``

Department of Electrical Engineering

MUHAMMAD YASIR

13122

Final Assignment

Electro Magnetic Field Theory

spring-2019-20 Dated 23-6-2020

Instructor: Dr. Rafiq Mansoor

``

PROBLEM 4.1. The value of E at P ($\rho = 2$, $\varphi = 40^\circ$, $z = 3$) is given as $E = 100a_\rho - 200a_\varphi + 300a_z$ V/m. Determine The incremental work required to move a 20 μC charge a distance of 6 μm in the direction of: (a) a_ρ ;(b) a_φ; (c) a_z; (d) E (d) $G = 2a_x - 3a_y + 4a_z$.

Solution.

a) In the direction of a_ρ : The incremental work is given by $dW = -q\,E \cdot dL$, where in this case, $dL = d\rho\, a_\rho = 6 \times 10^{-6}\, a_\rho$.
Thus
$dW = -(20 \times 10^{-6}\, C)(100\, V/m)(6 \times 10^{-6}\, m)$
$= -12 \times 10^{-9}$ J
$= -12$ nJ
b) In the direction of a_φ: In this case $dL = 2\, d\varphi\, a_\varphi = 6 \times 10^{-6}\, a_\varphi$, and so
$dW = -(20 \times 10^{-6})(-200)(6 \times 10^{-6})$
$= 2.4 \times 10^{-8}$ J
$= 24$ nJ
c) in the direction of a_z: Here, $dL = dz\, a_z = 6 \times 10^{-6}\, a_z$, and so
$dW = -(20 \times 10^{-6})(300)(6 \times 10^{-6})$
$= -3.6 \times 10^{-8}$ J
$= -36$ nJ
d) in the direction of E: Here, $dL = 6 \times 10^{-6}\, a_E$, where
$a_E = 100a_\rho - 200a_\varphi + 300a_z / [100^2 + 200^2 + 300^2]^{½}$
$= 0.267\, a_\rho - 0\ 535\, a_\varphi + 0.802\, a_z$

.

Thus
$dW = -(20 \times 10^{-6})[100a_\rho - 200a_\varphi + 300a_z] \cdot [0.267\, a_\rho - 0.535\, a_\varphi + 0.802\, a_z](6 \times 10^{-6})$
$dW = -44.9$ nJ
e) In the direction of $G = 2a_x - 3a_y + 4a_z$: In this case, $dL = 6 \times 10^{-6}\, a_G$, where
$a_G = 2a_x - 3a_y + 4a_z / [2^2 + 3^2 + 4^2]^{½}$
$= 0.371\, a_x - 0.557\, a_y + 0.743\, a_z$
So now
$dW = -(20 \times 10^{-6})[100a_\rho - 200a_\varphi + 300a_z] \cdot [0.371\, a_x - 0.557\, a_y + 0.743\, a_z](6 \times 10^{-6})$
$= -(20 \times 10^{-6})\ [37.1(a_\rho \cdot a_x) - 55.7(a_\rho \cdot a_y) - 74.2(a_\varphi \cdot a_x) + 111.4(a_\varphi \cdot a_y) + 222.9](6 \times 10^{-6})$

``

where, at P, $(a_\rho \cdot a_x) = (a_\varphi \cdot a_y) = \cos(40°) = 0.766$, $(a_\rho \cdot a_y) = \sin(40°) = 0.643$, and
$(a_\varphi \cdot a_x) = -\sin(40°) = -0.643$. Substituting these results in
$dW = -(20 \times 10^{-6})[28.4 - 35.8 + 47.7 + 85.3 + 222.9](6 \times 10^{-6})$
$= -41$ nJ

PROBLEM 4.2.
Let $E = 400a_x - 300a_y + 500a_z$ in the neighborhood of point P (6, 2, −3). Find the incremental work done in moving a 4-C charge a distance of 1 mm in the direction specified by:

Solution.

a) $a_x + a_y + a_z$: We write
$dW = -qE \cdot dL$
$= -4(400a_x - 300a_y + 500a_z) \cdot [(a_x + a_y + a_z) / \sqrt{3}](10^{-3})$
$= -(4 \times 10^{-3})\ \sqrt{3}\ (400 - 300 + 500)$
$= -1.39$ J
b) $-2a_x + 3a_y - a_z$: The computation is similar to that of part a, but we change the direction:
$dW = -qE \cdot dL$
$= -4(400a_x - 300a_y + 500a_z) \cdot [(-2a_x + 3a_y - a_z) / \sqrt{14}]\ (10^{-3})$
$= -(4 \times 10^{-3})\ \sqrt{14}\ (-800 - 900 - 500)$
$= 2.35$J

PROBLEM 44.3.
If $E = 120\ a_\rho$ V/m, find the incremental amount of work done in moving a 50 μm charge a distance of 2 mm from:
Solution.

a) P (1, 2, 3) toward Q(2, 1, 4): The vector along this direction will be Q− P = (1, −1, 1) from which $a_{PQ} = [a_x - a_y + a_z]/\sqrt{3}$. We now write
$dW = -qE \cdot dL$
$= -(50 \times 10^{-6})\ [120a_\rho \cdot \{(a_x - a_y + a_z) / \sqrt{3}\}]\ (2 \times 10^{-3})$
$= -(50 \times 10^{-6})(120)\ (a_\rho \cdot a_x) - (a_\rho \cdot a_y)\ [1/\sqrt{3}](2 \times 10^{-3})$
At P, $\varphi = \tan^{-1}(2/1) = 63.4°$. Thus $(a_\rho \cdot a_x)$
$= \cos(63.4) = 0.447$ and $(a_\rho \cdot a_y)$
$= \sin(63.4)$
$= 0.894$. Substituting these, we obtain
$dW = 3.1$μJ.
b) Q(2, 1, 4) toward P (1, 2, 3): A little thought is in order here: Note that the fifield has only a radial component and does not depend on φ or z. Note also that P and Q are at the same radius (√ 5) from the z axis, but have different φ

``

and z coordinates. We could just as well position the two points at the same z location and the problem would not change. If this were so, then moving along a straight line between P and Q would thus involve moving along a chord of a circle whose
radius is √ 5. Halfway along this line is a point of symmetry in the field (make a sketch to see
this). This means that when starting from either point, the initial force will be the same. Thus the
answer is dW = 3.1μJ as in part a. This is also found by going through the same procedure as
in part a, but with the direction (roles of P and Q reversed.

PROBLEM 4.5.

Compute the value of $\int_A^P$ G · dL for G = 2yax with A(1, −1, 2) and P (2, 1, 2)

using the path;
Solution.

a) straight-line segments A(1, −1, 2) to B(1, 1, 2) to P (2, 1, 2): In general we would have
_x0005_

$\int_A^P$ G · dL = _x0005_ $\int_A^P$ 2y dx

The change in x occurs when moving between B and P, during which y = 1. Thus

$\int_A^P$ G · dL = $\int_B^P$ 2y dx = $\int_1^2$ 2(1)dx = 2

b) straight-line segments A(1, −1, 2) to C(2, −1, 2) to P (2, 1, 2): In this case the change in x occurs when moving from A to C, during which y = −1. Thus

$\int_A^P$ G · dL = $\int_A^C$ A 2y dx = $\int_1^2$ _x0005_ 2(−1)dx = −2

PROBLEM 4.7.

Repeat Problem 4.6 for G = 3xy3ax + 2zay . Now things are different in that the path does matter:
Solution.

a) straight line: y = x − 1, z = 1: We obtain:

$\int$ G · dL = $\int_2^4$ 3xy² dx + $\int_1^3$ 2z dy = $\int_2^4$ 3x(x − 1)² dx + $\int_1^3$ 2(1) dy

= 90

b) parabola: 6y = x2 + 2, z = 1: We obtain:

``

$\int G \cdot dL = \int_{2}^{4} 3xy^2 \, dx + \int_{1}^{3} 2z \, dy =$ _x0005_ $\int_{2}^{4} 1/12x(x^2 + 2)^2 \, dx +$ _x0005_

$\int_{1}^{3} 2(1) \, dy$

$= 82$