Name # M. Arsalan Shah.

ID # 7830.

Submitted To : Mam. Shomaila

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Q NO# 1

Find the Fourier Series representation of

$$F(t) = 1+t, \; -\pi \le t \le \pi$$

Solution:

$$F(t) = 1+t \quad -\pi \le t \le \pi$$

Here we use the formula:

$$\Rightarrow f(x) = a_0 + \sum_{n=1} a_n \cos t + \sum_{n=-1} b_n \sin t \longrightarrow eq(i).$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t)\,dt.$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} (1+t)\,dt.$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left| t + \frac{t^2}{2} \right|_{-\pi}^{\pi}$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left(\pi - (-\pi) + \frac{\pi^2}{2} - \left(-\frac{\pi^2}{2}\right) \right).$$

$$\Rightarrow a_0 = \frac{1}{2\pi} \left(2\pi + \frac{2\pi}{2} \right)$$

$$\Rightarrow \boxed{a_0 = \frac{1}{2\pi} \left(2\pi + \pi^2 \right)}$$

$$\Rightarrow a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (1+t)(\cos nt)\,dt$$

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$$\Rightarrow a_n = \frac{1}{\pi}\left((1+t)\frac{\sin nt}{n}\Big|_{-\pi}^{\pi} - \int\left(\frac{\sin nt}{n}\frac{d}{dt}(1+t)\right)\right).$$

$$\Rightarrow a_n = \frac{1}{\pi}\left((1+t)\cancel{\frac{\sin nt}{n}}^{0} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi}\right).$$

$$\Rightarrow a_n = \frac{-1}{n^2\pi}\left(\cos n\pi - \cos n(-\pi)\right)$$

$$\Rightarrow a_n = -1/n^2\pi\left(-1 - (-1)\right)$$

$$\Rightarrow \boxed{a_n = 0}$$

Now !

$$\Rightarrow b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(1+t)\sin nt\,dt.$$

$$\Rightarrow b_n = \frac{1}{\pi}\left((1+t)\int_{-\pi}^{\pi}\sin nt - \int\left(\sin nt\,\frac{d}{dt}(1+t)\,dt\right)\right).$$

$$\Rightarrow b_n = \frac{1}{\pi}\left((1+t)\frac{(-\cos nt)}{n}\Big|_{-\pi}^{\pi} + \int\cancel{\frac{-\cos nt}{n}}^{0}(1)\right).$$

$$\Rightarrow b_n = -1/n\pi\left((1+t)(\cos n\pi) - (1+(\pi))(\cos nt)\right).$$

$$\Rightarrow b_n = -1/n\pi\left(\cancel{\cos n\pi} + \pi\cos n\pi - \cancel{\cos n\pi} + \pi\cos n\pi\right).$$

$$\Rightarrow b_n = -1/n\cancel{\pi}\left(2\cancel{\pi}\cos n\pi\right)$$

$$\Rightarrow \text{Here } \cos n\pi = \frac{(-1)^{n+1}}{n}$$

$$\Rightarrow b_n = 2/n\,(-1)^{n+1}$$

So eq. become:

$$\boxed{f(x) = 1/2\pi\,(2\pi+\pi^2) + 0 + \sum_{n=1}^{\infty} 2\frac{(-1)^{n+1}}{n}\sin t}$$ Ans.

QNO # 2

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix}$$

Eigen Values = ?

Solution :

Step # 1

We know that:

$$\Rightarrow \quad (A - \lambda I) X = 0$$

$\because A$ = Given Matrix .

Step # 2

The characteristic equation is given by :

$$\Rightarrow \quad |A - \lambda I| = 0$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad \begin{vmatrix} 1-\lambda & 0 & -1 \\ 3 & 1-\lambda & 4 \\ 0 & 2 & 2-\lambda \end{vmatrix} = 0$$

Step # 3

$$\Rightarrow \quad \lambda^3 - \left|\text{sum of Digonal element}\right| \lambda^2 + \left|\text{sum of Digonal minors}\right| \lambda - |A| = 0 \quad \text{—— B}$$

$\Rightarrow$ Sum of diagonal element $= 1 + 1 + 2 = 4$

$\Rightarrow$ Sum of Diagonal minors $= \begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} + \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 0 \\ 3 & 1 \end{vmatrix}$

$\Rightarrow (-6) + (2) + (1)$

$\Rightarrow -6 + 2 + 1$

$\Rightarrow \boxed{-3}$.

$\Rightarrow$ By Putting values in eq (B)

$\Rightarrow \lambda^3 - 4\lambda^2 - 3\lambda - |A| = 0$ —— (C)

$\Rightarrow |A| = \begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 4 \\ 0 & 2 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 2 & 2 \end{vmatrix} - 0\begin{vmatrix} 3 & 4 \\ 0 & 2 \end{vmatrix} + 1\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix}$

$\Rightarrow 1(2-8) - 0 + 1(6-0).$

$\Rightarrow -6+6$

$\Rightarrow \boxed{0}$.

$\Rightarrow$ By Putting value in (C)

$\Rightarrow \lambda^3 - 4\lambda^2 - 3\lambda - 0 = 0$

$\Rightarrow \lambda^3 - 4\lambda^2 - 3\lambda = 0$

$\Rightarrow \lambda(\lambda^2 - 4\lambda - 3) = 0$

$\boxed{\lambda = 0}$

$\Rightarrow \lambda^2 - 4\lambda = 0$

P.T.O

⇒ Using Quadratic formula:

Where:

$a = 1$

$b = -4$

$c = -3$

$$\Rightarrow \lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Put the value:

$$\Rightarrow \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-3)}}{2(1)}$$

$$\Rightarrow \frac{4 \pm \sqrt{16+12}}{2} \Rightarrow \frac{4 \pm \sqrt{28}}{2}$$

$$\Rightarrow \lambda = \frac{4+\sqrt{28}}{2}, \quad \lambda = \frac{4-\sqrt{28}}{2}$$

We have Eigen values:

$$\lambda = \left(0, \frac{4+\sqrt{28}}{2}, \frac{4-\sqrt{28}}{2}\right)$$

Requried solution:.

Q no # 3

Solve the following system of linear eq:

$$5x + 0 + 4z + 2m = 3$$
$$x - y + 2z + m = 1$$
$$4x + y + 2z + 0 = 1$$
$$x + y + z + m = 0$$

Solution:

We know that:

$$\Rightarrow \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{array}\right] \quad R_4 \times R_2$$

$$\Rightarrow \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 4 & 1 & 2 & 0 & 1 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & +6/5 & +4/5 & 3/5 \\ 0 & 2 & -1 & 0 & -1 \end{array}\right] \quad -1/5 \times R_2$$

$$\Rightarrow \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -1 & 6/5 & 4/5 & 3/5 \\ 0 & 0 & 7/5 & 3/5 & 1/5 \end{array}\right] \quad 5 \times R_3 \;\&\; 5 \times R_4$$

$$\Rightarrow \left[\begin{array}{cccc|c} 5 & 0 & 4 & 2 & 3 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 5R_3 \ \& \ 5R_4$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 1 & -1 & 2 & 1 & 1 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad 1/5 \times R_1$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -1 & 6/5 & 1/5 & 2/5 \\ 0 & -5 & 6 & 4 & 3 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_2 \times 5$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -6 & 6 & 1 & 2 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 7 & 8 & 1 \end{array}\right] \quad R_3 - R_2$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 3 & 1/7 \\ 0 & 0 & 0 & 8 & 1/3 \end{array}\right] \quad \begin{array}{l} R_3 \leftrightarrow R_4 \\ 1/7 \times R_3 \\ 1/3 \times R_4 \end{array}$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & -5 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad R_2 \times -5$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 6 & 1 & 2 \\ 0 & 0 & 1 & 1 & -4/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

P.T.O

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & -5 & 24/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 4/5 & 2/5 & 3/5 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1/2 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right] \quad 5/4 \times R_1$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 126/84 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right].$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 1/2 & 1/2 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right].$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 3/4 \\ 0 & 1 & 0 & 0 & 31/21 \\ 0 & 0 & 1 & 0 & -11/21 \\ 0 & 0 & 0 & 1 & 1/3 \end{array}\right].$$

$$\Rightarrow (x, y, z \;\&\; m) = \left(3/4, 31/21, -11/21, 1/3\right).$$

Where!

$x = 3/4$

$y = 31/21$

$z = -11/21$

$m = 1/3$ Answer:

Q No # 4

$$U(x,t) = \sin(x+2t)$$

is a solution of one-dimensional wave eq.

Solution :

$$u(x,t) = \sin(x+2t):$$

Differentiate w.r.t x partially.

$$\Rightarrow \frac{\partial u}{\partial u} = \frac{\partial}{\partial x}\sin(x+2t)$$

$$\Rightarrow \frac{\partial u}{\partial x} = \cos(x+2t)\frac{\partial}{\partial x}(x+2t)$$

$$\Rightarrow \partial u/\partial x = \cos(x+2t)(1+0).$$

$$\Rightarrow \partial u/\partial x = \cos(x+2t)$$

$$\Rightarrow \partial^2 u/\partial x^2 = \frac{\partial}{\partial x}\cos(x+2t)$$

$$\Rightarrow \frac{\partial^2 u}{\partial x^2} = -\sin(x+2t)\cdot\frac{\partial}{\partial x}(x+2t)$$

$$\Rightarrow \partial^2 u/\partial x^2 = -\sin(x+2t)(1+0)$$

$$\Rightarrow \boxed{\partial^2 u/\partial x^2 = -\sin(x+2t)}.$$

So :

$$u(x,t) = \sin(x+2t)$$

Again diff w.r.t "t".

$$\Rightarrow \partial u/\partial t = \partial/\partial t \sin(x+2t)$$

$$\Rightarrow \partial u/\partial t = 2\cos(x+2t)\ (0+2)$$

$$\Rightarrow \partial u/\partial t = 2\cos(x+2t)$$

$$\Rightarrow \partial^2 u/\partial t^2 = (2) - \sin(x+2t)\ (0+2)$$

$$\Rightarrow \boxed{\partial^2 u/\partial t^2 = -4\sin(x+2t)}$$

We know that One-dimensional wave eq:

$$\Rightarrow \partial^2 u/\partial t^2 = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$\Rightarrow -4\sin(x+2t) = c^2\left[-\sin(x+2t)\right].$$

$$\Rightarrow -4\sin(x+2t) = -c^2\sin(x+2t)$$

$$\Rightarrow -4\sin(x+2t) + c^2\sin(x+2t) = 0$$

$\Rightarrow$ For the arbitrary constant $c = \pm 2$

P.T.O

$$\Rightarrow \quad -4\sin(x+2t) + (\pm 2)^2 \sin(x+2t) = 0$$

$$\Rightarrow \quad -4\sin(x+2t) + 4\sin(x+2t) = 0$$

$$0 = 0$$

Then it will be verified for the Orbitrary constant

$$\boxed{c = 2}.$$

The End !