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Program	BS(CS)
Exam	final-term.
Course	Differential equation.

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Q # 4 (1)

Sol

$$\mathcal{L}\{y''\} - 10\,\mathcal{L}\{y'\} + 9\,\mathcal{L}\{y\} = \mathcal{L}\{5t\}$$

$$s^2Y(s) - sy(0) - y' - 10\{sy(s) - y(0)) + 9y(s) - \frac{5}{s^2}$$

$$(s^2 - 10s + 9)\,Y(s) + s - 12 = \frac{5}{s^2}$$

$$Y(s) = \frac{5}{s^2(s-9)\,s-1} + \frac{12 - s}{(s-9)(s-9)}$$

$$Y(s) = \frac{5 + 12s^2 - s^2}{s^2(s-9)(s-1)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s-9} + \frac{D}{s-1}$$

$$\Rightarrow 5 + 12s^2 - s^3 = As\,(s-9)(s-1) + B\cdot(s-9)(s-1) + Cs^2(s-1) + Ds^2(s-9)$$

$s = 0 \quad 5 = 9B \quad \Rightarrow \quad B = \frac{5}{9}$

$s = 1 \quad 16 = -8D \quad \Rightarrow \quad D = -2$

$s = 9 \quad 248 = 648C \quad \Rightarrow \quad C = \frac{31}{81}$

$s = 2 \quad 42 = -14A + \frac{4345}{81} \quad \Rightarrow A = \frac{50}{81}$

$$Y(s) = \frac{\frac{50}{81}}{s} + \frac{\frac{5}{9}}{s^2} + \frac{\frac{31}{81}}{s-9} - \frac{2}{s-1}$$

$$Y(t) = \frac{50}{81} + \frac{5}{9}t + \frac{31}{81}e^{9t} - 2e^{t}$$

Ans

Q #4 Part (ii).

sol

$$= s^2Y(s) - sy(0) - y'(0) - 6(sy(s)) - y(0))$$
$$+ 15y(s) = 2\frac{3}{s^2+9}$$

$$= (s^2 - 6s + 15)\ Y(s) + s - 2 = \frac{6}{s^2+9}$$

$$Y(s) = \frac{-s^3 + 2s^2 - 9s + 24}{(s^2+9)\ (s^2-6s+15)}$$

$$Y(s) = \frac{As+B}{s^2+9} + \frac{Cs+D}{s^2+6s+15}$$

$$-s^3 + 2s^2 - 9s + 24 = (As+B)\ (s^2 - 6s + 15) + (Cs+B) + (s^2+9)$$

$$= (A+C)s^2 + (-6A + B + B)s^2 + (15A - 6B + 9C)s$$
$$+ 15B + 9D.$$

$s^2 = A + C = 1 \Rightarrow\ \Rightarrow A = 1/10$

$s^2 = -6A + B + D = 2 \Rightarrow B = 1/10$

(4).

$s^1 = 15A - 6B + 9C = -9 \Rightarrow C = -\frac{11}{10}$

$s^0 = 15B + 9D = 24 \Rightarrow D = \frac{5}{2}$

$$Y(s) = \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11s+25}{s^2-6s+15}\right)$$

$$= \frac{1}{10}\left(\frac{s+1}{s^2+9} + \frac{-11(s-3+3)+25}{(s-3)^2+6}\right)$$

$$= \frac{1}{10}\ \frac{s}{s^2+9} + \frac{1\frac{3}{3}}{s^2+9} - \frac{11(s-3)}{(s-3)^2+6} - \frac{8\frac{\sqrt{6}}{\sqrt{6}}}{(s-3)^2+6}$$

$$= y(t) = \frac{1}{10}(\cos(3t) + \frac{1}{3}\sin(3t) - 11e^{3t}\cos(\sqrt{6}t)$$

$$- \frac{8}{\sqrt{6}} e^{3t} \sin(\sqrt{6}t))$$

Ans

Q # 2

i)

sol

$16r^2 - 40r + 25 = (4r-5)^2 = 0 \quad r_{1,2} = \frac{5}{4}$

$y(t) = c_1 e^{\frac{5t}{4}} + c_2 t e^{\frac{5t}{4}}$

$y'(t) = \frac{5}{4} c_1 e^{\frac{5t}{4}} + c_2 e^{\frac{5t}{4}} + \frac{5}{4} c_2 t e^{\frac{5t}{4}}$

$3 = y(0) = c_1$

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$-9/4 = y'(0) = 5/4 c_1 + c_2$

$y(t) = 3e^{5t/4} - 6te^{5t/4}$

Ans.

$\longleftrightarrow$

Q#2
(iii)

sol.

$r^2 - 4r + 9 = 0$

$y(t) = c_1 e^{2t} \cos(\sqrt{5}t) + c_2 e^{2t} \sin(\sqrt{5}t)$

$\Rightarrow 0 = y(0) = c_1$

$y(t) = c_2 e^{2t} \sin(\sqrt{5}t)$

$y'(t) = 2c_2 e^{2t} \sin(\sqrt{5}t) + \sqrt{5} c_2 e^{2t} \cos(\sqrt{5}t)$

$-8 = y'(0) = \sqrt{5} c_2 \Rightarrow c_2 = \frac{-8}{\sqrt{5}}$

$y(t) = \frac{-8}{\sqrt{5}} e^{2t} \sin(\sqrt{5}t)$

A.

$\longleftrightarrow$

Q#2
(iv)

sol.

$r^2 - 8r + 17 = 0$

$y(t) = c_1 e^{4t} \cos(t) + c_2 e^{4t} \sin(t)$

$\Rightarrow y'(t) = 4c_1 e^{4t} \cos(t) - c_1 e^{4t} \sin(t)$
$+ 4c_2 e^{4t} \sin(t) + c_2 e^{4t} \cos(t)$

$\Rightarrow -4 = y(0) = c_1$

$-1 = y'(0) = 4c_1 + c_2$

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$y(t) = -4e^{4t}\cos(t) + 15e^{4t}\sin(t)$.

Ans.

Q # 1 (B) Part (ii)

Sol

$y'' - 4y' - 12y = 0$

$r^2 - 4r - 12 = (r-6)(r+2) = 0 \Rightarrow r_1 = (-2), r_2 = 6$

$y_c(t) = c_1 e^{-2t} + c e^{6t}$

$y_p(t) = Ae^{5t}$

$\Rightarrow 25Ae^{5t} - 4(5Ae^{5t}) - 12(Ae^{5t}) = 3e^{5t}$

$\Rightarrow -7Ae^{5t} = 3e^{5t}$

$\Rightarrow -7A = 3 \Rightarrow A = -\frac{3}{7}$

$y_p(t) = -\frac{3}{7}e^{5t}$

Ans

Q# 1 (a).

Ans Therefore, this differential equation ins nonhomogenous.

Definition:-

A second order differential equation is linear if it can be written in the form. where and are real-valued function and not identically zero- if - in other words, if for every value of x - the equation is said to be a homogeneous linear equation.

Q # 3

Ans: (Definition of Laplace Transform)

The Laplace transform Provide a useful method of solving certain types of differential equation when certain initial condition are given especially when the initial value are zero

The Laplace Transform $\mathcal{L}$ of a function $f(t)$ for $t>0$ is defined by the following integral over 0 to ∞

$\mathcal{L}\{f(t)\} =$

Q # 3 (A): Find the Laplace transform

1)

sol. $f(t) = 6e^{-5t} + e^{3t} + 5t^3$

$F(s) = 6 \frac{1}{s-(-5)} + \frac{1}{s-3} + 5 \frac{3!}{s^{3+1}} - 9 \frac{1}{s}$

$= \frac{6}{s+5} + \frac{1}{s-3} \frac{30}{s^4} - \frac{9}{s}$ Ans.

2)

sol. $g(t) = 4\cos(4t) - 9\sin(4t) + 2\cos(10t)$

$G(s) = 4 \frac{s}{s^2+4^2} - 9 \frac{4}{s^2-4^2} + 2 \frac{s}{s^2+10^2}$

$= \frac{4s}{s^2+16} - \frac{36}{s^2+16} + \frac{2s}{s^2+100}$ Ans

3)

sol. $h(t) = e^{3t} + \cos(6t) - e^{3t}\cos(6t)$

$H(s) = \frac{1}{s-3} + \frac{s}{s^2+6^2} - \frac{s-3}{(s-3)^2+6^2}$

$= \frac{1}{s-3} + \frac{s}{s^2+36} - \frac{s-3}{(s-3)^2+36}$ Ans.