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Reg# 16549

Semester: 6th

Final Term Paper: PROBABILITY &
STATISTICS

Submitted to: Sir DAUD KHAN

Q1:- Construct a grouped distribution table for the following data. and Calculate Mean, Mode, Median & Quartiles.

423, 369, 387, 411, 390

Answer:- In Ascending:-

363, 369, 371, 372, 377, 381, 382, 386, 387, 389, 390, 391, 392
393, 394, 396, 390, 400, 401, 405, 408, 409, 410, 411, 415, 419, 422
423, 428, 431.

$$\text{Max value} = 431$$

$$\text{Min value} = 363$$

$$\begin{aligned} \text{Range} &= 431 - 363 \\ &= 68 \end{aligned}$$

Since range is 68 we will choose as interval length of 10.

class interval	frequency	Mid-points	f.m	cf
360-369	2	364.5	729	2
370-379	3	374.5	1123.5	5
380-389	5	384.5	1922.5	(10)
<u>390-399</u>	(7)	394.5	2761.5	17
400-409	5	404.5	2022.5	22
410-419	4	414.5	1658	26
420-429	3	424.5	1273.5	29
430-439	1	434.5	434.5	30
	<u>30</u>		<u>11925</u>	

$$\text{Mean} = \frac{\sum f \cdot m}{\sum f}$$

$$= \frac{11925}{30}$$

$$\text{Mean} = 397.5$$

$$\text{Median} = l_n + \left(\frac{\frac{\sum f}{2} - cf_{cb}}{fm} \right)$$

$$= 389.5 + \left(\frac{\frac{30}{2} - 10}{7} \right) 399.5 - 389.5$$

$$= 389.5 + \left(\frac{15 - 10}{7} \right) 10$$

$$= 389.5 \left(\frac{5}{7} \right) 10$$

$$= 389.5 + 50/7$$

$$= 389.5 + 7 \cdot 1428$$

$$\boxed{\text{Median} = 396.6}$$

$$\text{Mode} = l_m + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2} \right) C$$

$$= 389.5 + \left(\frac{2}{2+2} \right) 10$$

$$= 389.5 + \left(\frac{2 \cdot 1}{4/2} \right) 10$$

$$= 389.5 + \frac{16}{2} 5$$

$$= 389.5 + 5$$

$$\boxed{\text{Mode} = 394.5}$$

$$l_m = 389.5$$

$$\Delta_1 = 7 - 5 = 2$$

$$\Delta_2 = 7 - 5 = 2$$

$$C = 399.5 - 389.5 = 10$$

Quartile finding

Class Interval	frequency	C.f
360-369	2	2
370-379	3	5
380-389	5	10
390-399	7	17
400-409	5	22
410-419	4	26
420-429	3	29
430-439	1	30
	<u>30</u>	

$$Q.D = \frac{Q_3 - Q_1}{2}$$

$$Q_1 = l + \frac{h}{f} (\Sigma_1 - c) \quad \Sigma_1 = \frac{n}{4} = \frac{30}{4} = 7.5$$

$$l = 390 \quad f = 7 \quad h = 4 \quad c = 10$$

$$= 390 + \frac{4}{7} (7.5 - 10)$$

$$= 390 + 0.5(-2.5)$$

$$= 390 - 1.25$$

$$\text{Quartile} = 388.75$$

Q2:- By Multiplying each of the numbers 3, 6, 2, 1, 7, 5 by 2 & then adding 5, we obtain 11, 17, 9, 7, 19, 15. What is the relation between the Standard deviation & the means of the two sets.

x	$x_i - u$	$(x_i - u)^2$
7	-6	36
9	-4	16
11	-2	4
17	4	16
15	2	4
19	6	36
$\frac{\sum x = 78}{6}$	$\sum (x_i - u) = 0$	$\sum (x_i - u)^2 = 112$

$$\text{Mean} = \frac{\sum x}{N}$$

$$= \frac{78}{6}$$

$$= 13$$

$$S.D = \sqrt{\frac{\sum (x_i - u)^2}{N}}$$

$$= \sqrt{\frac{112}{6}}$$

The mean is a measure of central tendency. The standard deviation is a measure of dispersion. Both are appropriate descriptive statistics for normally distributed data sets using ratio or interval scaling. Both mean and S.D are used in calculating some correlation coefficients, effect sizes, t scores.

The mean is a measure of central tendency. The standard deviation is a measure of dispersion. Both are appropriate descriptive statistics for normally distributed data sets using ratio or interval scaling. Both mean and standard deviation are used in calculating some correlation coefficients, effect sizes, t scores, F scores (Analysis of variance).

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Q3:- For the following grouped distribution table
Calculate. The variance & Standard Deviation.

Class	64-84	85-104	105-124	125-144	145-164	165-184	185-204
Frequency	15	18	27	10	6	5	13

Ans:-

Class	f	x	(x- \bar{x})	(x- \bar{x}) ²	f(x-x) ²
64-84	15	74	-49	2401	36015
85-104	18	94.5	-28.5	812.25	14620.5
105-124	27	114.5	-8.5	72.25	1951
125-144	10	134	11	121	1210
145-164	6	154.5	31.5	992.25	5953.5
165-184	5	174.5	51.5	2652.25	13261
185-204	13	194.5	71.5	5112.25	66459
Total	94				139470

$$\bar{x} = 123$$

$$\text{Variance} = S^2 = \frac{\sum f(x-\bar{x})^2}{n} = \text{Standard Deviation}$$

$$\therefore n = \sum f = \frac{139470}{94} = 1484 = \sqrt{S^2} = \sqrt{1484}$$

$$= S = 38.5$$

- Q4. if two fair dice are thrown, what is the probability of getting
1. A double six
 2. A Sum of 8 or more dots.

Solution: The Sample Space S is represented by the following 36 outcomes

$$S = \{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6) \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6) \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6) \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6) \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \}$$

- ① Let A be the event that double six occurs

$$A = \{ (6,6) \} \text{ \& thus}$$

$$P(A) = 1/36$$

- ② Let B denotes that Sum of 8 or more dots occurs

$$B = \{ (2,6), (3,5), (3,6), (4,4), (4,5), (4,6), (5,3), (5,4), (5,5) \\ (5,6), (6,2), (6,3), (6,4), (6,5), (6,6) \}$$

Hence

$$P(B) = 15/36 = 5/12$$

Q5:- Let C_1, C_2, \dots, C_M be a partition of the Sample Space SS , & A & B be two events. Suppose we know that

- A & B are conditionally independent given C_i , for all $i \in \{1, 2, \dots, M\}$
- B is independent of all C_i .

Prove that A & B are independent.

Solution:- Since the C_i 's form a partition of the sample space. We can apply the law of total probability for $A \cap B$:

$$P(A \cap B) = \sum_{i=1}^M P(A \cap B | C_i) P(C_i)$$

$$= \sum_{i=1}^M P(A | C_i) P(B | C_i) P(C_i) \quad (A \text{ \& } B \text{ are conditionally independent})$$

$$= \sum_{i=1}^M P(A | C_i) P(B) P(C_i) \quad (B \text{ is independent of all } C_i\text{'s})$$

$$= P(B) \sum_{i=1}^M P(A | C_i) P(C_i)$$

$$= P(B) P(A) \quad (\text{law of total Probability.})$$