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Sec A

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Subject Introduction to Structural Dynamics and earthquacke Engineering.

Q:: 01

Ans

Given Data:.

$E = 29{,}000$ ksi

$I = 150\ in^4$

δ_{st} = Deflection due to 7756 lb

Static load Beam is pulled

1/2" downwards

Required

Natural time period of

system.

Develop and solve the equation

of motion

Draw graphs to show the variation of displacement with time and the variation of equivalent static forces with time.

Sollution

General EOM for SDOF System is $ku + c\dot{u} + m\ddot{u} = P(t)$

Since system is undamped $(c=0)$

Undergoing free vibration $P(t)=0$

Hence general EOM becomes

$$ku + m\ddot{u} = 0 \quad \text{—— ①}$$

$$k = \frac{3EI}{L^3} \Rightarrow \frac{3 \times 29000\ k/in^2 \times 150\ in^4}{(10 \times 12\ in)^3}$$

$k = 7.55 \; k/in$

$k = 7.55208 \; k/in$

→ In order to eliminate chances of mistake during calculation, it is more appropriate to use fundamental units like lb, ft sec or kg, m, sec

$k = 7.55208 \; k/in \Rightarrow 90625 \; lb/ft$

$$m = \frac{W}{g} \Rightarrow \frac{7756}{32.2} = 240.869 \; slug$$

$$W_n = \sqrt{\frac{k}{m}} \Rightarrow \sqrt{\frac{90625}{240.869}}$$

$W_n = 19.396 \; rad/sec$

$$T_n = \frac{2\pi}{W_n} = \frac{2\pi}{19.396} = 0.324 \; sec$$

Put m and k in eq (1)

$90625\,u + 240.869\,\ddot{u} = 0$

Where k is in lb/ft and m is in lb sec/ft^2

=> General solution to EOM for undamped free vibration is

$$u(t) = u(0)\cos(\omega_n t) + \frac{\dot{u}(0)}{\omega_n \sin(\omega_n t)}$$

$$u(0) = \frac{1}{2''} = \frac{1}{24}\text{ ft} \;\&\; \ddot{u}(0) = 0$$

$$u(t) = \left(\frac{1}{24}\right) \times \cos(19.396t) + 0$$

$$= \left(\frac{1}{24}\right) \times \cos(19.396)t$$

Equivalent static force at any time t is

$$f_s(t) = k \cdot u(t) = \frac{90625 \times \cos(19.396t)}{24}$$

$$= 3776 \cos(19.396t)$$

Amplitude of dynamic displacement, u_0 for undamped free vibration is

$$u_0 = \sqrt{\left[(u(0))^2 + \left(\frac{v(0)}{w_n}\right)^2\right]}$$

$$= \sqrt{\left(\left(\frac{1}{24}\right)^2 + 0\right)}$$

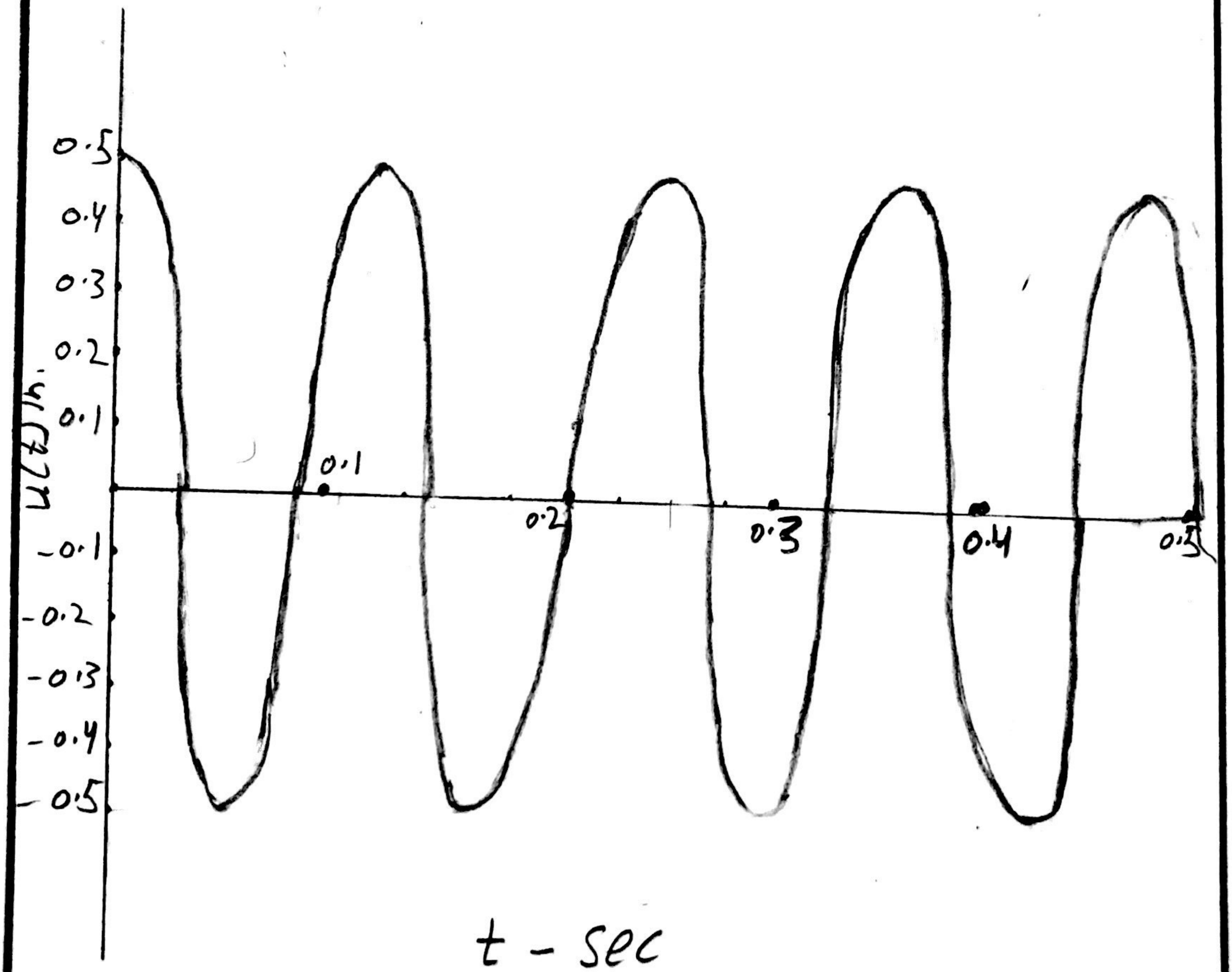
$$= \frac{1}{24} \text{ ft}$$

Amplitude of equivalent Static force, f_{so}

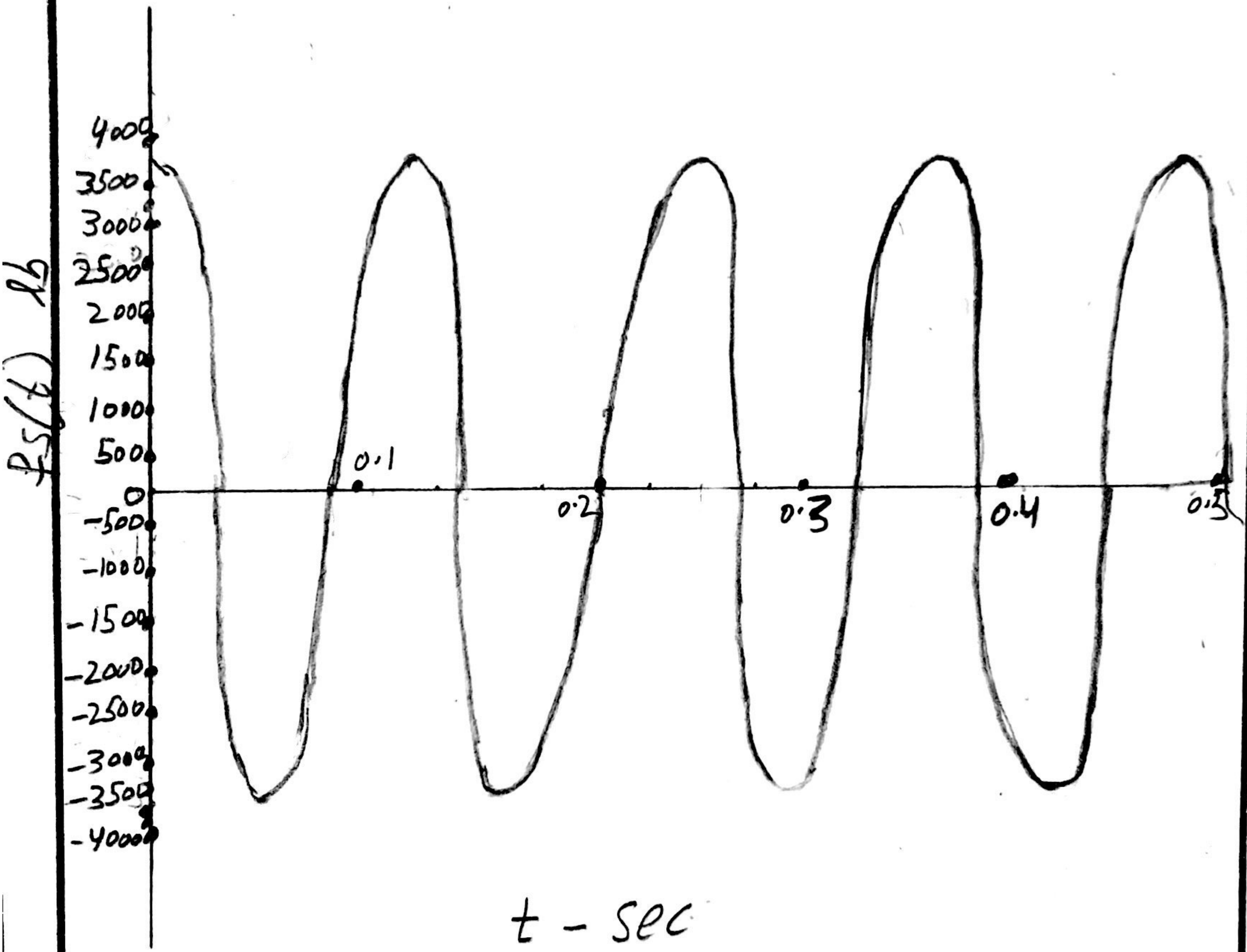
$$ku_0 = 90625 \times \frac{1}{24}$$

$k_{uo} = 3776.04$

6



variation of displacement with time



variation of Equivalent static forces with time.

Q2.

Ans:-

Given:-

ζ (Damping ration) of Reinforced concrete with considerable cracking = 3-5 %.

= 3 %

Using Data of beam given in Question I

Required:-

→ Develope and solve the equation showing variation in Equivalent static force with time.

→ Draw graph to show the variation of displacement with time and the variation of equilent static force with time

Solution:

EOM for damped free vibration is

$$ku + C\dot{u} + m\ddot{u} = 0 \quad \text{--- (1)}$$

From Question one.

$k = 90625$ lb/ft

$m = 240.869$ lb sec²/ft

$W_n = 19.396$ rad/sec.

$C = \xi \times 2 m W_n$

$C = (0.03) \times 2 (240.869)(19.396)$

$C = 280.31$ lb.sec/ft.

Put values in eq (1).

$$90625u + 280.31u + 240.869u = 0$$

Solution to the EOM for damped free vibration is.

$$u(t) = e^{-\xi w_n t}\left[u(0)\cos(w_D t) + \frac{1}{w_D}[\dot{u}(0) + u(0)\xi w_n] \times \sin w_D t\right]$$

$w_D = 19.396$ rad/sec.

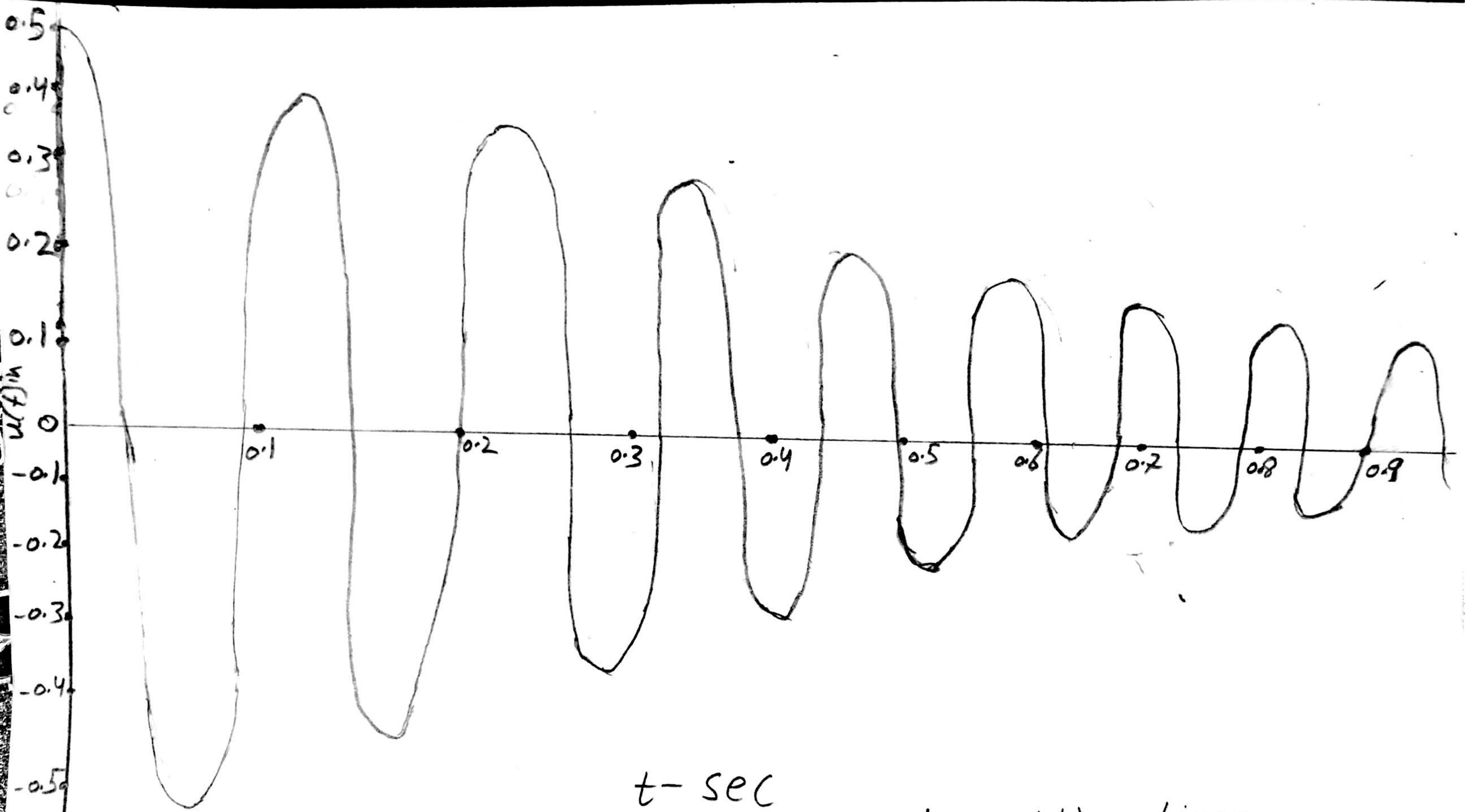
$$u(t) = e^{-0.582t}\left[0.041 \times \cos(19.396t) + 0.00125 \times \sin(19.396t)\right]$$

$$f_s(t) = k \cdot u(t) \Rightarrow 90625 \times u(t)$$

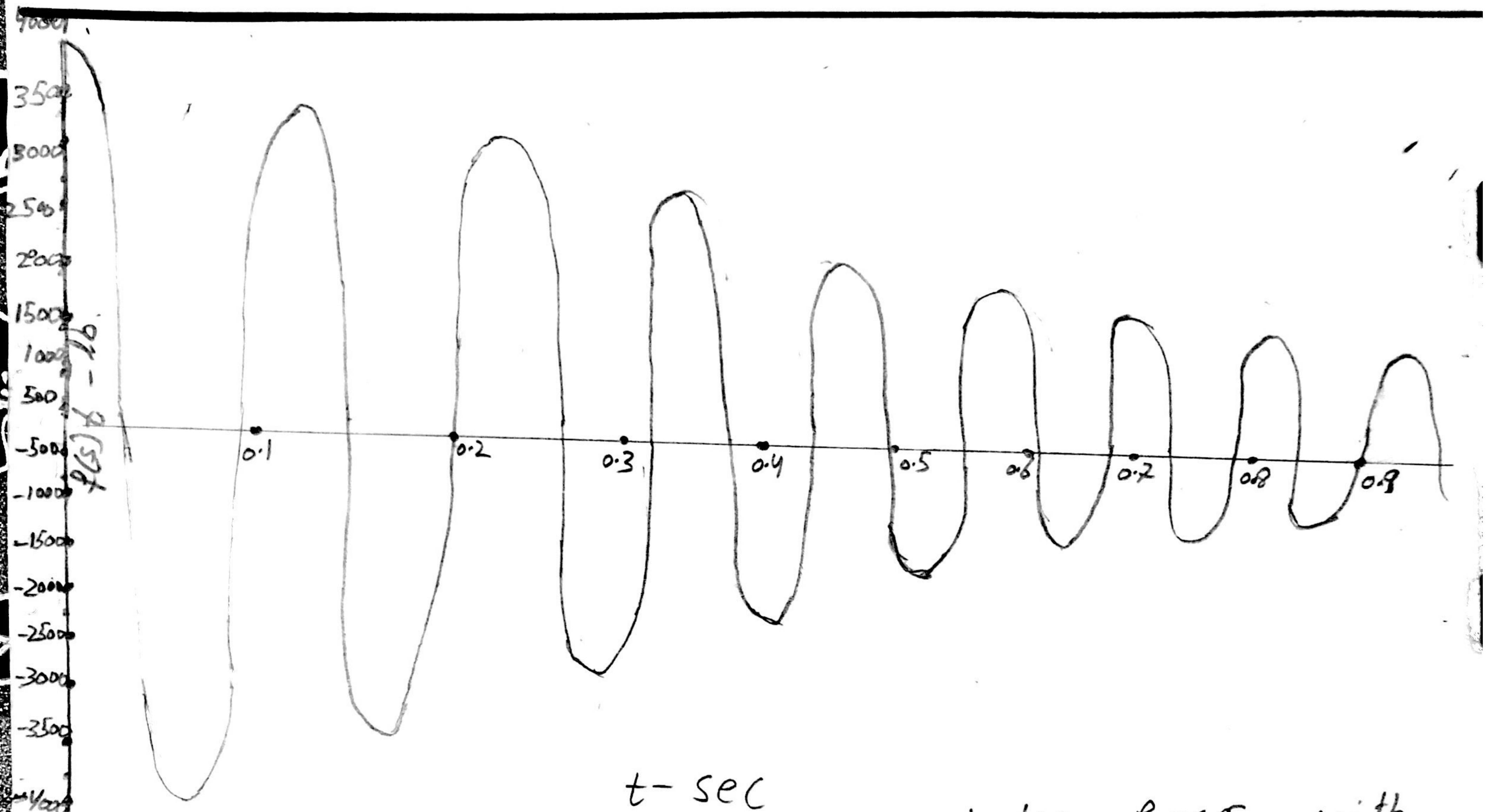
$$f_s(t) = e^{-0.582t}\left[(90625 \times 0.041)\cos(19.396t) + (90625 \times 0.00125)\sin(19.396t)\right]$$

$$f_s(t) = e^{-0.582t}\left[3715.62\cos(19.396t) + 113.28\sin(19.396t)\right]$$

Ans.



variation of displacement with time



variation of Equivalent static forces with time.

Q:: 3

Given data:

- Force = 60 kips.
- Displacement of tank = $\left(\frac{ID}{1000}\right)''$

$$= \left(\frac{7756}{1000}\right) = 7.756''$$

- Time taken to complete 7 cycles = 3.57 sec.
- Amplitude of displacement = 2.286 cm = 0.9''

Required data::

1. Damping ratios, ξ
2. Natural period of un-damped vibration.

3 Stiffness of structures.

4 Weight of tank.

5 Damping co-efficient.

6 Number of cycles to reduce the displacement amplitude to 0.5"

Solution:-

→ Displacement of tank, $u_1 = 7.756''$

→ After 7 cycles, i-e After $j = 7$

$$u_{j+1} = u_8 = 0.9''$$

1 **Damping ratio:**

Damping ratio is find as

$$\delta = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{1+1}}\right]$$

$$7 = \frac{1}{2\pi\zeta} \ln\left[\frac{7.756}{0.9}\right]$$

$$\zeta = 0.0489 = 4.89\ \%$$

2. Natural Period of Undamped Vibration, $T_n = ?$

As the 7 cycles of vibrations are completed in 3.57 sec

$\Rightarrow$ Time required to complete one cycle, $T_D = \frac{3.57}{7} = 0.51$ sec.

Now

$$W_D = W_n \sqrt{(1-\zeta^2)}$$

$$\frac{2\pi}{W_D} = \frac{2\pi}{(W_n\sqrt{1-\zeta^2})}$$

$$= T_D = \frac{T_n}{(1-\xi)^2}$$

$$\Rightarrow T_n = T_D \times \sqrt{1-\xi^2}$$

$$T_n = 0.51 \times \sqrt{1-(0.0488)^2}$$

$$T_n = 0.5094 = 0.51 \text{ sec}$$

$$T_n = 0.51 \text{ sec.}$$

So the natural period of undamped vibration, $T_n = 0.51$ sec.

3 **Stiffness of structure, k=?**

$$k = \frac{60 \times \cos 60^\circ}{7.756} = 3.86 \text{ k/in.}$$

$$k = 3.86 \text{ k/in} = 46.320 \text{ lb/ft.}$$

4 **Weight of tank:-**

weight of tank, w is find as,

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$$W_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{k}{W/g}} = \sqrt{\frac{k \cdot g}{W}}$$

$$\Rightarrow W_n^2 = \frac{k \times g}{W_n^2}$$

Also $W_n = \frac{2\pi}{T_n}$

$$W = \frac{kg}{\left(\frac{4\pi^2}{T_n^2}\right)}$$

$$W = kg \times \frac{T_n^2}{4\pi^2}$$

$$W = \left[\frac{46320 \text{ lb}}{\text{ft}} \times \frac{32.2 \text{ ft}}{\text{sec}^2}\right] \times \frac{(0.51 \text{ sec})^2}{4\pi^2}$$

$$W = 9826.93 \text{ lb} = 9.826k$$

5 Damping co-efficient, c=?

It is known that

$$\zeta = \frac{c}{2m W_n}$$

$$\Rightarrow c = \zeta \times 2m \times \omega_n$$

$$= \zeta \times 2m \times \left(\frac{2\pi}{T_n}\right)$$

$$c = \frac{(0.0488) \times 4 \times \pi \times \left(\frac{9826.93}{32.2}\right)}{0.51}$$

$$c = 366.90 \text{ lb} \cdot \text{sec/ft.}$$

6 Number of cycles to reduce the displacement amplitude to 0.5", j

$$j = \frac{1}{2\pi\zeta} \ln\left[\frac{u_1}{u_{j+1}}\right]$$

$$j = \frac{1}{2 \times \pi \times 0.049} \ln\left[\frac{7.756}{0.5}\right]$$

$$j = 8.94 \text{ or } 9 \text{ cycles.}$$