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Program: BS(SE-2)

Section: A

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Subject: Linear Algebra

Q1) i) A $\begin{bmatrix} 1 & 2 & \text{2nd. ID} \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$

ID = 15832

2nd number = 5

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$A^{adj} = ?$

Sol:-

Replace all the values with its cofactors and then take transpose which is also equals to adjoint of A.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} \Rightarrow (1) \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= (1)(6-1) \Rightarrow \boxed{5}$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} \Rightarrow (-1)(4-3) \Rightarrow \boxed{-1}$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ 3 & 1 \end{vmatrix} \Rightarrow (1)(2-9) \Rightarrow \boxed{-7}$$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 5 \\ 1 & 2 \end{vmatrix} = (-1)(4-5) = \boxed{1}$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix} = (1)(2-15) = \boxed{-13}$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = (-1)(1-6) = \boxed{5}$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 5 \\ 3 & 1 \end{vmatrix} = (1)(2-15) = \boxed{-13}$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} = (-1)(1-10) = \boxed{9}$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = (1)(3-4) = \boxed{-1}$

$$\text{Cofactors} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -1 & -7 \\ 1 & -13 & 5 \\ -13 & 9 & -1 \end{bmatrix}$$

Ans.

Q1) ii = $B\begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 8 \\ 5 & -2 & 8 \end{bmatrix}$

Sol: $A_{Adj} = ?$

$A_{11} = (-1)^{1+1}\begin{vmatrix} -1 & 8 \\ -2 & 8 \end{vmatrix} = (1)(-2+16) = \boxed{8}$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 2 & 8 \\ 5 & 8 \end{vmatrix} = (-1)(16-40) = \boxed{24}$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 2 & -1 \\ 5 & -2 \end{vmatrix} = (+1)(-4+5) = \boxed{1}$

$A_{21} = (-1)^{2+1}\begin{vmatrix} 4 & 5 \\ -2 & 8 \end{vmatrix} = (-1)(32+10) = \boxed{-42}$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 3 & 5 \\ 5 & 8 \end{vmatrix} = (1)(24-25) = \boxed{-1}$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 3 & 4 \\ 5 & -2 \end{vmatrix} = (-1)(-6-20) = \boxed{26}$

$A_{31} = (-1)^{3+1}\begin{vmatrix} 4 & 5 \\ -1 & 8 \end{vmatrix} = (1)(32+5) = \boxed{37}$

$A_{32} = (-1)^{3+2}\begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = (-1)(24-10)$

$= \boxed{-14}$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix}$$

$$= (1)(-3-8)$$

$$= \boxed{-11}$$

Now Replacing values with cofactors

$$B = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 24 & 1 \\ -42 & -1 & 26 \\ 37 & -14 & -11 \end{bmatrix}$$

Now Taking Transpose.

$$B_{adj} = B^t = \begin{bmatrix} 8 & -42 & 37 \\ 24 & -1 & -14 \\ 1 & 26 & -11 \end{bmatrix}$$

Ans

Q2) Find the cofactors of A_{21}, A_{31}, A_{33} if

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & 3 & 1 \\ 4 & -3 & 2 \end{bmatrix}$$

Sol:-

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -2 & 3 \\ -3 & 2 \end{vmatrix}$$

$$= (-1)(-4+9) = -5$$

$$\boxed{A_{21} = -5} \text{ Ans}$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -2 & 3 \\ 3 & 1 \end{vmatrix}$$

$$= (1)(-2-9) = -11$$

$$\boxed{A_{31} = -11} \text{ Ans}$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} \Rightarrow (1)(3-4)$$

$$= -1$$

$$\boxed{A_{33} = -1} \text{ Ans}$$

Q3) $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$ & $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Step 1 :
By using formula

$\therefore \quad |A - \lambda I| = 0$

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{vmatrix} - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 1-0 & 1-0 \\ 1-0 & 3-\lambda & 2-0 \\ -1-0 & 1-0 & 2-\lambda \end{vmatrix} = 0$$

$$\begin{vmatrix} 2-\lambda & 1 & 1 \\ 1 & 3-\lambda & 2 \\ -1 & 1 & 2-\lambda \end{vmatrix} = 0$$

Now Taking Determinent:

$$2-\lambda\left((3-\lambda)(2-\lambda)-2\right)-1\left(2-(2-\lambda)\right)+$$
$$1\left(1+1(3-\lambda)\right)=0$$

$2-\lambda(6-3\lambda-2\lambda+\lambda^2-2)-1(2-2+\lambda)$
$+1(1+3-\lambda)=0$

$2-\lambda(\lambda^2-5\lambda+4)-1(\lambda)+1(4-\lambda)=0$

$\therefore$ By Arranging.

$2\lambda^2-10\lambda+8-\lambda^3+5\lambda^2-4\lambda-\lambda+4-\lambda=0$

$\Rightarrow \lambda^3-7\lambda^2+16\lambda-12=0$ —— (1)

Now Put $\lambda-2=0$

Then put $\lambda=2$ in above eq (1)

$\lambda^3-7\lambda^2+16\lambda-12=0$

↓

Becomes.

$(2)^3-7(2)^2+16(2)-12=0$

$8-28+32-12=0$

$-20+20=0$

$0=0$

So Eigen value is $\lambda=2$

or $\lambda-2=0$ Ans-

$-\lambda)+$