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Subject:- Linear Algebra

2nd Semester Section A.

Software-engineering

Q1) $\begin{bmatrix} 1 & 7 & 3 & 0 & 5 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{bmatrix}$

$R1 \leftrightarrow 3R_3$

$\underset{\sim}{R} \left[\begin{array}{cccc|c} 1 & 7 & 0 & 0 & 11 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right]$

$R_1 - 7R_2$

$\underset{\sim}{R} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 7 \end{array}\right]$

Q2) a) $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$, $\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

$R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

Which is requied second matrix

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 0 & 3 & -5 \end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix} 1 & 3 & -1 & 5 \\ 0 & 1 & -4 & 2 \\ 0 & 2 & -5 & -1 \end{bmatrix}$$

this is required 1^{st} matrix

Q2b) A matrix that satisfies the following 4 properties is said to be in echelon form row

- All the leading entries in each of the rows of the matrix are 1.

- If a colum contain a Leading entry then all entries below that Leading entry are 0

- If any two consecutive non.zero rows, the leading entry in the upper row excess to the left of the Leading entry in the lower row

- All rows which consists entirely of zero appear at the bottom of the matrix.

C) $\begin{bmatrix} 5 & 0 & 0 & 7 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \end{bmatrix}$

it is not row echelon form because this matrix does not satisfies all the four properties of reduced Row echelon form

d) $\begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 \end{bmatrix} R_2 \leftrightarrow R_3 \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

It is reduced echelon form because it satisfies properties

a) $\begin{bmatrix} e & 0 & 0 & 0 \\ 0 & \pi & 0 & 0 \\ 0 & 0 & -\pi & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$ this is echelon form

b) $\begin{bmatrix} 1 & 0 & \pi \\ 0 & 1 & e \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ this is echelon form

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Q3b) echelon

echelon form of a matrix isn't unique which means there are infinite answers ~~to perform~~ possible when you perform row reduction

$$\begin{bmatrix} 1 & .5 & \cancel{5}.5 & 1.5 \\ 0 & 1 & 1.4 & -.6 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

reduced row echelon

is at the other end of the spectrum; it is unique, which means row-reduction on a matrix will produce the same answer no matter how you perform the same row operations.

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

The practical use of reduced row echelon form is that it has the advantage of being uniquely defined for a given matrix. it is worth to note that for an invertible matrix the reduced form is simply a unit matrix, the concept is only useful for degenerate cases.

Q3b) $\begin{bmatrix} 1 & 5 & 8 \\ 2 & 8 & -1 \\ -7 & 0 & 0 \\ 1 & -4 & 0 \end{bmatrix}$

$R_1 \leftrightarrow R_4$

$\begin{bmatrix} 1 & -4 & 0 \\ 2 & 8 & -1 \\ -7 & 0 & 0 \\ 1 & 5 & 8 \end{bmatrix}$

$R_2 + 7R_3$

$\begin{bmatrix} 1 & -4 & 0 \\ 2 & 8 & -1 \\ 0 & +35 & +56 \\ 1 & 5 & 8 \end{bmatrix}$

$R_4 - R_1$

$\begin{bmatrix} 1 & -4 & 0 \\ 2 & 8 & -1 \\ 0 & 35 & 56 \\ 0 & 9 & 8 \end{bmatrix}$

$R_4 - \frac{9}{35} R_3$

$$\begin{bmatrix} 1 & -4 & 0 \\ 2 & 8 & -1 \\ 0 & 35 & 56 \\ 0 & 0 & \frac{-224}{35} \end{bmatrix}$$