Date

Question 1(a):-

Determin the response.

Sol:-

$\lambda^2 - 4\lambda + 4 = 0$

$\lambda = 2, 2$ Hence

$y_h(n) = C_1 2^n + C_2 n 2^n$

The particular solution is

$y_p(n) = K(-1)^n u(n)$

subsituting this solution into the difference equation we obtain

$\Rightarrow K(-1)^n u(n) - 4K(-1)^{n-1} u(n-1) + 4K(-1)^{n-2} u(n-2) = (-1)^n u(n) - (-1)^{n-1} u(n-1)$

$\Rightarrow$ For $n = 2$, $K(1+4+4) = 2$

$K = 2/9$ the total solution is

$y(n) = [C_1 2^n + C_2 n 2^n + 2/9 (-1)^n] u(n)$

From the initial conditions we obtain, $y(0) = 1$, $y(1) = 2$

Than

$C_1 + 2/9 = 1$

$\Rightarrow C_1 = 7/9$

$2C_1 + 2C_2 - 2/9 = 2$

$C_2 = 1/3$

Date

Question 1(b) :- Determine the impulse response of system described

Sol :-

The characteristic equation is

$\lambda^2 - 0.7\lambda + 0.1 = 0$

$\lambda = 1/2 , 1/5$ Hence

$y_h(n) = C_1 \, 1/2^n + C_2 \, 1/5^n$

with $x(n) = \delta(n)$ we have

$y(0) = 2$

$y(1) - 0.7(0) = 0$

$\Rightarrow y(1) = 1.4$

Hence ; $C_1 + C_2 = 2$

And ,

$1/2\, C_1 + 1/5 = 1.4$

$1.4 = 7/5$

$\Rightarrow C_1 + 2/5\, C_2 = 14/5$

These equation yield

$C_1 = 10/3 \; , \; C_2 = -4/3$

$h(n) = \left[10/3 \, (1/2)^n - 4/3 \, (1/5)^n \right] u(n)$

The step response is

$S(n) = \sum_{k=0}^{n} h(n-k)$

$$S(n) = \frac{10}{3} \left(\frac{1}{2}\right)^n \sum_{k=0}^{n} 2k - \frac{4}{3} \left(\frac{1}{5}\right)^n \sum_{k=0}^{n} 5K$$

$$S(n) = \frac{10}{3} \left(\frac{1}{2}\right)^n (2^{n+1} - 1)\, u(n) - 1/3 \left(1/5\right)^n (5^{n+1} - 1)\, u(n$$

Welcome Notes

Date

Question 2(a) :-

Determine the causal Singnal

Sol :- Taking inverse and z-transform

$$\frac{A}{(1-2z^{-1})} + \frac{B}{(1-z^{-1})} + \frac{Cz^{-1}}{(1-z^{-1})^2}$$

$A = 4$, $B = -3$, $C = -1$

Hence

$$x(n) = [4(2)^n - 3 - n]\, u(n)$$

Question 2(b) :-

Evaluate the inverse z-transform.

Sol :-

Using the complex inversion integral we have,

$$x(n) = \frac{1}{2\pi j} \oint_c \frac{z^{n-1}}{1-az^{-1}}\, dz = \frac{1}{2\pi j} \oint_c \frac{z^n\, dz}{z-a}$$

Where C is circle at radius greater than $|a|$ we shall evaluate this integral using (3,4,2) with $f(z) = z^n$, we distinguish two cases. 1 If $n \geq 0$, $f(z)$ has radius only zero & hence

Welcome Notes

P.T.O

no poles inside C is $z=a$
Hence,

$$x(n)=f(z_0)=a^n \quad n\geq 0$$

(2)- if $n<0$ $f(z)=z^n$ has an n^{th} order pole at $z=0$, which is also inside C Thus there are contribution from both poles for $n=-1$ we have,

$$x(-1)=\frac{1}{2\pi j}\oint_c \frac{1}{z^2(z-a)}dz=\frac{1}{z-a}\Big|_{z=0}+\frac{1}{z}\Big|_{z=a}=0$$

if $n=-2$, we have

$$x(-2)=\frac{1}{2\pi j}\oint \frac{1}{z^2(z-a)}dz=\frac{d}{dz}\left(\frac{1}{z-a}\right)\Big|_{z=0}+\frac{1}{z^2}\Big|_{z=a}=0$$

By Continuing in same way we can show that $x(n)=0$
For $n<0$, Thus

$$x(n)=a^n u(n)$$

Date

Question 3 (a) :–

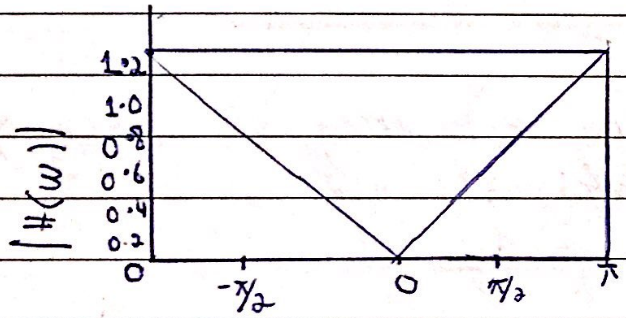
Sol :–

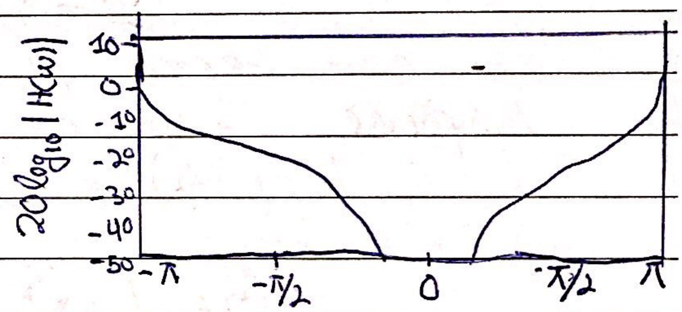
At $\omega = 0$ we have

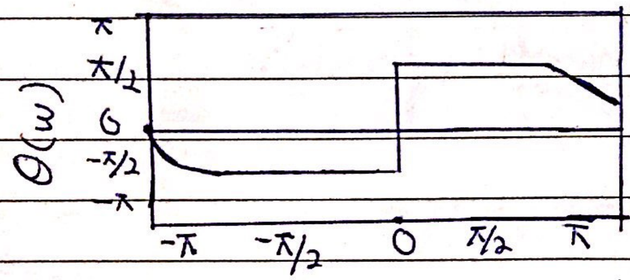
$$H(0) = \frac{b_0}{(1-p)^2} = 1$$

Hence

$$b_0 = (1-p)^2$$







At $\omega = \pi/4$

$$H(\pi/4) = (1-p)^2 / (1 - pe^{-j\pi/4})^2$$

$$= (1-p)^2 / [1 - p\cos(\pi/4) + jp\sin(\pi/4)^2]$$

$$= (1-p)^2 / [1 - p/\sqrt{2} + jp/\sqrt{2}]^2$$

Hence

$$= \frac{(1-p)^4}{[(1-p/\sqrt{2})^2 + p^2/2]^2}$$

$$= 1/2$$

Equivalently, $\sqrt{2}(1-p)^2 = 1 + p^2 - \sqrt{2}p$

The system function for desired filter

$$H(z) = \frac{0.46}{(1 - 0.32z^{-1})^2}$$

Date

Question 3 (b) :-

Sol :-

The filter must have poles a $P_{12} = r_c$
And zero at $z=1$ & $z=-1$.
Consequently the same system function

$$\Rightarrow H(z) = G\frac{(z-1)(z+1)}{(z-jr)(z+jr)}$$

$$H(z) = G\frac{z^2-1}{z^2+r^2}$$

the gain factor is determine evaluating the frequency response $H(w)$ of the filter at $w=\pi/2$

$$H(\pi/2) = G\frac{2}{1-r^2} = 1$$

$$G = \frac{1-r^2}{2}$$

The value of r is determined by evaluating $H(w)$ at $w = 4\pi/9$

We have,

$$\left|H\left(\frac{4\pi}{9}\right)\right|^2 = \frac{(1-r^2)^2}{4}\frac{2-2\cos(8\pi/9)}{1+r^4+2r^2\cos(8\pi/9)}$$

$$= 1/2$$

$\Rightarrow$ Equivalently

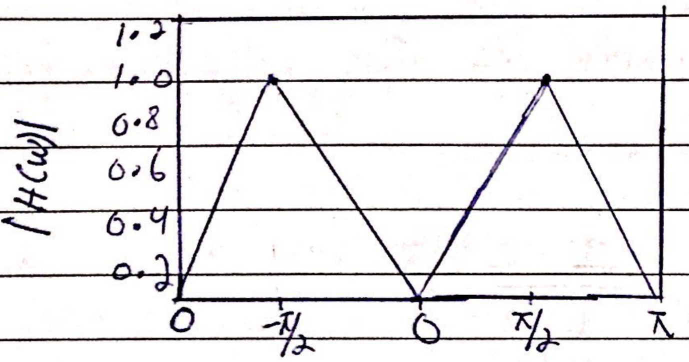
$$1.94(1-r^2)^2 = 1-1.88r^2+r^4$$

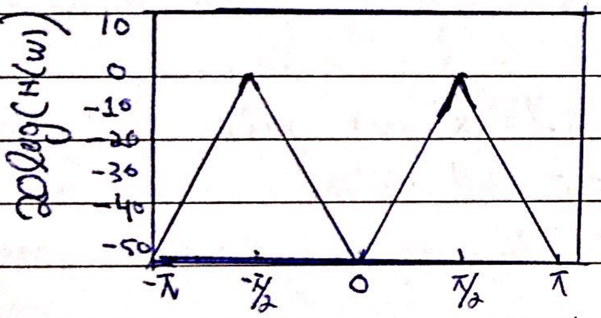
The value of $r^2 = 0.7$ satisfies this equation therefore the system function for desired filter is

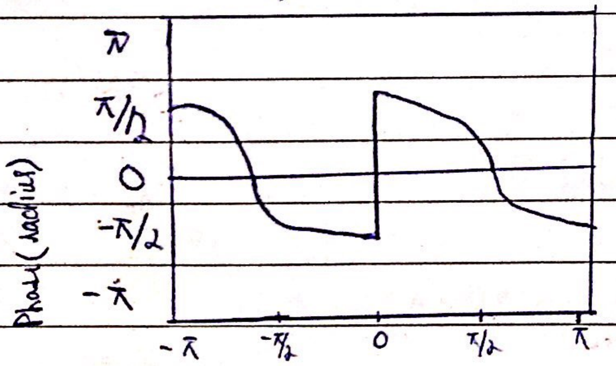
Date

$\Rightarrow$ $H(z) = 0.15 \dfrac{1-z^{-2}}{1+0.7z^{-2}}$

its Frequency responce is illustrated







magnitude and Phase response of a simple bandpass Filter is

$$H(z) = 0.15\left[(1-z^{-2}) / (1+0.7z^{-2})\right]$$

Date

Question 4(a) :-

Sol :-

The fourier transform of this sequence is

$$X(w) = \sum_{n=0}^{L-1} x(n)e^{-jwn}$$

$$x(w) = \sum_{n=0}^{L-1} e^{-jwn} = \frac{1-e^{-jwL}}{1-e^{-jw}}$$

$$= \frac{\sin(wL/2)}{\sin(w/2)} e^{-jw(L-1)/2}$$

The magnitude and phase of $x(w)$ are illustrated for $L=10$, The N point DFT of $x(n)$ is simply $x(w)$ evaluated at the set of N equally spaced frequency

$\Rightarrow w_K = 2\pi K/N,$

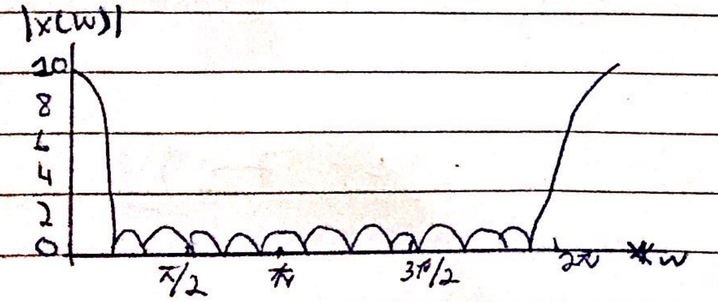
$K = 0, 1, 2 \ldots\ldots N-1.$

Hence,

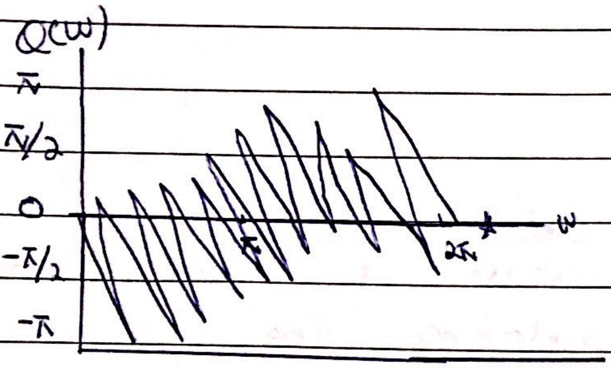
$$x(K) = \frac{1-e^{-j2\pi KL/N}}{1-e^{-j2\pi K/N}}$$

$K = 0, 1, \ldots\ldots, N-1$

$$x(K) = \frac{\sin(\pi KL/N)}{\sin(\pi K/N)} e^{-j\pi K(L-1)/N}$$



Date



If N is selected such that $N=L$ then the discrete Fourier transform becomes

$$x(K) = \begin{cases} L, & K=0 \\ 0, & K=1,2,\cdots L-1 \end{cases}$$

Thus there is only one non zero value in the DFT this apparent from observation of $X(\omega)$, Since $X(\omega)=0$ at the frequencies $\omega K = 2\pi K/L$, $K \neq 0$;

The reader should verify that $x(n)$ can be recovered from $X(K)$ by performing an L-point IDFT provides a plot of the N-Point DFT in magnitude phase for $L=10$ $N=50$ & $N=100$, Now the spectral characteristic of the sequence are more clearly evident.

Date

Question 4 (b) :-

Sol:- L

Each sequence consist of four non zero points for the purpose of illustrating the operation involved in circular convolution it is desired to graph each sequence, as point on a circle then the sequence $x_1(n)$ and $x_2(n)$ are graphed as illustrated we note that the sequence are grapheed in a counter clock wise direction on a circle

Now, $x_3(m)$ is obtained by circularly convolving $x_1(n)$ with $x_2(n)$ as specified. ~~Begin~~

Begining with $m=0$ we have ;

$$x_3(m) = \sum_{n=0}^{3} x_1(n)\, x_2[(-n)]N$$

$x_2(-n)_4$ is simply the sequence $x_2(n)$ folded & graphed on a circle.

The product sequence is obtained by multiplying $x_1(n)$ with $x_2(-n)_4$ point by point Finally we sum the values in the product sequence to obtain

$$x_3(0) = 14$$

For $m=1$ we have

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n)_4$$

it is easily verified that $x_2(1-n)_4$ is simply the sequence $x_2(-n)_4$ rotated counter clockwise by on unit in the time.

This rotated sequence multplies $x_1(n)$ to yield the product

Date

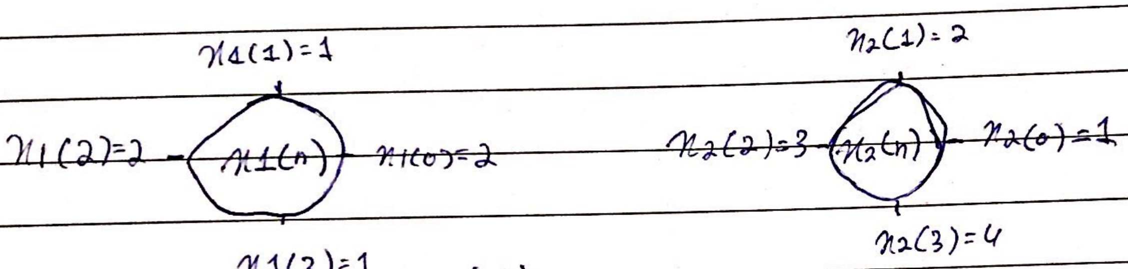
product sequence to obtain $x_3(1)$ Thus

$x_3(1) = 16$

For $m = 2$, we have

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2(2-n)_4$$

Now $x_2(2-n)_4$ is the folded sequence.



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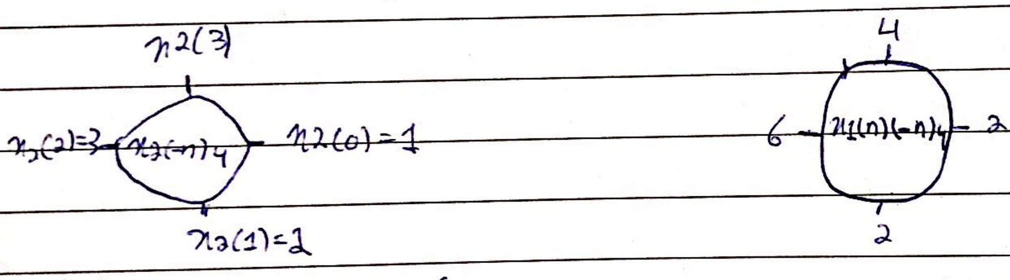
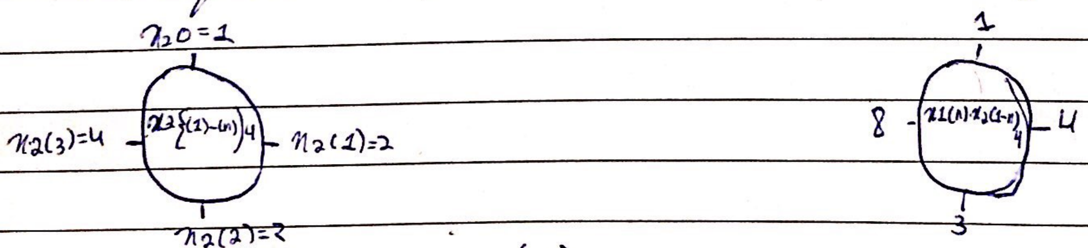
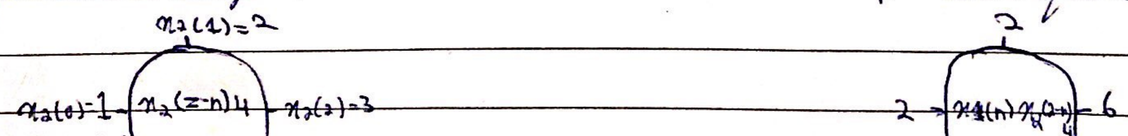


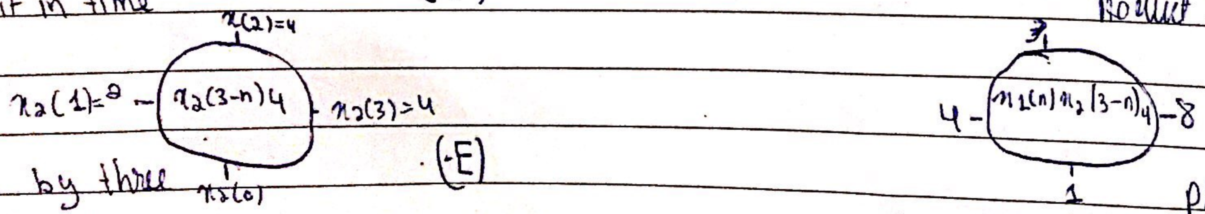
Folded sequence (B) product sequence



folded seque rotated by one unit in time (c) product sequence



Folded sequence rotated by two unit in time (D) Product Sequence.



rotated by three unit in time (E) product seq.

Welcome Notes