Department of Electrical Engineering
Assignment
Date: 13/04/2020

Course Details

Course Title: Digital Signal Processing **Module:** 6th
Instructor: **Total Marks:** 30

Student Details

Name: owais afridi **Student ID:** 13686

Q1.	(a)	Consider the following analog signal $$x_a(t) = 3cos100\pi t + 4sin200\pi t$$ i. Determine the minimum sampling rate required to avoid aliasing. ii. Suppose that the signal is sampled at the rate $F_s = 100Hz$. What is the discrete-time signal obtained after sampling? Also explain the effect of this sampling rate on the newly generated discrete time signal. iii. What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?	Marks 5 CLO 1
	(b)	Consider a discrete time signal which is given by $$x(n) = \begin{cases} 0.5^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$ This is signal is sampled at the rate $F_s = 2Hz$. i. Draw the sampled signal. ii. The samples of the signals are intended to carry 3 bits per sample. Determine the quantization level and quantization resolution to quantized the sampled signal achieved in part i . iii. Perform the process of truncation and rounding off on all the values of the sampled signal and find the quantization error for each of the sampled data. Express your answer in tabular form.	Marks 5 CLO 1
Q2.	(a)	Determine the response of the system to the following input signal with given impulse response $$x[n] = \left\{2, \underset{\uparrow}{1}, -2, 3, -4\right\} \quad , h[n] = \left\{\underset{\uparrow}{3}, 1, 2, 1, 4\right\}$$	Marks 5 CLO 2

	(b)	Compute the convolution y(n) of the following signal $$x(n) = \begin{cases} \alpha^{n+1}, & -3 \le n \le 5 \\ 0, & elsewhere \end{cases}$$ $$h(n) = \begin{cases} 2^{n}, & 0 \le n \le 4 \\ 0, & elsewhere \end{cases}$$	Marks 5 CLO 2
Q3.		Determine the z- transform of the following signals and also sketch its Region of Convergence (ROC). i. $x(n) = \begin{cases} (\frac{1}{4})^{n}, & n \ge 0 \\ (\frac{1}{3})^{-n}, & n < 0 \end{cases}$ ii. $x(n) = \begin{cases} (\frac{1}{2})^{n} - 3^{n}, & n \ge 0 \\ 0, & elsewhere \end{cases}$	Marks 10 CLO 2

Owais Afridi ID 13686 P# 1

Q1: Consider the following analog Signal

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

Part (a)

(i) Determine the minimum Sampling rate required to avoid aliasing.

Minimum Sampling rate:

$x_a(t) = 3\cos 100\pi t + 4\sin 200\pi t$

$F_1 = 50\,Hz$

$F_2 = 100\,Hz$

$F_s = 100\,Hz$

F_s is maximum than F_1

$F_1 = 50$ is minimum sampling rate to avoid aliasing.

$f_b = 2 f_{max}$

F_2 is max, $F_2 = 100\,Hz$

$F_b = 2 \times 100$

$F_b = 200$

(ii) We have

$$F_s = 100\ Hz$$

So

$$f_1' = \frac{f_1}{F_s} = \frac{50}{100} = 0.5\ Hz$$

F_2 becomes

$$f_2' = \frac{F_2}{F_s} = \frac{100}{100} = 1\ Hz$$

So $W_1' = 2\pi f_1'$

$$W_1' = 2\pi \times 0.5$$

$$W_1' = \pi$$

and,

$$W_2' = 2\pi f_2'$$

$$W_2' = 2\pi \times 1$$

$$W_2' = 2\pi$$

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$x[n] = 3\cos 100\pi t + 4\sin 200\pi t$

$x[n] = 3\cos \pi t + 4\sin 2\pi t.$

(iii) We can construct the orignal signal and also frequency component. Since at 50 Hz and 100Hz are present in the sampled signal.
The signal we can recover is

$y_a(t) = 3\cos \pi t + 4\sin 2\pi t$

$x_a(t)$, Orignal signal is different from it. due to low samping rate used, distortion of the orignal analog signal was caused by the aliasing effect.

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Q#1

Part(b) Consider a discreate time Signal which is given by

$$x(n) = \begin{cases} 0.5^n & , n \geq 0 \\ 0 & , n < 0 \end{cases}$$

This is Signal is Sampled at the rate $F_s = 2Hz$

As We Know that

$$F_s = \frac{1}{T}$$

$$\Rightarrow T = \frac{1}{F_s}$$

$$T = \frac{1}{2}$$

$$T = 0.5 \text{ Sec}$$

(i) Draw the Sampled Signal.

x_n	0.5^n
0	1
0.5	0.707
1	0.5
1.5	0.25

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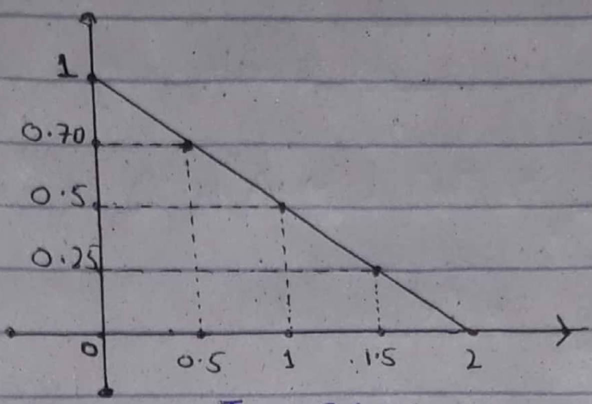

T = 0.5 sec

(ii) The Samples of the Signals are intended to Carray 3 bits per Samples. Determine the quantization level and quantization resolution to quantized the Sampled Signal achived in part (i).

$n = 3$ bits/sample

first we have to find qountization level:

$L = 2^n$

$L = 2^3$

$L = 8$

Quantization resoclution $(\Delta) = \frac{x_{max} - x_{min}}{L}$

$\Delta = \frac{1-0}{8}$

$\Delta = \frac{1}{8}$

$\Delta = 0.142$

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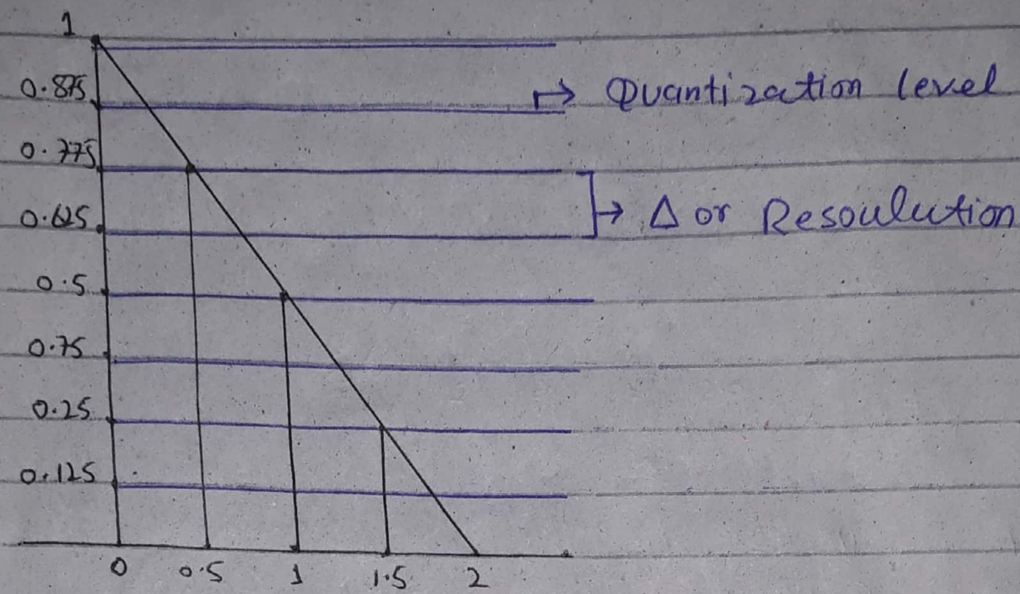

(iii) Perform the process of truncation and rounding off number of the sampled signal and find the Quantization error for each of the sampled data. Express your answer in tabuler form.

n	Discreati Time Signal	Rounding	Truncation	$e_q[n] = x_q[n] - x[n]$
0	1	1	1	0
1	0.87	0.8	0.8	0.0
2	0.75	0.7	0.7	0.0
3	0.65	0.6	0.6	0.0
4	0.5	0.5	0.5	0.0
5	0.37	0.3	0.4	-0.1
6	0.25	0.2	0.3	-0.1
7	0.12	0.1	0.1	0.0

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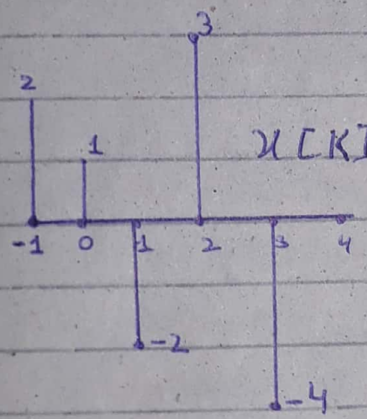
Q # 2

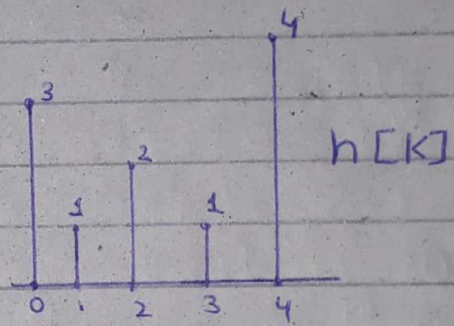
part(a): Determine the response of the System to the following input with given impulse response

$$x[n] = \{2, \underset{\uparrow}{1}, -2, 3, -4\}$$

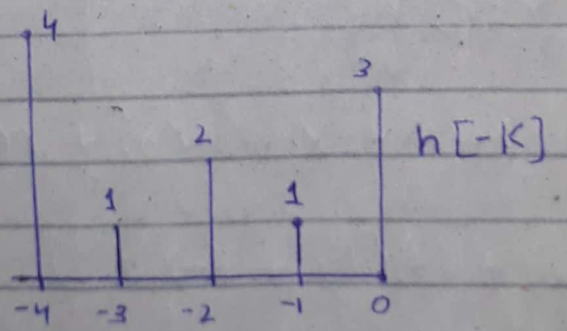
$$h[n] = \{\underset{\uparrow}{3}, 1, 2, 1, 4\}.$$

Solution:





Convert h[K] into h[-K]:



As,

$$y[n] = \sum_{K=-\infty}^{\infty} x[K]\,h[n-K]$$

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For $n=0$:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k].$$

$$y[0] = \sum_{k=-1}^{0} x[k]\,h[-k].$$

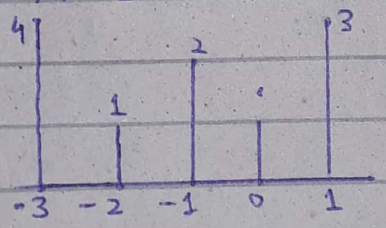
$$= x[-1]\,h(-1) + x(0)\,h(0)$$

$$= (2)(1) + (1)(3)$$

$$= 2+3$$

$$y[0] = 5$$

For $n=1$:



$$y[1] = \sum_{k=-1}^{1} x[n]\,h[1-k]$$

$$= x(-1)\,h(-1) + x(0)\,h(0) + x(1)\,h(1)$$

$$= (2)(2) + (1)(1) + (-2)(3)$$

$$= 4+1-6$$

$$= 5-6$$

$$y[1] = -1$$

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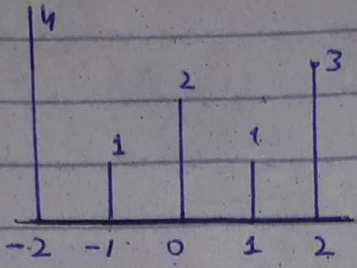
For $n=2$:



$h[2-k]$.

$$y[2] = \sum_{k=-1}^{2} x[k]\,h[2-k].$$

$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$

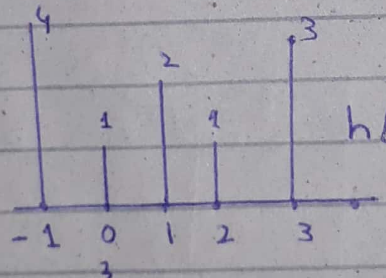
$= (2)(1) + (1)(2) + (-2)(1) + (3)(3).$

$= 2 + 2 - 2 + 9$

$= 2 + 9$

$y[2] = 11$

For $n=3$:



$h[3-k]$.

$$y[3] = \sum_{k=-1}^{3} x[n]\,h[3-k]$$

$= x(-1)h(-1) + x(0)h(0) + x(1)h(1) + x(2)h(2)$
$+ x(3)h(3)$

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$y[3] = (2)(4) + (1)(1) + (-2)(2) + (3)(1) + (-4)(3)$

$= 8 + 1 - 4 - 12 + 3$

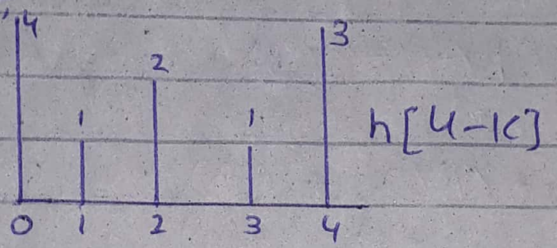
$= 1 - 4 - 4 + 3$

$= 1 - 8 + 3$

$y(3] = -7 + 3$

$= -4$

$n = 4$:



$$y[4] = \sum_{K=0}^{3} x[K]\,h[4-K]$$

$= x(0)h(0) + x(1)h(1) + x(2)h(2) + x(3)h(3)$.

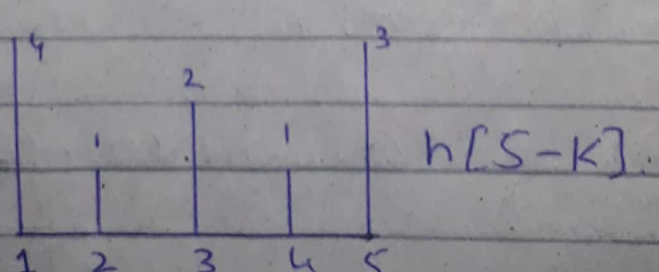
$= (1)(4) + (-2)(1) + (3)(2) + (-4)(1)$

$= 4 - 2 + 6 - 4$

$= -2 + 6$

$y[4] = 4$

$n = 5$:



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$$y[5] = \sum_{k=1}^{3} x[k]\, h[5-k].$$

$= x(1)h(1) + x(2)h(2) + x(3)h(3).$

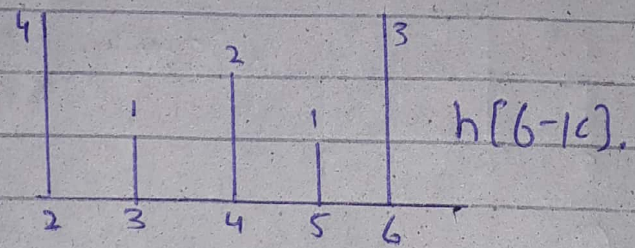
$= (-2)(4) + (3)(1) + (-4)(2)$

$= -8 + 3 - 8$

$= -13$

$n = 6:$

$$y[6] = \sum_{k=2}^{3} x[k]\, h[6-k].$$



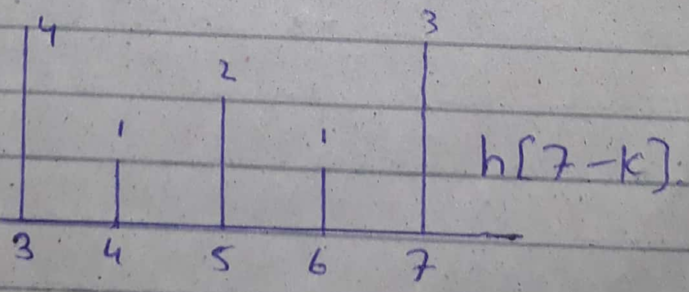
$y[6] = x(2)h(2) + x(3)h(3)$

$= (3)(4) + (-4)(1)$

$= 12 - 4$

$= 8$

$n = 7:$



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$$y[7] = \sum_{k=3}^{3} x(k)\, h[7-k].$$

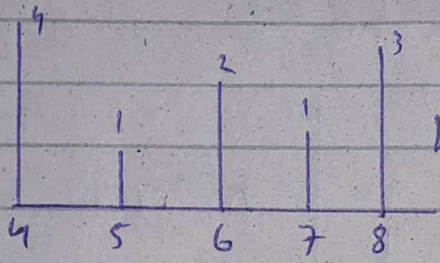
$$= x(3)\, h(3)$$

$$= (-4)(4)$$

$$y[7] = -16$$

$n = 8$:

$$y[8] = x(k)\, h[8-k].$$



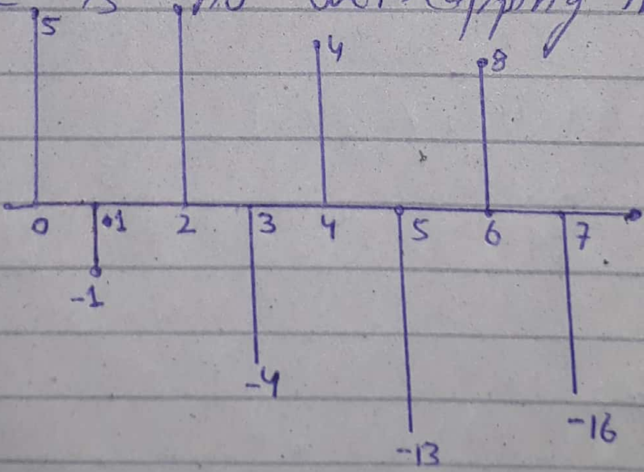
$h[8-k]$

$$y[8] = 0$$

There is no over-lapping in $n=8$

So,

$y[n] =$



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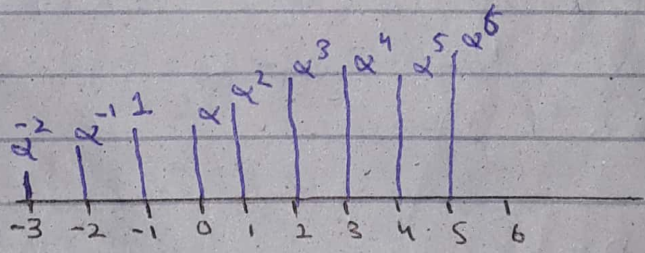
Q # 2

part (b): Compute the Convolution $y(n)$ of the following signal

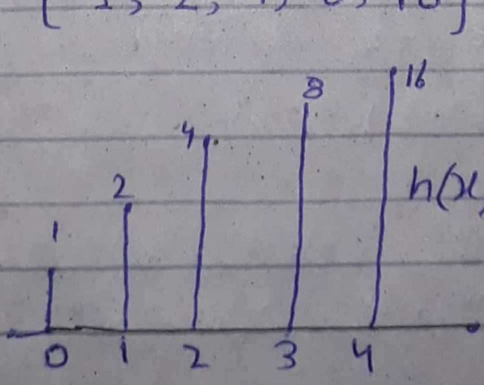
$$x(n) \begin{cases} \alpha^{n+1} & , -3 \leq n \leq 5 \\ 0 & , \text{else} \end{cases}$$

$$h(n) = \begin{cases} 2^n , & 0 \leq n \leq 4 \\ 0 & \text{else} \end{cases}$$

Sol: $x(n) = \{\alpha^{-2}, \alpha^{-1}, 1, \alpha, \alpha^2, \alpha^3, \alpha^4, \alpha^5, \alpha^6\}$



$h(x) = \{1, 2, 4, 8, 16\}$



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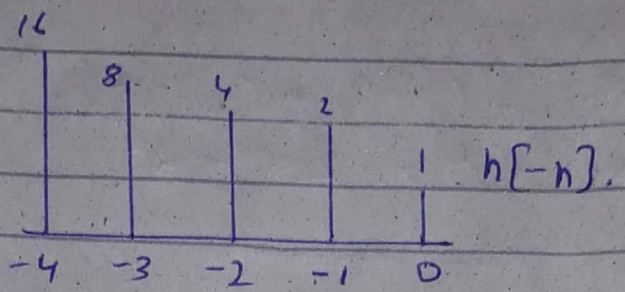

$y[-3] = (\alpha^{-2})(1)$

$y[-3] = \alpha^{-2}$

$y[-2] = (\alpha^{-1})(1) + (\alpha^{-2})(2)$

$y[-2] = \alpha^{-1} + \alpha^{-2}$

$y[-2] = \alpha^{-3}$

$y[-1] = (\alpha^{1})(1) + (2)(4) + (\alpha^{-1})(4) + (\alpha^{-2})(8)$

$y[-1] = (1\times1) + 8\alpha^{-1} + 4\alpha^{-2}$

$y[-1] = 12\alpha^{-3} + 1$

$y[0] = (8)(\alpha^{-2}) + (\alpha^{-1})(4) + (1)(2) + (\alpha)(1)$

$y[0] = 8\alpha^{-2} + 4\alpha^{-1} + 2 + \alpha$

$y[0] = 12\alpha^{-3} + 2 + \alpha.$

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$y[1] = (\alpha^{-1})(8) + (1)(4) + (\alpha)(2) + (\alpha^2)(1)$
$= 8\alpha^{-1} + 4 + 2\alpha + \alpha^2$

$y[2] = (\alpha^{-1})(16) + (1)(8) + (\alpha)(4) + (\alpha^2)(2)$
$+ (\alpha^3)(1)$

$y[2] = 16\alpha^{-1} + 8 + 4\alpha + 2\alpha^2 + \alpha^3$

$y[3] = (1)(16) + (\alpha)(8) + (\alpha^2)(4) + (\alpha^3)(2) + (\alpha^4)(1)$
$= 16 + 8\alpha + 4\alpha^2 + 2\alpha^3 + \alpha^4$

$y[4] = (\alpha)(16) + (\alpha^2)(8) + (\alpha^3)(4) + (\alpha^4)(2) + (\alpha^5)(1)$

$y[4] = 16\alpha + 8\alpha^2 + 4\alpha^3 + 2\alpha^4 + \alpha^5$

$y[5] = (\alpha^2)(16) + (\alpha^3)(8) + (\alpha^4)(4) + (\alpha^5)(2) + (\alpha^6)(1)$

$y[5] = 16\alpha^2 + 8\alpha^3 + 4\alpha^4 + 2\alpha^5 + \alpha^6$

$y[6] = (\alpha^3)(16) + (\alpha^4)(8) + (\alpha^5)(4) + (\alpha^6)(2) +$

$y[6] = 16\alpha^3 + 8\alpha^4 + 4\alpha^5 + 2\alpha^6$

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$y[7] = (16)(\alpha^4) + (\alpha^5)(8) + (\alpha^6)(4)$

$y[7] = 16\alpha^4 + 8\alpha^5 + 4\alpha^6$

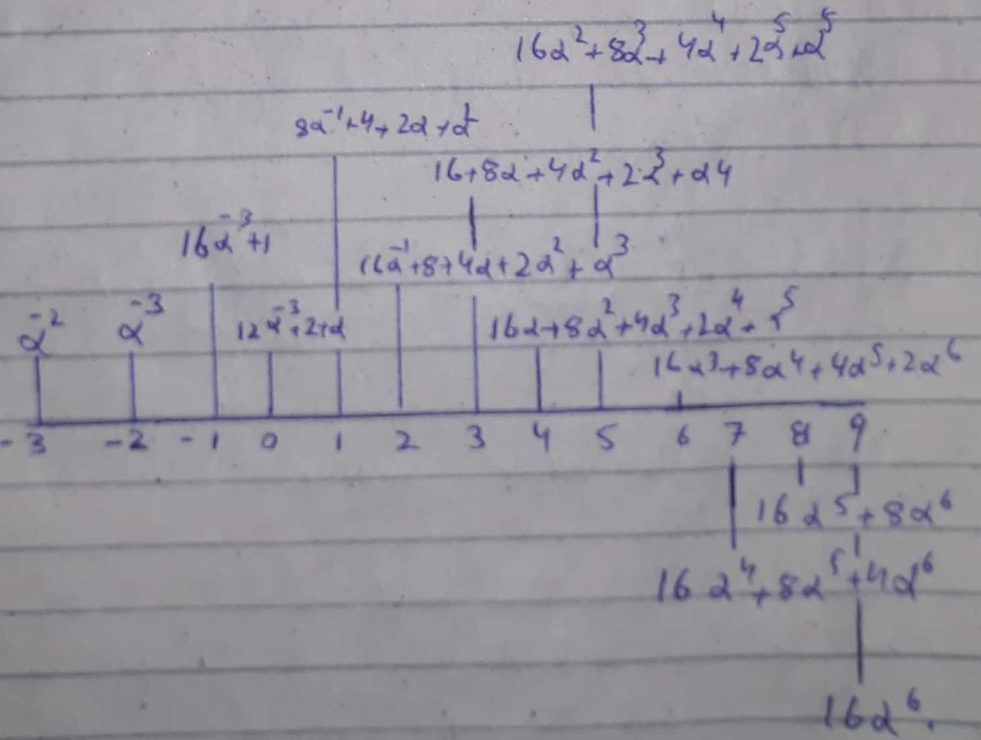
$y[8] = (\alpha^5)(16) + (\alpha^6)(8)$

$y[8] = 16\alpha^5 + 8\alpha^6$

$y[9] = 16\alpha^6$

$y[10] = 0$

There is no overlap in $y[10]$.



Owais Afridi 13686. P#17

Question#

(3) Determine the z-transform of the following Signals and also sketch its Region of Convergence

(i) $x(n) = \begin{cases} \left(\frac{1}{4}\right)^n, & n \geq 0 \\ \left(\frac{1}{3}\right)^{-n}, & n < 0 \end{cases}$

Sol: The z-Tranform is:

$x(n) = a^n u(n)$

$X(z) = \frac{1}{1-\alpha z^{-1}}$ ROC $|z|$

$X(z) = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n z^{-n} + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n z^{-n} - 1$

Using geometric Series

$= \frac{1}{1-\frac{1}{4}z^{-1}} + \frac{1}{1-\frac{1}{3}z} - 1$

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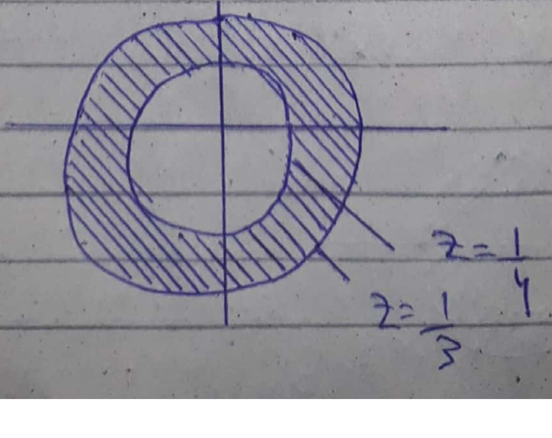
$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - (1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - \left(1-\frac{1}{3}z - \frac{1}{4}z^{-1} + \frac{1}{12}z^{-1}z\right)}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1-\frac{1}{3}z + 1 - \frac{1}{4}z^{-1} - 1 + \frac{1}{3}z + \frac{1}{4}z^{-1} + \frac{1}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{1+1/12}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$

$$= \frac{\frac{13}{12}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z)}$$, So ROC $\frac{1}{4} < |z| < 3$



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Q#03

part(ii) $x(n) = \begin{cases} \left(\frac{1}{2}\right)^n - 3^n , & n \geq 0 \\ 0 , & \text{else} \end{cases}$

Sol: So the z-transform is

$$x(n) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n z^{-n} - \sum_{n=0}^{\infty} 3^n z^{-n}$$

$$\frac{1}{1-\frac{1}{2}z^{-1}} - \frac{1}{1-3z^{-1}}$$

$$\frac{1-3z^{-1} - 1-\frac{1}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}$$

$$\frac{-\frac{5}{2}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-3z^{-1}\right)}$$

$\because -3z^{-1} - \frac{1}{2}z^{-1}$

$\because \frac{-6z^{-1} - z^{-1}}{2}$

$\because -\frac{5z^{-1}}{2}$

Hence the ROC is $|z| > 3$.