Depertment of Civil Engineering.

Summer 2020: Duration: 4 hours.

Subject : Calculus

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Course code: NS-112.

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betch : 2016.

QNo#01?　　7792

Sol:⇒ Cordinate of $P = (4, 1, 3)$

$$OP = 4i + 1j + 3K$$

or $OQ = \vec{OQ} - \vec{OP}$

$$= (i + 2j + 4K) - (4i + 1j + 3K)$$

$$= -3i + 1j + 1K \longrightarrow ①$$

now distance between P & $Q = |PQ|$

$$= \sqrt{(-3)^2 + (1)^2 + (1)^2}$$

$$= \sqrt{9 + 1 + 1}$$

$$= \sqrt{11} \longrightarrow ②$$

let M be the point wich divided PQ in ratio 1:3, then by ratio theorm

Position vector of $M = \vec{OM}$

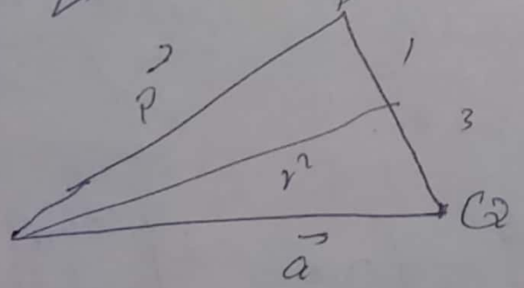
$$= \frac{3(4i + 1j + 3K) + (1)(i + 2j + 4K)}{1 + 3}$$

7722

$$\cancel{=} = \frac{12i+3j+9k+i+2j+4k}{4}$$

$$= \frac{13i+5j+13k}{4} \quad \text{——} ③$$

Hence eq (1), (2) & (3) are the required Solution.



$a : b = 1 : 3$

$$\vec{r} = \frac{b\vec{p} + q\vec{a}}{b+a}$$

$$= \frac{3(4i+j+3k)+1(\hat{i}+2j+4\hat{k})}{3+1}$$

$$= \frac{12\hat{i}+3\hat{j}+9k+\hat{i}+2\hat{j}+4k}{4}$$

$$= \frac{13\hat{i}+5\hat{j}+13\hat{k}}{4}$$

$$\boxed{\vec{r} = \frac{13}{4}\hat{i} + \frac{5}{4}\hat{j} + \frac{13}{4}\hat{k}}$$ Ans.

Qno#02?

$$\int \frac{4x^2+10x+4}{2x^2+x}\,dx$$

Sol:→

$$\begin{array}{r} 2x-1 \\ 2x^2+x\,\overline{)\,4x^3+10x+4} \\ 4x^3 \quad \pm 2x^2 \\ \hline -2x^2+10x+4 \\ \overset{+}{-}2x^2 \mp x \\ \hline 11x+4 \end{array}$$

So:

$$2x-1+\frac{11x+4}{2x^2+x}=\frac{4x^3+10x+4}{2x^2+x}$$

$$\int\frac{4x^2+10x+4}{2x^2+x}=\int 2x-1+\int\frac{11x+4}{2x^2+x} \text{ —— ①}$$

$$=2\int x\,dx-\int 1\,dx+\int\frac{11x+4}{2x^2+2}\,dx$$

$$\frac{2x^2}{2}-x+\int\frac{11x+4}{x(2x+1)}\,dx \text{ —— ②}$$

now find.

$$\int\frac{11x+4}{x(2x+1}\,dx=?$$

$$\frac{11\cdot x+4}{x(2x+1)}=\frac{A}{x}+\frac{B}{(2x+1)}$$

$$\frac{11x+4}{x(2x+1)} = \frac{A(2x+1)+Bx}{x(2x+1}$$

$$11x+4 = A(2x+1)+Bx \quad \text{——} ③$$

$$\boxed{4 = A}$$

Now put $x = -\frac{1}{2}$ in —— ③

$$11\left(-\frac{1}{2}\right)+4 = B\left(-\frac{1}{2}\right)$$

$$\frac{-11}{\cancel{2}}+4 = \frac{-B}{\cancel{2}}$$

$$-3 = -B \Longrightarrow \boxed{B=3}$$

put the value of A & B in Ⓐ

$$\frac{11x+4}{x(2x+1)} = \frac{4}{x}+\frac{3}{2x+1}$$

taking integral on both side

$$\cancel{\int \frac{11x+4}{x(2x}} \quad \int \frac{11x+4}{x(2x+1}dx = \int \frac{4x+}{x} \int \frac{3}{2x+1}dx$$

$$4\int\frac{1}{x}\,dx+3\int\frac{1}{2x+1}\,dx$$

$$4\ln(x)+\frac{3}{2}\ln(2x+1)$$

puting these value in — ②

$$x^2-x+4\ln|x|+\frac{3}{2}\ln|2x+1)$$

puting value in — eq ①

$$\int\frac{4x^3+10x+4}{2x^2+x}\,dx=x^2-x+4\ln|x|+\frac{3}{2}\ln(2x+1)+C$$

Ans

QNO# 03

par a) $\int_0^2 x^2 e^x \, dx$

Sol:- now Integration ~~by pa~~rts. 1st find $\int x^2 e^x dx$

~~$\int x^2 e^x d$~~

$x^2 \int e^x - \int \left(e^x dx \cdot \frac{d}{dx} x^2 \right) dx$

$x^2 e^x - \int e^x (2x) dx$

$x^2 e^x - \int 2x e^x dx$

$x^2 e^x - 2 \int x e^x dx$

Use integrating by parts

$x^2 e^x - 2 \left[x \int e^x dx - \int \left(\int e^x dx \cdot \frac{dx}{dx} x \right) d \right.$

$x^2 e^x - 2 \left[x e^x - \int e^x dx \right]$

$x^2 e^x - 2x e^x + 2 e^x$

$(x^2 - 2x + 2) e^x$

applying limit

~~$(2^2 - 2(2) + 2) e^2 - (0^2 - 2(0) + 2) e^0$~~

$\boxed{\cancel{2e^2 - 2}}$ Ans. (pto)

$$\left(x^2e^x - 2xe^x + 2e^x\right|_0^2$$

$$2^2e^2 - 2(2)e^2 + 2e^2 - (0 - 0 + 2e^0)$$

$$(4e^2 - 4e^2 + 2e^2 - 2)$$

$$\boxed{2e^2 - 2 \text{ Ans}}$$

Qno 3

part b) $\int_1^2 \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$. ?

Sol: First find integration.

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \text{——} ①$$

let $y = \sqrt{x}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

$$2\,dy = \frac{1}{\sqrt{x}}\,dx \quad \text{put in} — ①$$

$$\int \sin(y)(2dy) = 2\int \sin(y)\,dy$$

$= 2(-\cos y)$

$= -2\cos y$

put $y = \sqrt{x}$

$= -2\cos\sqrt{x}$

put limits

$= -2\left| \cos\sqrt{x} \right|_{1}^{2} = -2(\cos\sqrt{2} - \cos 1)$

$= -2\cos\sqrt{2} + 2\cos(1)$ Ans

Q No # 04?

= the laplace equation in 3.d is

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} = 0 \longrightarrow Ⓐ$$

So $u(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$

$$u(x, y, z) = (x^2+y^2+z^2)^{-1/2}$$

$$\frac{\partial u}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2x)$$

$$\frac{\partial u}{\partial x} = -x(x^2+y^2+z^2)^{\frac{-3}{2}}$$

$$\frac{\partial^2 u}{\partial x^2} = -\left(x\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)^{-5/2}(2x) + (x^2+y^2+z^2)^{-3/2}\right)$$

$$\frac{\partial^2 u}{\partial x^2} = 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} \longrightarrow ①$$

Now $\frac{\partial u}{\partial y} = -\frac{1}{x}(x^2+y^2+z^2)^{-3/2}(\partial y)$

~~2y~~ $\frac{\partial u}{\partial y} = -y(x^2+y^2+z^2)^{\frac{-3}{2}}$

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$$\frac{\partial^2 u}{\partial y^2} = -\left[y\left(\frac{-3}{\cancel{2}}\right)(x^2+y^2+z^2)^{\frac{-5}{2}}(2y) + (x^2+y^2+z^2)^{\frac{-3}{2}}\right]$$

$$\boxed{\frac{\partial^2 u}{\partial y^2} = 3y^2(x^2+y^2+z^2)^{\frac{-5}{2}} - (x^2+y^2+z^2)^{\frac{-3}{2}}} \text{ — } ②$$

$$\frac{\partial U}{\partial z} = \frac{-1}{\cancel{2}}(x^2+y^2+z^2)^{\frac{-3}{2}}(\cancel{2}z)$$

$$\frac{\partial u}{\partial z} = -z(x^2+y^2+z^2)^{\frac{-3}{2}}$$

$$\boxed{\frac{\partial^2 u}{\partial z^2} = 3z^2(x^2+y^2+z^2)^{\frac{-5}{2}} - (x^2+y^2+z^2)^{\frac{-3}{2}}} \text{ — } 3$$

puting (1) (2) & (3) in Ⓐ

$$3z^2(x^2+y^2+z^2)^{\frac{-5}{2}} - (x^2+y^2+z^2)^{\frac{-3}{2}} + 3y^2(x^2+y^2+z^2)^{\frac{-5}{2}} - (x^2+y^2+z^2)^{\frac{-3}{2}}$$

$$+ \rightarrow + 3z^2(x^2+y^2+z^2)^{\frac{-5}{2}} - (x^2+y^2+z^2)^{\frac{-3}{2}}$$

$$= (x^2+y^2+z^2)^{\frac{-5}{2}}\left[3x^2 - (x^2+y^2+z^2) + 3y^2 - (x^2+y^2+z^2) + 3z^2 - (x^2+y^2+z^2)\right.$$

$$= (x^2+y^2+z^2)^{\frac{-5}{2}}(3\cancel{x^2} - \cancel{x^2} - \cancel{y^2} - z^2 + 3y^2 - \cancel{x^2} - y^2 - z^2 + 3x^2 - \cancel{x^2} - y^2 - z^2)$$

$$= (x^2+y^2+z^2)^{\frac{-5}{2}}(0) = 0$$

So the given $U(x, y, z)$ is solution

Of laplace Equation

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