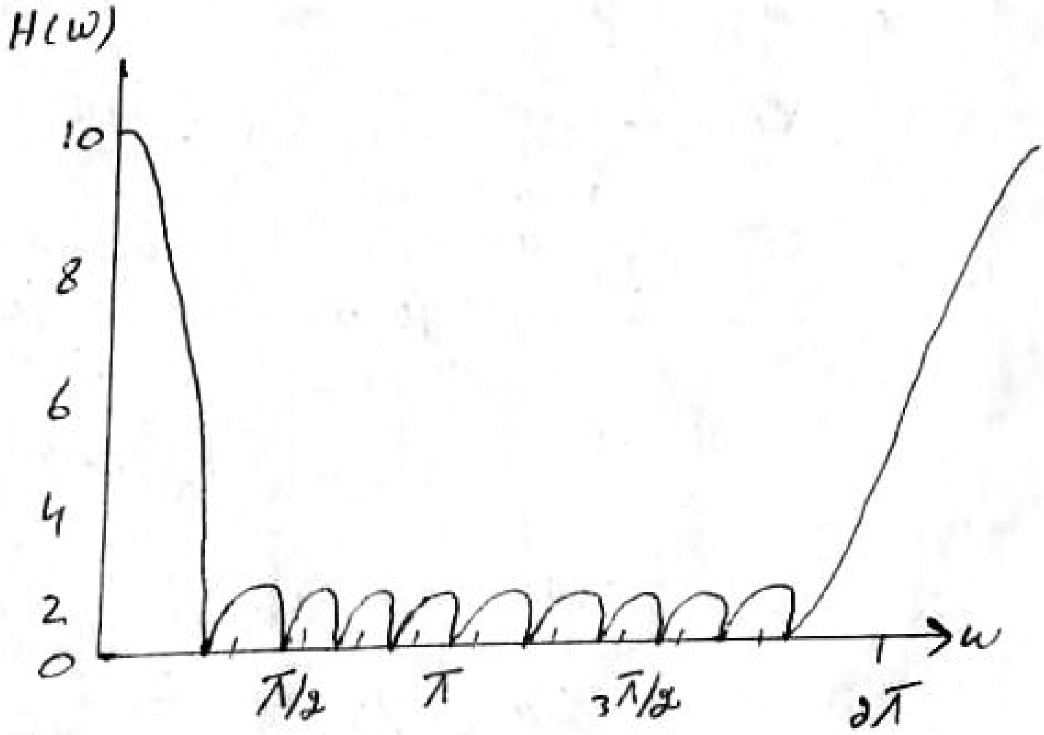
so,

$$w_K = 2\pi K/N, \quad K = 0, 1 \ldots N-1.$$

Hence.

$$X(K) = \frac{1 - e^{-j2\pi KL/N}}{1 - e^{-j2\pi K/N}} \quad K = 0, 1 \dots N-1$$

$$\frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$



If N is selected such that $N = L$ then the DFT becomes

$$X(K) = \begin{cases} L & K = 0 \\ 0 & K = 1, 2 \dots L-1 \end{cases}$$

Thus there is only one non-zero value in the DFT.

since $X(\omega) = 0$ at the frequencies $\omega_K = 2\pi K/L$, $K \neq 0$.

The reader should verify that $x(n)$ can be recovered from $X(k)$ by performing an L-point IDFT.
In magnitude and phase for $L=10$, $N=50$ and $N=100$
as one will conclude by comparing these spectra with the continious spectrum $X(\omega)$.

Q4 (b) perform the cirular convolution of the following two sequences.

$$x_1(n) = \{ \underset{\uparrow}{2}, 1, 2, 1 \}$$

$$x_2(n) = \{ \underset{\uparrow}{1}, 2, 3, 4 \}$$

Soln: ~~Each~~ Each sequence consists of four nonzero points. For the purpose of illustrating the operations involved in circular convolution, it is desirable to graph each sequence as point on circle.

Thus the sequence $x_1(n)$ and $x_2(n)$ are graphed as. we note the ~~sequences $x_1(n)$ and $x_2(n)$ are~~ sequences are graped in counterclock-wise direction on a circle This establishes the reference direction in rotating one of the sequence relative to the other.

Now $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as specified by. Begining $m=0$ we have.

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2((-n))_4$$

The product sequence is obtained by multiplying $x_1(n)$ with $x_2((-n))_4$ point by point. Finally we sum the values in the product sequence to obtain.

$$x_3(0) = 14$$

From $m=1$ we have.

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2((1-n))_4$$

verified that $x_2((1-n))_4$ is simply the sequence $x_2((-n))_4$ rotated counterclock wise by one unit in time.

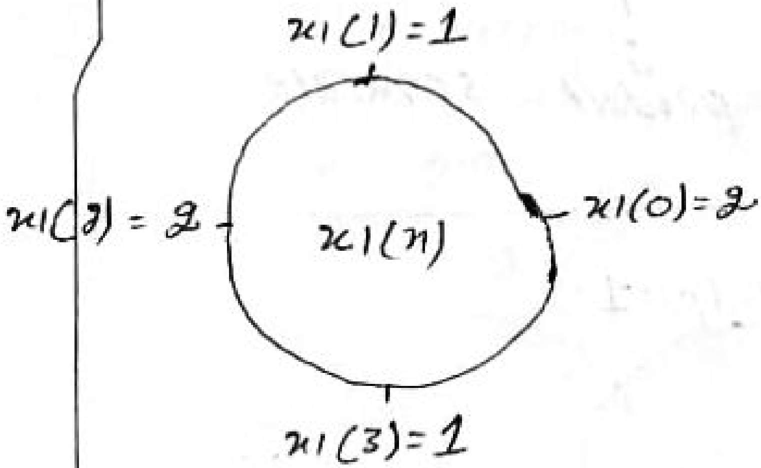
Finally we sum the values in in the product sequence to obtain $x_3(1)$ Thus.

$$x_3(1) = 16$$

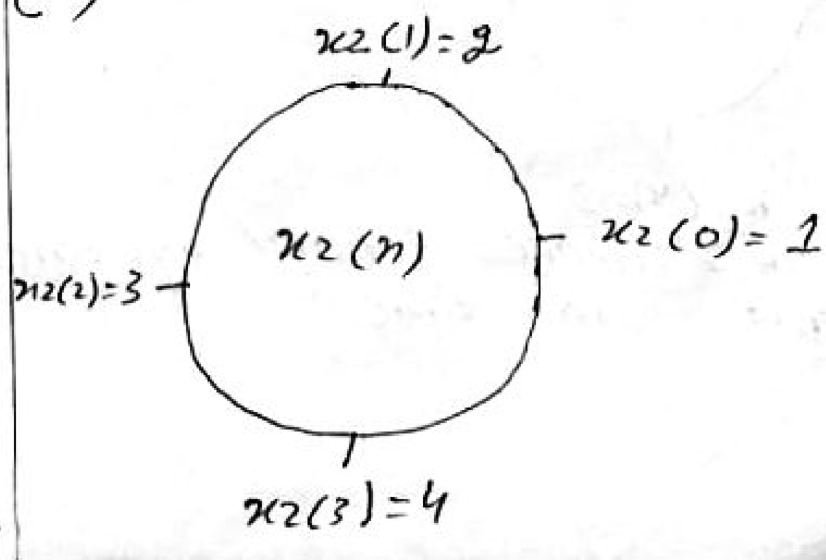
From $m = 2$ we have

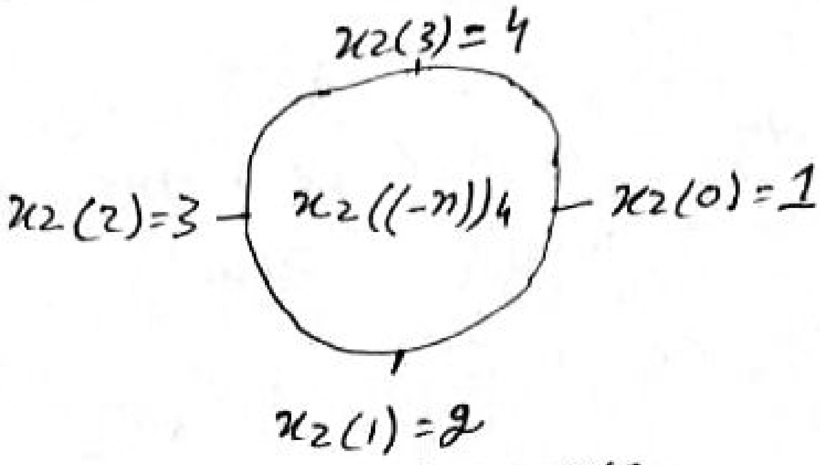
$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2((2-n))_4$$

Now $x_2((2-n))_4$ is the folded sequence. rotated two times in the counterclockwise direction.



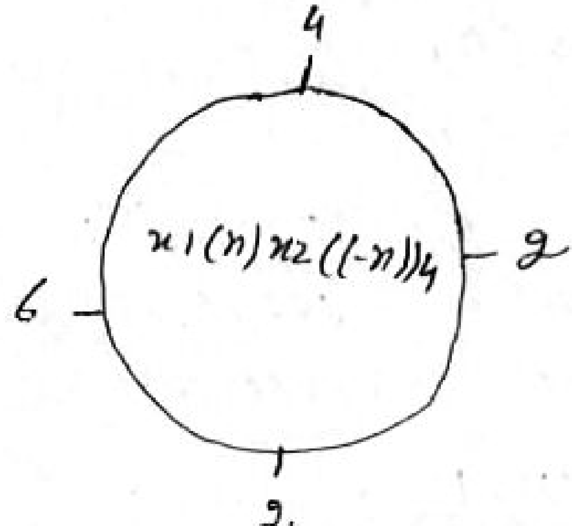
(a)





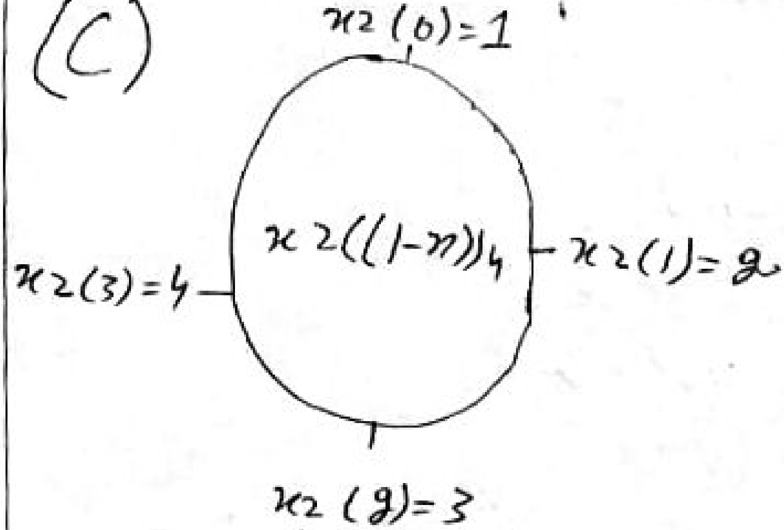
Folded sequence.

(b)

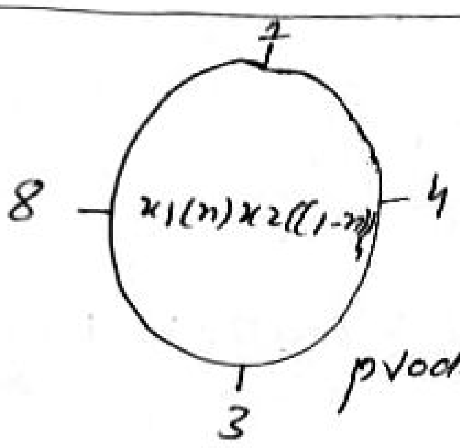


product sequence.

(C)

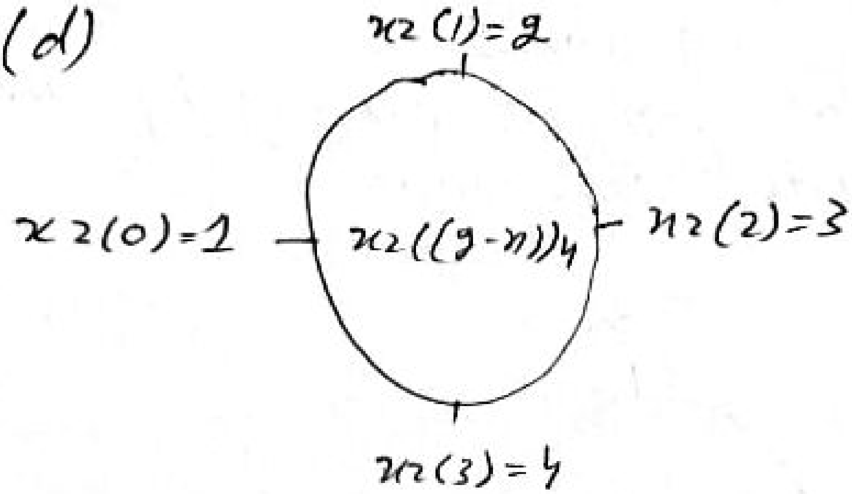


Folded sequence rotated by one unit in time.

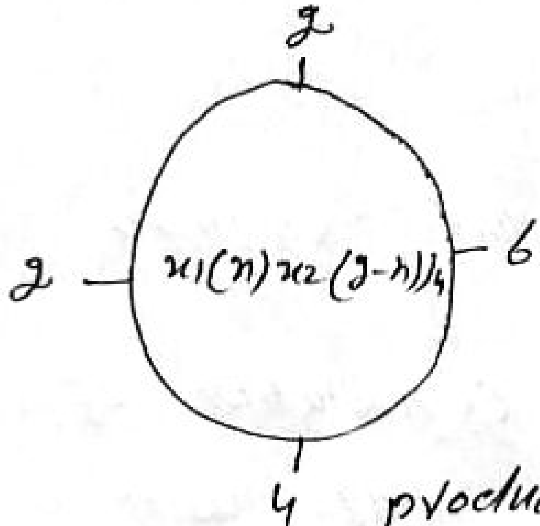


product sequence

(d)

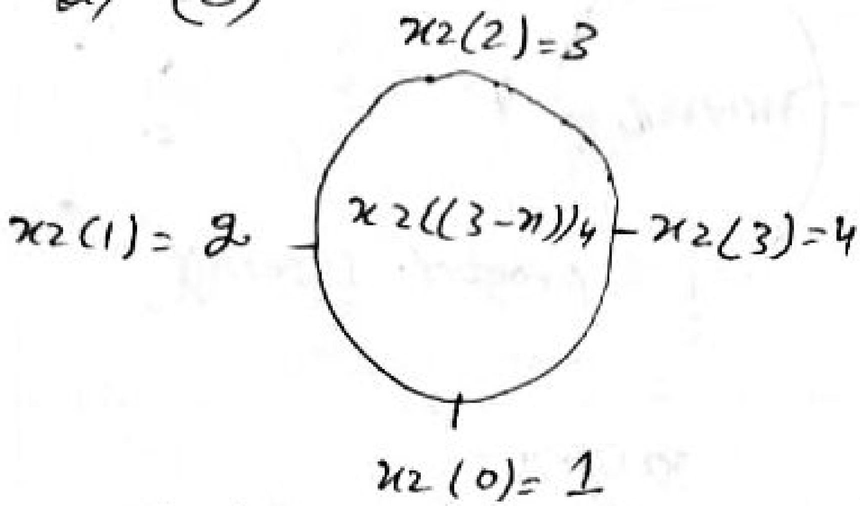


Folded sequence rotated by two units in time

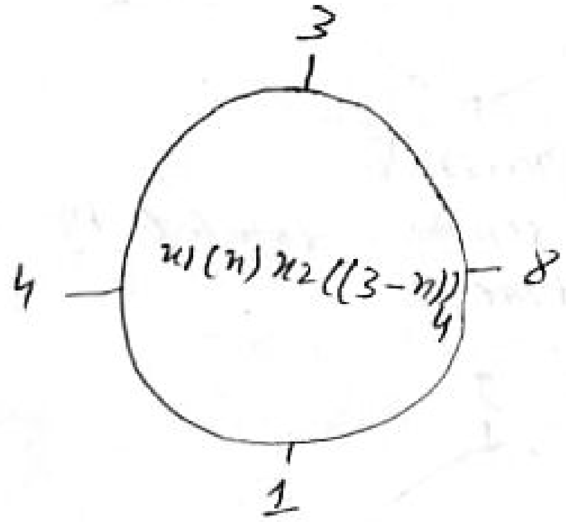


product sequence

~~d)~~ (e)



Folded sequence rotated by three units in time.



product sequence

Circular convolution of two sequences along with the product sequence.

$x_1(n)\, x_2((3-n))_4$ By summing the four terms in the product sequence, we obtain.

$$x_3(2) = 14$$

For $m = 3$, we have

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2((3-n))_4.$$

The folded sequence $x_2((-n))_4$ is now rotated by three units in time to yield $x_2((3-n))_4$ and the resultant sequence is multiplied by $x_1(n)$ to yield the product sequence. The sum of the values in the product sequence is

$$x_3(3) = 16$$

we observe that if the computation above is continued beyond $m=3$ we simply repeat the sequence of four values obtained above. Therefore the circular convolution of two sequences $x_1(n)$ and $x_2(n)$ yields the sequence

$$x_3(n) = \{\underset{\uparrow}{14}, 16, 14, 16\}$$

we observe that circular convolution involves basically the same four steps as the ordinary linear convolution.

The reader can easily show from our previous development that either one of the two sequences may be folded and rotated without changing the result of the circular convolution. Thus.

$$x_3(m) = \sum_{n=0}^{N-1} x_2(n)\, x_1((m-n))_N$$

$$m = 0, \underline{1} \ldots N - \underline{1}.$$

X —— X —— X —— X.